DÚ 10 Panák

1) $y=x^2$ kolem osy y

$y=\sqrt{x}$ — // — x

$$V=\pi\int_0^1(\sqrt{x})^2dx=\pi\int_0^1 x\,dx=\pi\left[\frac{x^2}{2}\right]_0^1=\pi\,\frac{1}{2}=\underline{\underline{\frac{\pi}{2}}}$$

2) délka křivky

$y=3x^{\frac{3}{2}}-1 \qquad x\in\langle 0,4\rangle$

$y'=\frac{9}{2}x^{\frac{1}{2}}$

$$l=\int_0^4\sqrt{1+\tfrac{81}{4}x}\,dx=\left|\begin{array}{ll} t=1+\frac{81}{4}x & x=0\to t=1\\ dt=\frac{81}{4}dx & x=4\to t=82\\ \frac{4}{81}dt=dx & \end{array}\right|=\int_1^{82}\underbrace{\sqrt{t}}_{\frac{1}{2}}\cdot\frac{4}{81}\,dt=\frac{4}{81}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{82}=$$

$$=\underline{\underline{\frac{8}{243}\left(82\sqrt{82}-1\right)}}\quad(\doteq 24{,}41)$$

3) $$\int_0^1\ln x\,dx=\left|\begin{array}{ll} u'=1 & v=\ln x\\ u=x & v'=\frac{1}{x}\end{array}\right|=[x\ln x]_0^1-\int_0^1 1\,dx=[x\ln x-x]_0^1=(0-1)-\left(\lim_{x\to0^+}x\ln x-0\right)=$$

$$=-1-\lim_{x\to0^+}x\ln x=\|0\cdot\infty\|=-1-\lim_{x\to0^+}\frac{\ln x}{\frac{1}{x}}=\left\|\frac{\infty}{\infty}\right\|\overset{L.P.}{=}-1-\lim_{x\to0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=-1-\lim_{x\to0^+}(-x)=-1-0=\underline{\underline{-1}}$$

$$\int_0^\infty e^{-x}dx=-\left[e^{-x}\right]_0^\infty=\left\|-\left(e^{-\infty}-e^0\right)\right\|=-(0-1)=\underline{\underline{1}}$$