v $\mathbb{Z}_4$ je $\underline{\underline{2\cdot 2 = 0}}$

$\mathbb{K} = \{0\} = \{1\}$ $\qquad$ $\mathbb{C} = \mathbb{R} + i\mathbb{R}$

$\mathbb{K} = \mathbb{Z}_2 = \{0, 1\}$

$\mathbb{Z}_k \supset \{i;\, i+k\}$ $\qquad$ $j \neq 0$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\mathbb{H} = \mathbb{C} + j\,\mathbb{C}$ $\qquad$ $i\cdot j = -j\cdot i$

$$\left.\begin{aligned} a\cdot(c+0) &= a\cdot c \\ &= a\cdot c + \underline{\underline{a\cdot 0}} \end{aligned}\right\} \Rightarrow a\cdot 0 = 0$$

$$0 = c\cdot 0 = c\cdot(1+(-1)) = c + \underset{-c}{\underset{\uparrow}{c(-1)}}$$

$$\Rightarrow (-1)\cdot c = -c$$

$$\left.\begin{aligned} (x^2+x)(0) &= 0+0 = 0 \\ (x^2+x)(1) &= 1+1 = 0 \end{aligned}\right] \text{ v } \mathbb{Z}_2$$

$$(a_0 + a_1 x)\cdot(b_0 + b_1 x) = a_0 b_0 + (a_1 b_0 + a_0 b_1)\cdot x$$
$$+ \underbrace{a_1 b_1}\, x^2$$

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$$f(c) = 0 \iff f(x) = (x - c)\cdot g(x)$$

---

1) $b = a\cdot d_1,\ c = b\cdot d_2 = (a\cdot d_1)\cdot d_2 = a\cdot(d_1\cdot d_2)$

2) $b = a\cdot d_1,\ c = a\cdot d_2$

$$\alpha\cdot b + \beta\cdot c = \alpha\cdot a\cdot d_1 + \beta\cdot a\cdot d_2$$
$$= a\cdot(\alpha d_1 + \beta\cdot d_2)$$

4) $e\cdot e^{-1} = 1 \qquad e\cdot(e^{-1}\cdot a) = a$

$2 \cdot (x^2+1)$ nad $\mathbb{Q}$ nerozložitý

$2x^2+2$

nad $\mathbb{Z}$

nad $\mathbb{Q}$

$$3x^2-3 = (3x-3)\cdot(x+1)$$
$$= (x-1)(3x+3)$$
$$= (qx-q)\cdot\left(\frac{3}{q}x+\frac{3}{q}\right)$$

$q \in \mathbb{Q} \quad q \neq 0$

$$\underline{\underline{x = x^{1/2}}} \cdot x^{1/2} = x^{1/4} \cdot x^{1/4} \cdot x^{1/4} \cdot x^{1/4}$$

$\deg f < \deg g$ : $a = 1,\ q = 0,\ r = f$

$\deg f \geq \deg g$ ?

příkl $\mathbb{K} = \mathbb{Z}$ $\quad f = (x+1)^2 \quad g = 2x+2$

$$2 \cdot f = (x+1) \cdot g$$

Důkaz: indukcí přes stupeň $f$.

$f = a_0 + \dots + a_n x^n$, $a_n \neq 0$, $g = b_0 + \dots + b_m x^m$, $b_m \neq 0$, $n \geq m$

$b_m \cdot f - a_n \cdot x^{n-m} \cdot g = \begin{cases} 0 & \leftarrow \text{konec důkazu} \\ \text{něco s menším deg} & \end{cases}$

$\Rightarrow$ indukce

$\Rightarrow \exists\, a', q', r' : \underbrace{a'(b_m \cdot f - a_n x^{n-m} \cdot g)}_{\text{správně vlastní}} = q' \cdot g + r'$

$$(a' \cdot b_m) \cdot f = (a_n \cdot x^{n-m} + q') \cdot g + r'$$

$$\overset{\shortparallel}{1}$$

$(f-g)(x)$ má zbytky $\mathbb{K}$

dělení se zbytkem:

$$f = q_1 g + r_1$$

$$g = q_2 r_1 + r_2$$

$$r_1 = q_3 r_2 + r_3$$

$$\vdots$$

$$r_{p-1} = q_{p+1} r_p + 0$$

$$r_p = h$$

$$h \mid f \wedge h \mid g$$

$$\Downarrow$$

$$h \mid r_1$$

$$h = r_p = r_{p-2} - q\, r_{p-1}$$

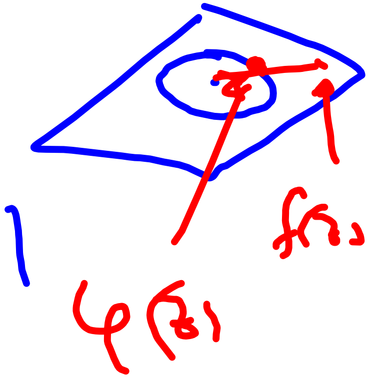
$$\vdots$$

$$= Af + Bg$$

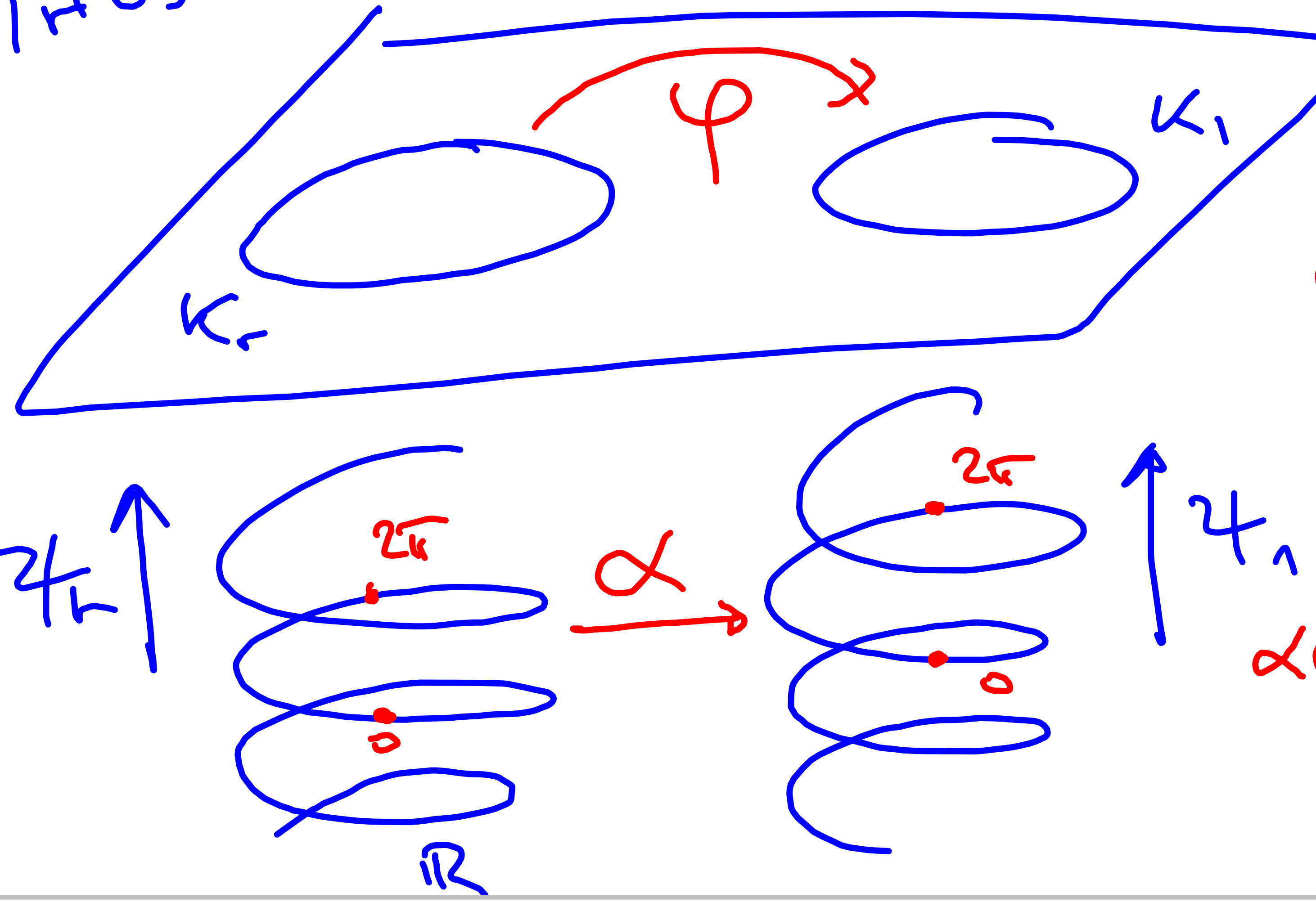
Důkaz

$f(z) \neq 0 \quad \forall z \in \mathbb{C}$

$\varphi : \mathbb{C} \to \mathbb{C} \qquad \varphi(z) = \dfrac{f(z)}{|f(z)|}$




$\psi_r(t) = r e^{it}$




$\varphi \circ \psi_r$

$\parallel$

$\psi_1 \circ \alpha$

$\alpha(0) \in \langle 0, 2\pi)$

$$\frac{1}{2\pi}\left(\alpha_r(2\pi) - \alpha_r(0)\right) = n_r \in \mathbb{Z}$$

$\Rightarrow$ konstanta nezávislá na $r$

trocha analýzy $\Rightarrow$ $n_r = 0$

$\Rightarrow$ stupeň $f = 0$.

$\blacksquare$