$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{2} & 5 \\ 0 & 1 & 0 & \frac{3}{4} & 0 & 2 \\ 0 & 0 & 1 & -4 & -\frac{2}{8} & -2 \end{array}\right)$$

$x_1 \quad x_2 \quad x_3 \quad a \quad b$

$$x_1 = 5 - \frac{2}{3}a + \frac{1}{2}b$$

$$x_2 = 2 - \frac{3}{4}a$$

$$x_3 = -2 + 4a + \frac{2}{8}b$$

$$\left(\begin{array}{cccc|c} 2 & 3 & 0 & -1 & 1 \\ 3 & 2 & 4 & -2 & 0 \\ 1 & -1 & 4 & -1 & 2 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 4 & -1 & 2 \\ 3 & 2 & 4 & -2 & 0 \\ 2 & 3 & 0 & -1 & 1 \end{array}\right) \begin{array}{l} \\ \vert -3 \\ \vert -2 \end{array} \sim$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 4 & -1 & 2 \\ 0 & 5 & -8 & 1 & -6 \\ 0 & 5 & -8 & 1 & -3 \end{array}\right)$$

$$(0 \quad 0 \quad 0 \quad 0 \mid 3)$$

$Ax = 0 \quad m < n$

má m ved. prvků

$\exists\, j$ tak, že $s_j(A)$ neobsahuje

vedoucí prvek $\qquad A' x' = -s_j(A)$

$A \sim RST \qquad x' = -s_j(A)$

$J = \{ j_1 < j_2 < \dots < j_k \}$ indexy ved.

$J' = \{ l_1 < l_2 < \dots < l_n \}$ ne

$x_{l_i} = 0 \quad l_i \neq j \quad x_j = 1$

$Ax = b \quad m < n$

$\exists x_0 \quad Ax_0 = b \Rightarrow$

$\exists x_1 \neq x_0 \quad Ax_1 = b$

dle a) $\exists z \neq 0 \quad Az = 0$

$$\frac{Ax_0 = b}{A(\underbrace{x_0 + z}_{x_1}) = b}$$

$(I) \Rightarrow (III)$ $\quad S \neq \emptyset$

$x, y \in S,\ a, b \in K \overset{?}{\Rightarrow} ax + by \in S$

$ax \in S,\ by \in S$

$ax + by \in S$

$(II) \Rightarrow (III)$ $\quad n = 0 \quad$ $0 \in S$

$n \Rightarrow n+1$

$x_1, \ldots, x_{n+1} \in S,\ a_1, \ldots, a_{n+1} \in K$

$(a_1 x_1 + \ldots + a_n x_n) + a_{n+1} x_{n+1} \in S$

$\in S \qquad\qquad \in S$

$X \quad [X] = \{ \sum a_i x_i \}$

1) $[X]$ je VP

$u, v \in [X] \stackrel{?}{\Rightarrow} u + v \in [X]$

$u = \sum_{i=1}^{m} a_i x_i \qquad v = \sum_{j=m+1}^{m} a_j x_j$

$x_i \in X \qquad x_j \in X$

$u + v = \sum_{i=1}^{m} a_i x_i \in [X]$

$c \cdot u = \sum_{i=1}^{m} (c a_i) x_i \in [X]$

$X \subseteq S$, S je VP $\stackrel{?}{\Rightarrow}$ $[X] \subseteq S$

$u \in [X]$

$$u = \sum_{i=1}^{n} a_i x_i \in S$$

$x_i \in X \subseteq S$

$X \subseteq Y \Rightarrow [X] \subseteq [Y]$

$u \in [X]$, $u = \sum_{i=1}^{n} a_i x_i \in [Y]$

$x_i \in X \subseteq Y$

$$[X] = [[X]]$$

$$\Leftarrow$$

$$X \subseteq [X]$$

$$\Downarrow$$

$$[X] \subseteq [[X]]$$

$[X]$ je VP $\Rightarrow$

$$[X] = [[X]]$$

$$v \in [X], \quad v = \sum_{i=1}^{m} a_i x_i, \quad x_i \in X$$

$$? \quad [X] = [X \cup \{v\}]$$

$$\subseteq$$

$$u \in [X \cup \{v\}]$$

$$u = \sum_{i=1}^{m} b_i y_i + b\, v =$$

$$y_i \in X$$

$$= \sum_{i=1}^{m} b_i \cdot \underbrace{y_i}_{\in X} + b \cdot \underbrace{\sum_{i=1}^{m} a_i x_i}_{\in X} \in [X]$$

$$[X] = [X \cup \{v\}] \Rightarrow$$

$$v \in [X]$$

$$v \in X \cup \{v\} \subseteq [X \cup \{v\}] = [X]$$

S, T VP $\Rightarrow S \cap T$ je VP

$o \in S,\ o \in T \Rightarrow o \in S \cap T \neq \emptyset$

$u \in S \cap T,\ v \in S \cap T$

$\overset{?}{\Rightarrow} u + v \in S \cap T$

$u \in S,\ v \in S \Rightarrow u + v \in S$

$u \in T,\ v \in T \Rightarrow u + v \in T$

$\Rightarrow u + v \in S \cap T$

$u \in S \cap T \overset{?}{\Rightarrow} cu \in S \cap T \quad \forall c \in K$

$\underline{cu \in S}$ $\quad \underline{cu \in T} \Rightarrow cu \in S \cap T$

ST, VP

$$0 \in S+T = [S \cup T] \,!!$$

$$u, v \in S+T \overset{?}{\Rightarrow} u+v \in S+T$$

$$u = s_1 + t_1, \quad v = s_2 + t_2$$

$$s_1, s_2 \in S, \quad t_1, t_2 \in T$$

$$u + v = s_1 + t_1 + s_2 + t_2 =$$

$$= \underbrace{(s_1 + s_2)}_{\in S} + \underbrace{(t_1 + t_2)}_{\in T} \in S+T$$

$$cu = \underbrace{cs_1}_{\in S} + \underbrace{ct_1}_{\in T} \in S+T$$

$$S + T \overset{?}{\supseteq} S \cup T$$

$$u \in S, \quad u = \underset{\in S}{u} + \underset{\in T}{0}$$

$$v \in T, \quad \Rightarrow v \in S + T$$

$$\Rightarrow \quad S + T \supseteq [S \cup T]$$

$$\subseteq \quad x \in S + T$$

$$x = \underset{\in S}{a_1} + \underset{\in T}{b_1} = 1 \cdot a_1 + 1 \cdot b_1 \Rightarrow x \in [S \cup T]$$

(i) $\Rightarrow$ (ii)

$$\mathbb{R} = x_1 + y_1 = x_2 + y_2$$

$$x_1, x_2 \in S, \quad y_1, y_2 \in T$$

$$\overset{?}{\Longrightarrow} \quad x_1 = x_2, \quad y_1 = y_2$$

$$S \ni \underbrace{x_1 - x_2}_{= 0} = \underbrace{y_2 - y_1}_{= 0} \in T$$

(ii) $\Rightarrow$ (i)

$$\mathbb{R} \in S \cap T \overset{?}{\Longrightarrow} \mathbb{R} = 0$$

$$\mathbb{R} = \underset{\in S}{\mathbb{R}} + \underset{\in T}{0} = \underset{\in S}{0} + \underset{\in T}{\mathbb{R}}$$

$$u = 0 \quad \text{ý } L2$$

$$c\,0 = 0$$

$$u \neq 0$$

$$c\,u = 0 \qquad c \neq 0$$

$$\left(\frac{1}{c}\,c\right) u = 0$$

$$1$$

$$u = 0$$

$u = k v$

$1 \cdot u + (-k) v = 0$

$\neq 0$

u, v LZ $\quad \exists c, d \in K$

$\quad \quad \quad$ aspoň $\neq 0$

$c u + d v = 0 \quad c \neq 0$

$c u = -d v$

$u = \left(-\frac{d}{c}\right) v$

$$u_1, \dots, (0), \dots u_n$$

$$0, \dots, 0, 1, \dots, 0$$

$$0 a_1 + \dots + (1) 0 + \quad 0 a_n = 0$$

$$u_1, \dots, u_i, \dots, u_i, \dots u_n$$

$$\| \; u_i$$

$$0 \dots 0, 1, 0, -1, \quad 0$$

$$0 \quad + u_i \quad - u_i \quad + 0 = 0$$

$u_1, \ldots, u_m$ jsou LZ (I) ⇒ (II) (II')

$\exists c_1, \ldots, c_m \in K$ $\exists c_i \neq 0$

$$\sum_{i=1}^{m} c_i u_i = 0$$

index $k = \max\{i : c_i \neq 0\}$

$k+1$ $c_{k+1} = 0, \ldots, c_m = 0$

$$\sum_{i=1}^{k} c_i u_i = 0 \quad \Big| \; \frac{1}{c_k}$$

$$u_k = \sum_{i=1}^{k-1} \left(-\frac{c_i}{c_k}\right) u_i$$

(ii) ⇒ (iii) ~~good~~ (ii)' ⇒ (iii)

(iii) ⇒ (i) $u_k = \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i u_i \quad / (-u_k)$

$$0 = \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i u_i + (-1) u_k$$

$$\Rightarrow u_1 \ldots u_n \text{ LZ}$$

$u_1, \dots, u_n$ LN

$x \in [u_1, \dots, u_n]$

$x = \sum_{i=1}^{n} c_i u_i = \sum_{i=1}^{n} d_i u_i$

?

$\Rightarrow$ $c_i = d_i$ $\forall i$

$\sum_{i=1}^{n} (c_i - d_i) u_i = 0$

$c_i - d_i = 0$

je-li $\overset{?}{\Longrightarrow}$ LN

$$\sum_{i=1}^{m} c_i u_i = 0 \Rightarrow c_1 = c_2 \quad = c_m = 0$$

$$\sum_{i=1}^{m} 0 \cdot u_i = 0$$

$u_1 \; \dots \; u_n \; v_1 \; \dots \; v_m$

LN

$W_1 \qquad W_1 \in [u_1 \dots u_n]$ NIC

$W_1 \notin \qquad i_1 = 1$

$W_2 \in [u_1 \dots u_n \, v_1]$

$i_1 \dots \quad i_k \qquad j \neq i_1 \dots i_k$

$W_{i_1} \qquad W_{i_k} \qquad i_p < j < i_{p+1}$

$W_j \in [u_1 \dots u_n \, v_{i_1}, \; v_{i_p}]$

$$\left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 1 & 1 & 1 & 0 & 5 & 1 \\ -1 & 0 & -3 & 1 & -2 & 1 \\ -1 & 1 & -5 & 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & -2 & 2 & 4 & 2 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$y \in [x_1, \ldots, x_4]$$

$$\mathbb{Z} \notin [x_1, \ldots, x_4]$$