

$$u_1, \dots, u_r \quad LN$$

$$e_1, \dots, e_r \quad e_i \perp e_j$$

$$q_r = \sum_{i=1}^{r-1} h_{ir} e_i + u_r \quad \|e_i\|=1$$

$$= \frac{\sum_{i=1}^{r-1} h_{ir} e_i + u_r}{\sqrt{\sum_{i=1}^{r-1} h_{ir}^2 + 1}}$$

$$\|q_r\|=1 \quad h_{ir} = \langle e_i, u_r \rangle$$

$$u_r \in \mathbb{R}^m \quad u_r = v_r e_r + \sum_{i=1}^{r-1} \langle e_i, u_r \rangle e_i$$

$$\begin{pmatrix} u_1 & \dots & u_r \end{pmatrix} = \begin{pmatrix} e_1 & \dots & e_r \end{pmatrix} \begin{pmatrix} v_1 \langle e_1, u_1 \rangle & & \\ & \ddots & \\ & & v_r \langle e_r, u_r \rangle \end{pmatrix}$$

$$\begin{matrix} U & = & QG & \cdot & T \\ m \times r & & m \times r & & r \times r \end{matrix}$$

$$e_1 = \frac{(0, 1, 2, 1)}{\| (0, 1, 2, 1) \|} = \frac{1}{\sqrt{6}} (0, 1, 2, 1)$$

$$\bar{e}_2 = -\frac{1}{\sqrt{6}} (e_1 \cdot u_2) e_1 + u_2$$

$$\begin{aligned} & (-1, 1, 1, 1) \cdot \frac{(0, 1, 2, 1)}{\sqrt{6}} \\ &= (0 + 1 + 2 + 1) \frac{1}{\sqrt{6}} = \sqrt{6} \end{aligned}$$

$$\begin{aligned} \bar{e}_2 &= -e_1 + u_2 = \\ &= \left(-1, 1 - \frac{1}{\sqrt{6}}, 1 - \frac{2}{\sqrt{6}}, 1 - \frac{1}{\sqrt{6}} \right) \\ &= \frac{1}{\sqrt{6}} (-\sqrt{6}, \sqrt{6} - 1, \sqrt{6} - 2, \sqrt{6} - 1) \end{aligned}$$

TAKE NOTE!

$$\begin{aligned} & \sqrt{6} + \sqrt{6} - 2\sqrt{6} + 1 + \sqrt{6} + 4 - 4\sqrt{6} \\ & + \sqrt{6} - 2\sqrt{6} + 1 = 30 - 8\sqrt{6} \end{aligned}$$

$$e_1 = (0, 1, 2, 1)$$

$$e_2 = n_1 e_1 + u_2$$

$$n_1 = - \frac{\langle e_1, u_2 \rangle}{\langle e_1, e_1 \rangle} = - \frac{4}{6}$$

$$(0, 1, 2, 1) (-1, 1, 1, 1) = \\ = (0 + 1 + 2 + 1) = 4$$

$$e_2 = - \frac{4}{6} (0, 1, 2, 1) + \\ (-1, 1, 1, 1) =$$

$$= (-1, 1 - \frac{4}{6}, 1 - \frac{8}{6}, 1 - \frac{4}{6})$$

$$e_3 = n_1 e_1 + n_2 e_2 + u_3$$

$$n_1 = - \frac{\langle e_1, u_3 \rangle}{\langle e_1, e_1 \rangle} = - \frac{2}{6}$$

$$e_2 = (-1, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$$

$$(0, 1, 2, 1) (1, 0, 1, 0) = \\ = (0 + 0 + 2 + 0) = 2$$

$$n_2 = - \frac{\langle e_2, u_3 \rangle}{\langle e_2, e_2 \rangle} = 1$$

$$= - \frac{(-1 + 0 - \frac{1}{3} + 0)}{1 + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \frac{4}{3} = 1$$

$$e_3 = - \frac{1}{3} (0, 1, 2, 1) +$$

$$+ 1 (-1, \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}) + u_3$$

$$= (-1, 0, -1, 0) + u_3 = \underline{\underline{0}}$$

$$V = W \oplus W^\perp \quad W \cap W^\perp = \{0\}$$

$$W \oplus W^\perp \subseteq V$$

$$\dim V = n$$

$$\dim W = k$$

$$\dim W^\perp = n - k$$

$$\begin{matrix} e_1, \dots, e_k \\ b_1, \dots, b_{n-k} \end{matrix}$$

~~$e_1, \dots, e_k, b_1, \dots, b_{n-k}, u_1, \dots, u_p$~~
basis $W \oplus W^\perp$

$$e_1, e_2, b_1, \dots, b_{n-k}, \underline{h_1}, \underline{h_p}$$

$$h_1 \perp e_i, h_1 \perp b_j$$

$$h_1 \in W^\perp$$

$$\underline{h_1 \perp h_1} \Rightarrow \underline{h_1 = 0}$$

$$x \in (W + S)^\perp \iff$$

$$x \perp W + S \iff$$

$$x \perp W \wedge x \perp S \iff$$

$$x \in W^\perp \wedge x \in S^\perp \iff$$

$$x \in W^\perp \cap S^\perp$$