$j_1 \quad \dots \quad j_k$

$x_{j_1}, \quad , x_{j_k} \quad$ LN ?

$c_1 x_{j_1} + \quad + c_2 x_{j_k} = 0$

$X \sim Y$

$X' \sim Y'$

$\underline{j_1, \dots, j_k}$

$$X' \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} = 0$$

$$\left(\begin{array}{ccc} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ \hline & 0 & \end{array} \right) \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} = 0$$

$\underline{c_1 = c_2 = \quad = c_k = 0}$

Název: III 14-10:58 (1 z 16)

$x_j \notin \{x_{j_1}, \dots, x_{j_k}\}$

$$x_j = \sum_{i=1}^{k} c_i x_{j_i}, \quad j_\ell < j < j_{\ell+1}$$

$X'' \sim Y''$

$j_1 \dots j_\ell \; j$

$$m_{j} = \begin{pmatrix} c_1 \\ \vdots \\ c_k \\ 0 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \dots + c_\ell \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$[x_1, \ldots, x_n] = [x_{j_1}, \ldots, x_{j_k}]$$

$$\Rightarrow$$

$$x_j \in [x_{j_1}, \ldots, x_{j_k}]$$

$$[\{x_1, \ldots, x_n\}] \subseteq [[x_{j_1}, \ldots, x_{j_k}]]$$

$$\shortparallel$$

$$[x_{j_1}, \ldots, x_{j_k}]$$

$$\begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 2 & 0 & 2 & 1 & 3 \\ 1 & 1 & 3 & 2 & 4 \\ 3 & 0 & 2 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & -2 & -4 & 3 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 0 & 2 & 4 & -3 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_3 = 1 \cdot x_1 + 2x_2$$

$\boxed{x_1, x_2, x_4}$ jsa LN

$x_3, x_5 \in [\underline{x_1, x_2}, x_4]$

$m \overset{?}{\leq} n$

SPOREM

$m < n$

$$u_j = \sum_{i=1}^{m} c_{ij} v_i^T$$

$\exists x$

$\not\exists x \quad C^T x = 0$

$x \neq 0$

$m \times n$

$m < n$

$$(c_{1j} \; \dots \; c_{mj}) \begin{pmatrix} v_1^T \\ \vdots \\ v_m^T \end{pmatrix} = u_j$$

$$x^T C \begin{pmatrix} v_1^T \\ \vdots \\ v_m^T \end{pmatrix} = 0$$

$$\underset{n}{\overset{m}{C}} \begin{pmatrix} v_1^T \\ \vdots \\ v_m^T \end{pmatrix} = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}$$

$\neq 0$

$$x^T \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix} = 0$$

$(I) \Rightarrow (II)$ $[X] = V$

$y_1, \ldots, y_n, \ldots$

$y_j \in [X]$ $|X| = n$

$y_{n+1}, \ldots, y_1 \in [X]$

$n + 1 \leq n$ SPOR

$(II) \Rightarrow (I)$

$\ulcorner$ $\mathbb{V}$ nelze vyj konci za n

$x_1 \in \mathbb{V}$ $\qquad \{x_1\} \subsetneqq \mathbb{V}$

$x_2 \in \mathbb{V} - \{x_1\}$

$\qquad \{x_1, x_2\} \subsetneqq \mathbb{V}$

$x_3 \in \mathbb{V} - \{x_1, x_2\}$

$\vdots$

$x_n \in \mathbb{V} - \{x_1, \dots, x_{n-1}\}$

$\{x_1 \dots x_n, \dots\}$ $\underline{\underline{LU}}$

$u_1, \ldots, u_k \quad V = [v_1, \ldots, v_n]$

$\exists\, u_{k+1}, \ldots, u_n$

$(u_1, \ldots, u_n)$ báze

$[u_1, \ldots, u_k, v_1, \ldots, v_n] = V$

$[u_1, \ldots, u_k, v_{i_1}, \ldots, v_{i_l}] = V$

$\underbrace{\hspace{8cm}}_{\text{LN}}$

a ok

b) $[v_1, \ldots, v_n] = V$

$$\Downarrow$$

$[v_{i_1}, \ldots, v_{i_\ell}] = V$

a) $V = [v_1, \dots, v_m] =$

$= [v_{i_1}, \dots, v_{i_k}]$

LN

b) STEINITZ

$u_1, \dots, u_n$ $v_1, \dots, v_m$

báze

$u_j \in [v_1, \dots, v_m] = V$

$n \leq m \leq n$ $m = n$

$u_1, \dots, u_n$ báze

$x \in V = [u_1, \dots, u_n]$

$$x = \sum_{i=1}^{n} c_i u_i = \sum_{i=1}^{n} d_i u_i$$

$$0 = \sum_{i=1}^{n} (c_i - d_i) u_i$$

$$c_i - d_i = 0$$

$$W = [u_1, \dots, u_m]$$

$$0 = c_1 u_1 + \quad + c_m u_m$$

$$0 = 0 u_1 + \quad + 0 u_m$$

$$(x)_2 = (y)_2 \Rightarrow x = y$$

$$x = 2\cdot(x)_2 = 2(y)_2 = y$$

$$\forall c \in \mathbb{R}^n \ \exists x \ (x)_2 = c$$

$$2c = x$$

$$(x)_2 = c$$

$$2\cdot(ax+by)_2 = ax+by$$

$$2\big(a(x)_2 + b(y)_2\big) = a\,2(x)_2 + b\,2(y)_2$$

S, T P.n.

$\dim(S+T) = \dim(S) + \dim T - \dim(S \cap T)$

$S \cap T$ P.n.

$u_1, \dots, u_l$ báze $S \cap T$

$u_1, \quad , u_l, u_{l+1}, \quad , u_m$ $\quad S$

$u_1, \quad , u_l, v_{l+1}, \quad , v_n$ $\quad T$

$u_1 \dots u_l\ u_{l+1} \ . \ u_m\ v_{l+1}, \ v_n$ $\quad S+T$

$2 \subset U$

$$c_1 u_1 + \dots + c_k u_k + c_{k+1} u_{k+1} + \dots + c_m u_m$$

$$+ d_{k+1} v_{k+1} + \dots + d_m v_m = 0$$

$$\sum_{i=1}^{m} c_i u_i = -\sum_{j=k+1}^{m} d_j v_j \in T$$

$$\cap \; S \cap T$$

$$c_{k+1} = \dots = c_m = 0$$

$$\sum_{i=1}^{k} \underbrace{c_i}_{0} u_i + \sum_{j=k+1}^{m} \underbrace{d_j}_{0} v_j = 0$$