$$\check{F}(u_1, \dots, u_m) \doteq 0$$

$$\underbrace{}_{i \quad j}$$

$$\parallel$$

$$-F(u_1, \dots, u_m)$$

$$\boxed{1 + 1 \neq 0}$$

$$F(u_1, \dots, i, \dots, j, \dots, u_m)$$

$$-F(u_1, \dots, u_j, u_i, u_m)$$

$$F(u_1, \dots, \underbrace{u_i + u_j, \dots, u_i + u_j}, \dots) = 0$$

$$0 = F(u_1, \dots, u_i, \dots, \underline{u_i + u_j}, \dots) +$$

$$+ F(u_1, \dots, u_j, \underline{u_i + u_j}, \dots) +$$

$$= \underset{\overset{\parallel}{0}}{F(u_1, \dots, u_i, u_i)} +$$

$$+ F(u_1, \dots, u_i, u_j) +$$

$$+ F(u_1, \dots, u_j, u_i) +$$

$$+ \underset{\overset{\parallel}{0}}{F(u_1, \dots, u_j, u_j)}$$

$$\det A = \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{\sigma(1)1} \dots a_{\sigma(n)n}$$

$$A^T = (b_{ji}) = B \quad b_{ji} = a_{ij}$$

$$\det B = \sum_{\sigma \in S_n} (-1)^{|\sigma|} b_{\sigma(1)1} \dots b_{\sigma(n)n}$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{1\sigma(1)} \dots a_{n\sigma(n)}$$

$$a_{j\sigma(j)} = a_{\sigma^{-1}(i)i}$$

$$i = \sigma(j)$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{\sigma^{-1}(1)1} \dots a_{\sigma^{-1}(n)n}$$

$$= \sum_{\sigma^{-1} \in S_n} (-1)^{|\sigma^{-1}|} a_{\sigma^{-1}(1)1} \dots \dots$$

$$\sigma^{-1} = \rho \qquad \qquad \underline{\underline{\det A}}$$

$$= \sum_{\rho \in S_n} (-1)^{|\rho|} a_{\rho(1)1} \dots a_{\rho(n)n}$$

$1 \le i \le n$ $\quad j$: fixní

$S_n(i,j) = \{\sigma \in S_n : i = \sigma(j)\}$

$$\tilde a_{ij} = \sum_{\sigma \in S_n(i,j)} (-1)^{|\sigma|} a_{\sigma(1)1} \cdots a_{\sigma(n)n} \qquad \forall i,j$$

$$\det A = \sum_{i=1}^{n} a_{ij} \tilde a_{ij} =$$

$$= \quad S_n = \bigcup_{i=1}^{n} S_n(i,j)$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} \cdots = \sum_{i=1}^{n} \sum_{\sigma \in S_n(i,j)} \cdots$$

$$\det A = (\tilde a_{1j}, \ldots, \tilde a_{nj}) \begin{pmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{pmatrix}$$

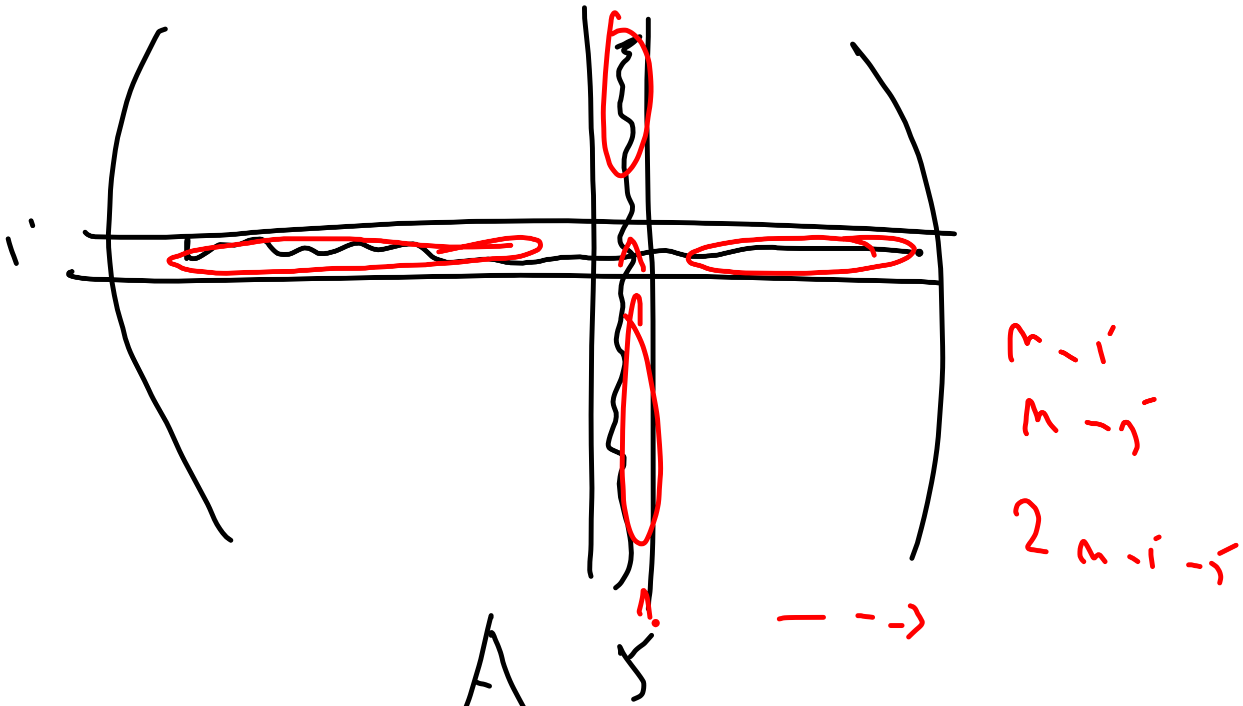
$$A^i = A^j$$

$$\det A = (\underline{\tilde a_{1i}, \ldots, \tilde a_{ni}}) \begin{pmatrix} a_{1i} \\ \vdots \\ a_{ni} \end{pmatrix}$$

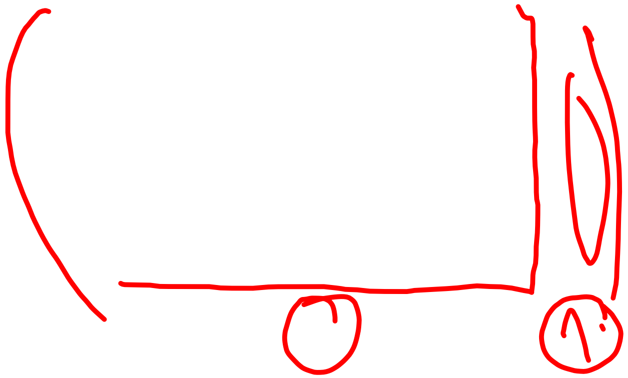
$$\parallel$$

$$(-\tilde a_{1j}, \ldots, -\tilde a_{nj})$$

$\det A = -\det A$



$$\tilde{a}_{ij} = (-1)^{i+j} |A_{ij}|$$



$$\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & x_2 - x_1 & x_2^2 - x_1 x_2 & \cdots & x_2^{n-1} - x_1 x_2^{n-2} \\ 1 & x_3 - x_1 & \vdots & & \\ \vdots & & & & \\ 1 & x_n - x_1 & x_n^2 - x_1 x_n & \cdots & x_n^{n-1} - x_1 x_n^{n-2} \end{vmatrix}$$

$-x_1$

$$= 1 \cdot (x_2 - x_1) \cdots (x_n - x_1) \cdot \begin{vmatrix} 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} \\ 1 & \vdots & & & \\ 1 & \vdots & & & \\ 1 & & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} \end{vmatrix}$$

$$= \prod_{1 \le i < j \le n} (x_j - x_i)$$