$x \neq 0, \ x \in \mathbb{R}^n \Longrightarrow$

$$\langle x, x \rangle = \sum_{i=1}^{n} x_i x_i > 0$$

$$\| $$

$$\sum x_i^2$$

$$\geq 0$$

$$\exists j \quad x_j^2 > 0$$

V nad $\mathbb{R}$ $\dim V = n$

$u_1, \dots, u_n$ báze

$$\langle u_i, u_j \rangle = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$u = \sum c_i u_i$$

$$v = \sum d_i u_i$$

$$\langle u, v \rangle = \langle \sum c_i u_i, v \rangle =$$

$$= \sum c_i \langle u_i, v \rangle = \sum c_i \langle u_i, \sum_j d_j u_j \rangle =$$

$$= \sum_i \sum_j c_i d_j \langle u_i, u_j \rangle$$

$$= \sum_i c_i d_i \underbrace{\langle u_i, v_i \rangle}_{=1}^{\delta_{ij}}$$

$$= \sum_{i=1}^{n} c_i d_i$$

$$\begin{pmatrix} \langle u_1, u_1\rangle & \langle u_1, u_2\rangle & \dots & \langle u_1, u_k\rangle \\ \langle u_2, u_1\rangle & \langle u_2, u_2\rangle & \dots & \end{pmatrix}$$

$$\langle u_i, u_j\rangle = \langle u_j, u_i\rangle$$

$$c \cdot G \cdot c^T = (c_1, \dots, c_k) \cdot$$

$$G \cdot \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} = ((c_1, \dots, c_k) \cdot$$

$$\begin{pmatrix} \langle u_1, u_1\rangle c_1 + \langle u_1, u_2\rangle c_2 + \dots \\ \\ \dots \end{pmatrix}$$

$$= (c_1, \dots, c_k) \cdot \begin{pmatrix} \langle u_1, \sum_{j=1}^{k} c_j u_j\rangle \\ \langle u_2, \sum_{j=1}^{k} c_j u_j\rangle \end{pmatrix}$$

$$= \langle \sum_{i=1}^{k} c_i u_i, \sum_{j=1}^{k} c_j u_j\rangle \geq 0$$

$$\| \quad c\, G\, c^T$$

$$> 0 \iff \sum_{i=1}^{k} c_i u_i \neq 0 \quad \text{pro } c \neq 0$$

$$\iff u_1, \dots, u_k \ \underline{LN}$$

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$$u_j = \frac{\sum_{i \neq j} c_i u_i}{\quad}$$

$$0 = \sum_{i=1}^{m} c_i \begin{pmatrix} (u_1, u_i) \\ (u_m, u_i) \end{pmatrix}, \quad c_j = -1$$

$$0 = \begin{pmatrix} (u_1, \sum c_i u_i) \\ \vdots \\ (u_m, \sum c_i u_i) \end{pmatrix}$$

$$0 = (c_1, \ldots, c_m) \begin{pmatrix} \quad \end{pmatrix} = cGc^T$$

$$\langle u + \lambda v, u + \lambda v \rangle \geq 0$$

$$\overset{c}{\langle u, u \rangle} + \overset{b}{2\lambda \langle u, v \rangle} + \lambda^2 \overset{a}{\langle v, v \rangle}$$

$$\leq 0$$

$$D = 4 \langle u, v \rangle^2 - 4 \langle v, v \rangle \langle u, u \rangle$$

$$\langle u, v \rangle^2 \leq \langle u, u \rangle \langle v, v \rangle$$

$$\left| \sum x_i \cdot y_i \right| \leq \sqrt{\sum x_i^2} \cdot \sqrt{\sum y_i^2}$$

$$\|x+y\| \leq \|x\| + \|y\|$$

$$\langle x+y, x+y\rangle =$$

$$\langle x, x\rangle + 2\langle x, y\rangle + \langle y, y\rangle$$

$$\left(\langle x, y\rangle \leq \|x\| \cdot \|y\|\right)$$

$$\|x\|^2 + 2\|x\|\|y\| + \|y\|^2$$

$$\langle x \mp y,\ x \mp y \rangle = \langle x, x\rangle \mp 2\langle x, y\rangle + \langle y, y\rangle =$$

$$= \|x\|^2 + \|y\|^2 \mp 2\,\|x\|\,\|y\|\,\underbrace{\cos(x,y)}_{=0}$$

$$\|x+y\|^2 + \|x-y\|^2 =$$

$$\|$$

$$\|x\|^2 + \|y\|^2 + 2\,\|x\|\,\|y\| \cos\alpha$$

$$\|x\|^2 + \|y\|^2 - 2\,\|x\|\,\|y\| \cos\alpha$$

$$= 2\left(\|x\|^2 + \|y\|^2\right)$$

$$\langle x+y,\, x-y\rangle = \langle x, x\rangle$$

$$\|$$

$$+ \cancel{\langle x, y\rangle} + \cancel{\langle y, x\rangle} - \langle y, y\rangle$$

$$= 0.$$

$u_1 \dots u_k \qquad e_1, \dots, e_k$

$\langle u_1 \rangle \qquad e_1 = u_1 \quad \langle e_1 \rangle$

$\langle u_1, u_2 \rangle \qquad ?\ e_2 \to \langle e_1, e_2 \rangle =$

$= \langle u_1, u_2 \rangle$

$e_2 = h_1 e_1 + u_2 \quad | \; e_1 \quad e_2^{\perp}$

$\langle u_1 u_2 \rangle = \langle u_1, e_2 \rangle = \langle e_1, e_2 \rangle$

$\langle e_1, e_2 \rangle = 0 = h_1 \langle e_1, e_1 \rangle + \langle u_2, e_1 \rangle$

$$h_1 = - \frac{\langle u_2, e_1 \rangle}{\langle e_1, e_1 \rangle}$$

$\langle u_1 \dots u_{k-1} \rangle = \langle e_1, \dots, e_{k-1} \rangle$

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$$\boxed{e_k = \sum_{i=1}^{k-1} h_i e_i + u_k} \quad | \; e_i$$

$\langle u_1 \dots u_k \rangle \overset{?}{=} \langle e_1 \dots, e_k \rangle$

$\supseteq$

$u_1 \dots u_{k-1} \subseteq \langle e_1 \dots e_{k-1} \rangle \subseteq$

$\subseteq \langle e_1 \dots, e_k \rangle$

$u_k = e_k - \sum h_i e_i \ \in$

$\langle e_k, e_i \rangle = h_i \langle e_i, e_i \rangle + \langle u_k, e_i \rangle$

$\overset{\shortparallel}{0}$

$$h_i = - \frac{\langle u_k, e_i \rangle}{\langle e_i, e_i \rangle}$$