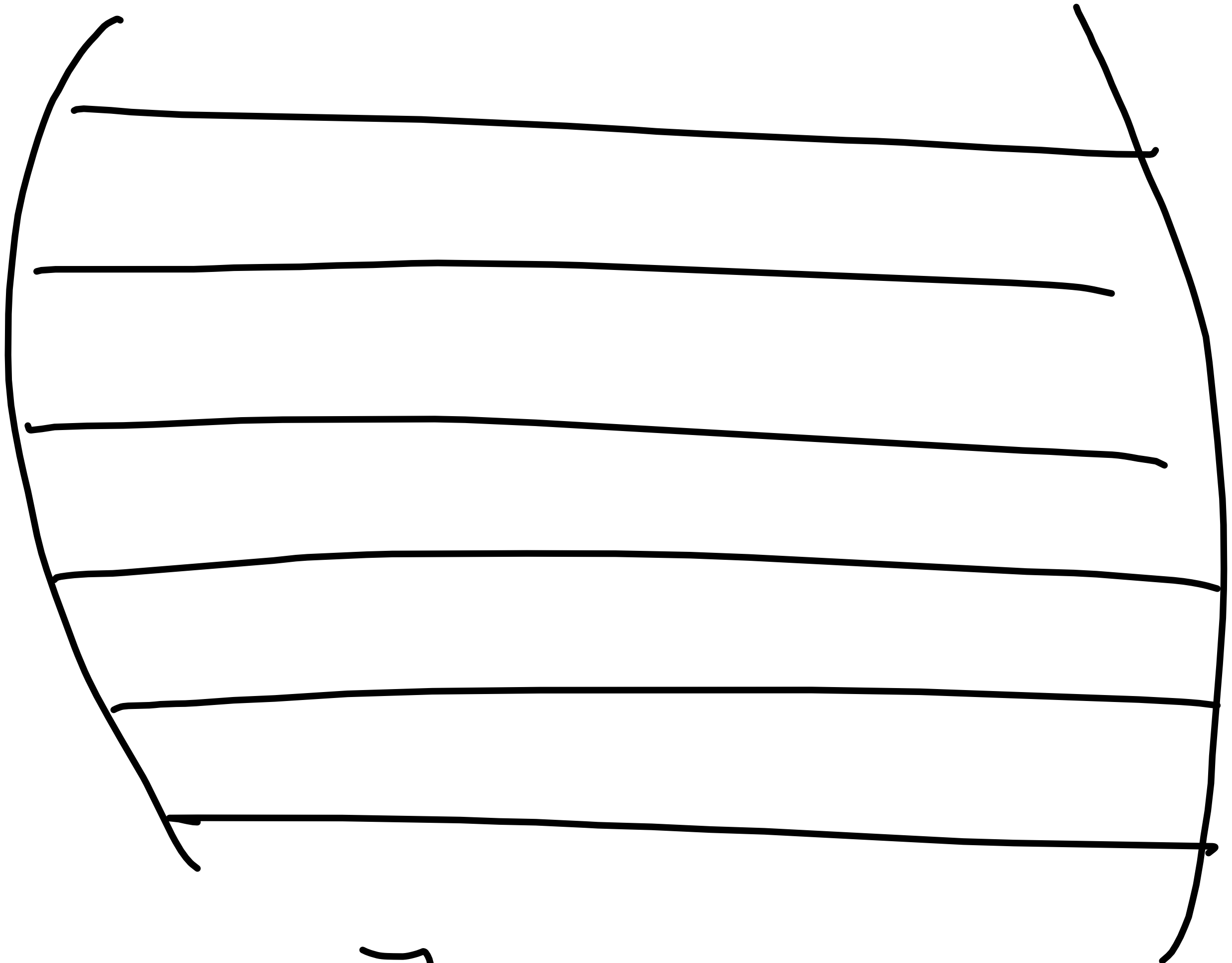


$$\mathrm{lin}[r_i(A)] = R_n(A)$$

$$h(A \cdot B) \le \min\big(h(A),\ h(B)\big)$$

$$h_r(A) = h_s(A)$$

$$h(A) = h(A^T)$$

$$\dim[\mathcal{R}_j(A)] = h_r(A)$$

$$AB = I_n = BA$$

$$\Uparrow$$

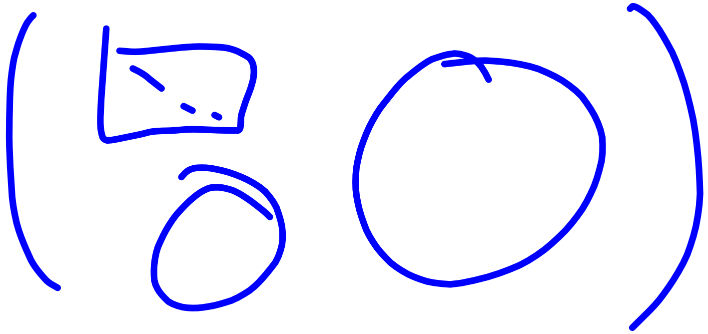
$$AB = I_n$$

$$\Uparrow$$

$$B = A^{-1} \qquad BA = I_n$$

$$(A \mid I_n) \overset{?}{\sim} (I_n \mid B)$$

$$B = A^{-1}$$



$E_1 A = A_1$

$E_2 A_1 = A_2$

$\vdots$

$E_n A_{n-1} = A_n = I_n$ $\quad A^{-1}$

$E_n E_{n-1} \dots E_1 A = I_n$

$E_1 I_n = E_1$

$E_2 E_1 = \dots$

A je reg. $\Leftrightarrow$ $\exists A^{-1}$

$\Leftrightarrow$ & A^{-1} $\exists A$

$A = E_n \cdots E_1$

$$A \sim B \iff \exists P \text{ reg.}$$

$$A = PB$$

$\Rightarrow$: $P = E_n \cdots E_1$

$B_1 = E_1 \cdot B$ $\qquad B_1 \sim B$

$\vdots$ $\qquad\qquad \vdots$

$A = B_n = E_n B_{n-1}$ $\qquad B_n \sim B_{n-1}$

$A \sim B$

$$h(A) \stackrel{?}{=} h(PA) \leq \min(h(A), h(P))$$

$$\leq h(A)$$

$$h(A) \leq h(PA)$$

$$\parallel$$

$$h(I_n A) = h(P^{-1} \cdot (P \cdot A))$$

$$\leq \min(h(P^{-1}), h(P \cdot A))$$

$$h(A) \leq h(PA)$$

$$(A \mid B) \sim (I_n \mid A^{-1}B)$$

$$E_1 A = A_1 \qquad E_1 B = B_1$$

$$\vdots$$

$$E_n A_{n-1} = I_n \qquad E_n B_{n-1} = B_n$$

$$B_n = \underbrace{E_n \cdot \ldots \cdot E_1}_{A^{-1}} \cdot B = A^{-1} B$$

$P, \ \alpha = (u_1, \dots, u_n)$

$v_j = \alpha \, s_j(P)$

$\underbrace{(v_1, \dots, v_n)}_{\beta} = \alpha \cdot P_{\alpha, \beta}$

$w_j = \alpha \, s_j(P^{-1})$

$\underbrace{(w_1 \dots w_n)}_{\gamma} = \alpha \cdot P^{-1} \quad / P$

$\gamma \, P_{\gamma \alpha} = \alpha$

$\alpha = (u_1, \dots, u_m)$, $\beta = (v_1, \dots, v_m)$

v $\mathbb{K}^m$

$$P_{\alpha,\beta} = \alpha^{-1}\beta$$

$$\alpha^{-1} = P_{\alpha,\varepsilon}, \quad P_{\alpha,\beta} = P_{\alpha,\varepsilon} P_{\varepsilon,\beta}, \quad P_{\varepsilon,\beta} = \beta$$

$$s_j(P_{\varepsilon,\beta}) = v_j$$

$$s_j(P_{\alpha,\varepsilon}) = s_j(\alpha^{-1})$$

$$(\alpha \mid \beta) \sim (I_m \mid \alpha^{-1}\beta)$$

$(I) \Rightarrow (II)$

$h(A) = h(B)$

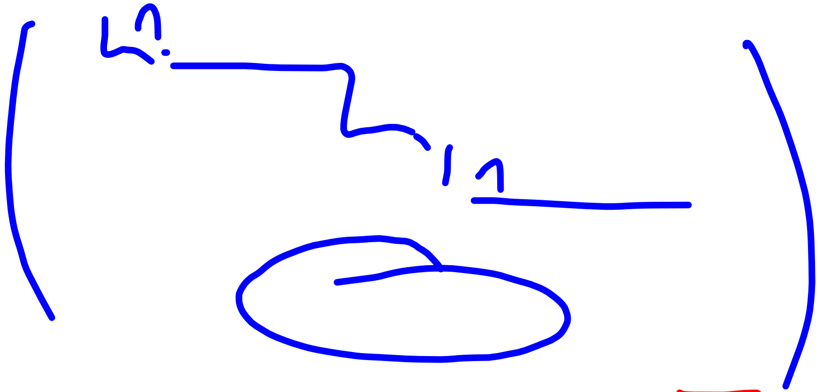
$$(\varphi)_{\alpha_2,\alpha_1} = A$$

$$\cong \qquad =$$

$$(\varphi)_{\beta_2,\beta_1} = B$$

$$(\varphi)_{\beta_2,\beta_1} = \underbrace{P_{\beta_2,\alpha_2}}_{= P} \, A \, \underbrace{P_{\alpha_1,\beta_1}}_{= Q}$$

$$h(A) = h(B) = r$$



$$P_2 B Q_2$$

$$B = P_2^{-1} P_1 A Q_1 Q_2^{-1}$$

$$P_1 A Q_1 = \left(\begin{array}{ccc|c} 1 & & 0 & \\ & \ddots & & 0 \\ 0 & & 1 & \\ \hline & 0 & & \end{array}\right) = \left(\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right)$$

$$A \quad (11) \Rightarrow (1)$$

(i) M je AP

$\Downarrow$ $p, q \in M \Rightarrow \ell(p,q) \subseteq M$

(ii)

$$r = \alpha p + (1-\alpha) q \in M$$

(ii) $\Rightarrow$ (iii) $\quad n = 0 \quad \underline{\underline{}} \quad h_0 = 1$

$p_0 \in M \qquad h_0 \, p_0 = p_0 \in M$

$n \Rightarrow n+1 \quad p_0, \ldots, p_{n+1} \in M$

$k_0 + \ldots + k_{n+1} + k_{n+1} = 1$

$k_0 p_0 + \ldots + k_{n+1} p_{n+1}$

$\exists\, k_j = 0$ jasné

$k_j \neq 0 \quad k_{n+1} \neq 0$

$$0 \neq 1 - k_{n+1} = \sum_{i=0}^{n} k_i$$

$$\frac{\lambda_0}{1-\lambda_{n+1}} + \ldots + \frac{\lambda_n}{1-\lambda_{n+1}} = 1$$

$$\underbrace{\frac{\lambda_0}{1-\lambda_{n+1}} P_0 + \ldots + \frac{\lambda_n}{1-\lambda_{n+1}} P_n}_{P} \in M$$

$$(1-\lambda_{n+1})\, P + \lambda_{n+1} P_{n+1} \in M$$

$$1-\lambda_{n+1} \neq 0 \qquad 1-\lambda_i = 0$$

$$1 - k_i = 0$$

$$k_i = 1$$

$$(n+2)\,1 = 0$$

$$M = P + S$$

$$S = M - P$$ [illegible]

$$+ 1 - a + b$$

$$ST_1 = q_1 - P$$

$$ST_2 = q_2 - P$$

$$a(q_1 - P) + b(q_2 - P)$$

$$= a q_1 + b q_2 - (a + b) P$$

$$= -P + a q_1 + b q_2 + (1 - a - b) P$$

$$= -P + q \in S \qquad q \in M$$