DÚ 10

1) $$\int \frac{4x^5-16x^3-2x^2-6x+8}{x^2-4}\,dx = \int\left(4x^3-2+\frac{-6x}{x^2-4}\right)dx =$$

$$(4x^5-16x^3-2x^2-6x+8):(x^2-4) = 4x^3-2$$
$$\frac{-(4x^5-16x^3)}{\quad -2x^2-6x+8}$$
$$\frac{-(-2x^2+8)}{-6x}$$

$$\frac{-6x}{x^2-4} = \frac{A}{x+2}+\frac{B}{x-2} \quad /\cdot(x+2)(x-2)$$
$$-6x = A(x-2)+B(x+2)$$
$$x^1:\ -6 = A+B \quad /\cdot 2$$
$$x^0:\ 0 = -2A+2B$$
$$-12 = 4B \qquad A=-6-B$$
$$B=-3 \qquad A=-3$$

$$= \int\left(4x^3-2-\frac{3}{x+2}-\frac{3}{x-2}\right)dx = 4\frac{x^4}{4}-2x-3\ln|x+2|-3\ln|x-2|+c =$$
$$= x^4-2x-3\left(\ln|x+2|+\ln|x-2|\right)+c = \underline{\underline{x^4-2x-3\ln|x^2-4|+c}}$$

2) $$\int \frac{4x^2-3x+11}{x^3-x^2+3x-3}\,dx = *$$

jeden kořen polynomu x^3-x^2+3x-3 je 1 (stačí uhádnout či Hornerovým schematem):

$$(x^3-x^2+3x-3):(x-1) = x^2+3$$
$$\frac{-(x^3-x^2)}{3x-3}$$
$$\frac{-(3x-3)}{0}$$

$$\Rightarrow \frac{4x^2-3x+11}{x^3-x^2+3x-3} = \frac{A}{x-1}+\frac{Bx+C}{x^2+3} \quad /\cdot(x-1)(x^2+3)$$
$$4x^2-3x+11 = A(x^2+3)+(Bx+C)(x-1)$$
$$4x^2-3x+11 = Ax^2+3A+Bx^2-Bx+Cx-C$$

$$x^2:\ 4 = A+B \qquad (1)$$
$$x^1:\ -3 = -B+C \qquad (2)$$
$$x^0:\ 11 = 3A-C \qquad (3)$$

$$(1)+(2)+(3):\ 12 = 4A \qquad B=4-A \qquad C=3A-11$$
$$A=3 \qquad B=4-3 \qquad C=9-11$$
$$B=1 \qquad C=-2$$

$$* = \int\left(\frac{3}{x-1}+\frac{x-2}{x^2+3}\right)dx = \int\left(\frac{3}{x-1}+\frac{1}{2}\frac{2x}{x^2+3}-2\frac{1}{x^2+3}\right)dx =$$
$$= \underline{\underline{3\ln|x-1|+\frac{1}{2}\ln(x^2+3)-\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}+c}}$$

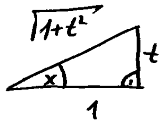
$$3)\ \int \frac{\sin^3 x}{\cos^2 x}\,dx = \left| \frac{(-\sin x)^3}{(\cos x)^2} = -\frac{\sin^3 x}{\cos^2 x} \Rightarrow \text{lichá vůči } \sin x \Rightarrow \begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \\ -dt = \sin x\,dx \end{array} \right| = \int \frac{\sin^2 x \,\sin x\,dx}{\cos^2 x} =$$

$$= \int \frac{(1-\cos^2 x)\sin x\,dx}{\cos^2 x} = \int \frac{1-t^2}{t^2}(-dt) = \int\left(1-\frac{1}{t^2}\right)dt = t + \frac{1}{t} + c = \underline{\underline{\cos x + \frac{1}{\cos x} + c}}$$

$$4)\ \int \frac{1}{\sin^2 x - 5\sin x\cos x}\,dx = \left| \frac{1}{(-\sin x)^2 - 5(-\sin x)\cdot(-\cos x)} = \frac{1}{\sin^2 x - 5\sin x\cos x} \Rightarrow \text{sudá vůči} \right.$$

$\sin x$ a $\cos x$ zároveň $\Rightarrow$

$$\left.\begin{array}{l} t = \operatorname{tg} x \\ \operatorname{arctg} t = x \\ \frac{1}{1+t^2}\,dt = dx \end{array} \quad \begin{array}{l} \sin x = \frac{t}{\sqrt{1+t^2}} \\ \cos x = \frac{1}{\sqrt{1+t^2}} \end{array} \right| =$$



$$= \int \frac{1}{\left(\frac{t}{\sqrt{1+t^2}}\right)^2 - 5\cdot\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}}}\cdot\frac{1}{1+t^2}\,dt = \int \frac{1}{\left(\frac{t^2}{1+t^2} - \frac{5t}{1+t^2}\right)\cdot(1+t^2)}\,dt = \int \frac{1}{t^2-5t}\,dt$$

$$\frac{1}{t^2-5t} = \frac{1}{t(t-5)} = \frac{A}{t} + \frac{B}{t-5} \quad \Big/ \cdot t(t-5)$$

$$1 = A(t-5) + Bt$$

$$\left.\begin{array}{l} t^1:\ 0 = A + B \\ t^0:\ 1 = -5A \end{array}\right\} \Rightarrow \begin{array}{l} A = -\frac{1}{5} \\ B = \frac{1}{5} \end{array}$$

$$= \int \frac{-\frac{1}{5}}{t} + \frac{\frac{1}{5}}{t-5}\,dt = \frac{1}{5}\left(-\ln|t| + \ln|t-5|\right) + c = \frac{1}{5}\ln\left|\frac{t-5}{t}\right| + c =$$

$$= \frac{1}{5}\ln\left|\frac{\operatorname{tg} x - 5}{\operatorname{tg} x}\right| + c = \frac{1}{5}\ln\left|\frac{\frac{\sin x}{\cos x} - 5 \ \ /\cdot\cos x}{\frac{\sin x}{\cos x} \ \ /\cdot \cos x}\right| + c = \underline{\underline{\frac{1}{5}\ln\left|\frac{\sin x - 5\cos x}{\sin x}\right| + c}}$$

$$5)\ \int \sqrt{x^2+12x}\,dx = \int \frac{x^2+12x}{\sqrt{x^2+12x}}\,dx = (Ax+B)\sqrt{x^2+12x} + k\cdot\int\frac{1}{\sqrt{x^2+12x}}\,dx \quad \Big/'$$

$$\frac{x^2+12x}{\sqrt{x^2+12x}} = A\sqrt{x^2+12x} + (Ax+B)\frac{2x+12}{2\sqrt{x^2+12x}} + k\cdot\frac{1}{\sqrt{x^2+12x}} \quad \Big/\cdot\sqrt{x^2+12x}$$

$$x^2+12x = A(x^2+12x) + (Ax+B)(x+6) + k$$

$$x^2+12x = Ax^2 + 12Ax + Ax^2 + 6Ax + Bx + 6B + k$$

$$\left.\begin{array}{l} x^2:\ 1 = 2A \\ x^1:\ 12 = 18A + B \\ x^0:\ 0 = 6B + k \end{array}\right\} \Rightarrow \begin{array}{l} A = \frac{1}{2} \\ B = 3 \\ k = -18 \end{array}$$

$$\int \frac{1}{\sqrt{x^2+12x}}\,dx = \int\frac{1}{\sqrt{(x+6)^2-36}}\,dx = \left|\begin{array}{l} t = x+6 \\ dt = dx \end{array}\right| = \int\frac{1}{\sqrt{t^2-36}}\,dt = \ln\left|t+\sqrt{t^2-36}\right| =$$

$$= \ln\left|x+6+\sqrt{x^2+12x}\right|$$

celkem:

$$\underline{\underline{\int\sqrt{x^2+12x}\,dx = \frac{1}{2}(x+6)\sqrt{x^2+12x} - 18\ln\left|x+6+\sqrt{x^2+12x}\right| + c}}$$