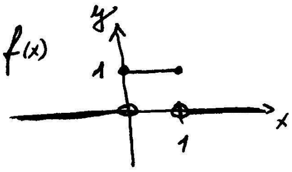
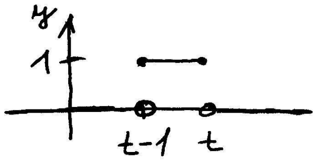
DÚ 13 Panák

Př. 1 $\quad f = \begin{cases} 1 & x \in \langle 0,1) \\ 0 & \text{jinak} \end{cases}$, $\quad f * f = ?$

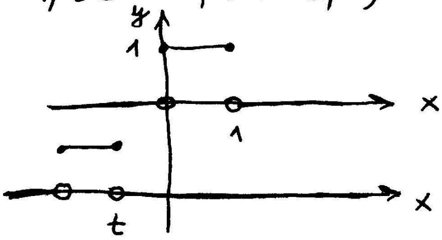
$f(x)$



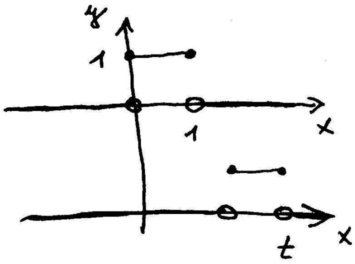
$f(t-x)$



1) $t \in (-\infty, 0) \cup \langle 2, \infty)$

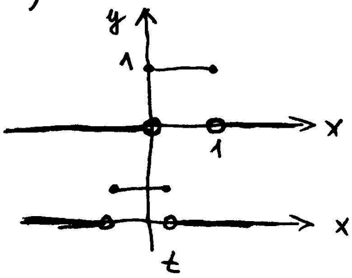


nebo



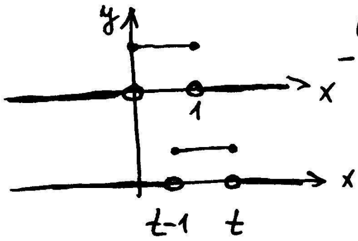
$$\int_{-\infty}^{\infty} f(t-x) \cdot f(x)\, dx = \int_{-\infty}^{\infty} 0\, dx = \underline{\underline{0}}$$

2) $t \in \langle 0,1)$



$$\int_{-\infty}^{\infty} f(t-x) f(x) dx = \int_0^t 1 \cdot 1\, dx = [x]_0^t = t - 0 = \underline{\underline{t}}$$

3) $t \in \langle 1,2)$

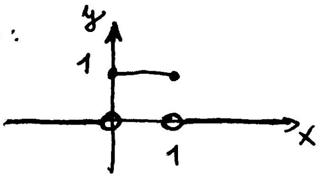


$$\int_{-\infty}^{\infty} f(t-x) \cdot f(x) dx = \int_{t-1}^{1} 1 \cdot 1\, dx = [x]_{t-1}^{1} = 1 - (t-1) = \underline{\underline{2-t}}$$

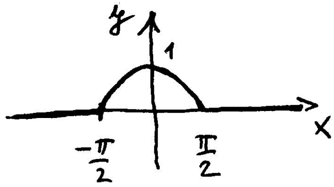
$$\text{celkem: } f * f = \begin{cases} x & x \in \langle 0,1) \\ 2-x & x \in \langle 1,2) \\ 0 & \text{jinak} \end{cases}$$

pr. 2 $\quad f = \begin{cases} 1 & x \in \langle 0, 1 \rangle \\ 0 & \text{jinak} \end{cases}, \quad g = \begin{cases} \cos x & x \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \\ 0 & \text{jinak} \end{cases} \qquad f * g = ?$

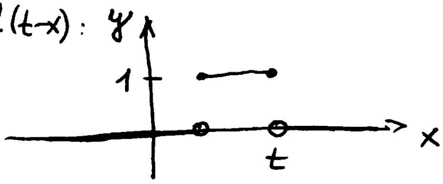
$f(x)$:



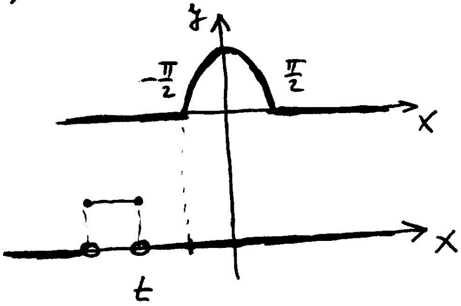
$g(x)$:



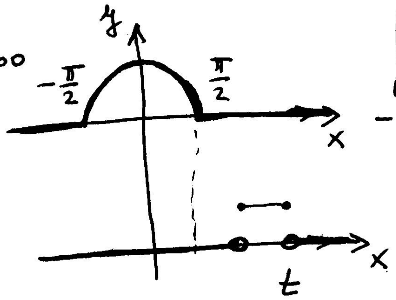
$f(t-x)$:



1) $t \in (-\infty, -\frac{\pi}{2}) \cup \langle \frac{\pi}{2}+1, \infty)$

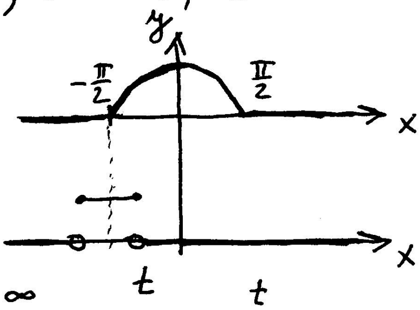


nebo



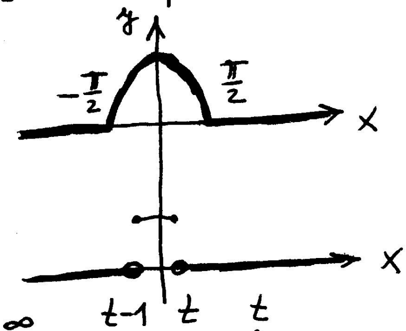
$$\int_{-\infty}^{\infty} f(t-x) \cdot g(x)\,dx = \int_{-\infty}^{\infty} 0\,dx = \underline{\underline{0}}$$

2) $t \in \langle -\frac{\pi}{2}, -\frac{\pi}{2}+1 \rangle$



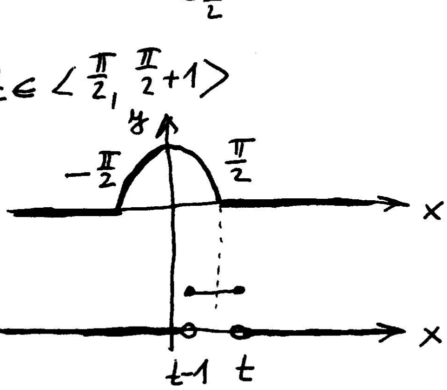
$$\int_{-\infty}^{\infty} f(t-x) \cdot g(x)\,dx = \int_{-\frac{\pi}{2}}^{t} 1 \cdot \cos x\,dx = [\sin x]_{-\frac{\pi}{2}}^{t} = \underline{\underline{\sin t + 1}}$$

3) $t \in \langle -\frac{\pi}{2}+1, \frac{\pi}{2} \rangle$



$$\int_{-\infty}^{\infty} f(t-x)\, g(x)\,dx = \int_{t-1}^{t} 1 \cdot \cos x\,dx = [\sin x]_{t-1}^{t} = \underline{\underline{\sin t - \sin(t-1)}}$$

3) $t \in \langle \frac{\pi}{2}, \frac{\pi}{2}+1 \rangle$



$$\int_{-\infty}^{\infty} f(t-x)\, g(x)\,dx = \int_{t-1}^{\frac{\pi}{2}} 1 \cdot \cos x\,dx = [\sin x]_{t-1}^{\frac{\pi}{2}} = \underline{\underline{1 - \sin(t-1)}}$$

celkem:

$$f * g = \begin{cases} \sin x + 1 & x \in \langle -\frac{\pi}{2}, -\frac{\pi}{2}+1 \rangle \\ \sin x - \sin(x-1) & x \in \langle -\frac{\pi}{2}+1, \frac{\pi}{2} \rangle \\ 1 - \sin(x-1) & x \in \langle \frac{\pi}{2}, \frac{\pi}{2}+1 \rangle \\ 0 & \text{jinak} \end{cases}$$

<u>př. 3</u> $f(x) = x^2$ sudá fce : $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx = *$$

$$\int x^2 \cos nx \, dx = \left| \begin{matrix} u = x^2 & v' = \cos nx \\ u' = 2x & v = \frac{1}{n} \sin nx \end{matrix} \right| = x^2 \frac{1}{n} \sin nx - \frac{1}{n} \int 2x \sin nx \, dx =$$

$$= \left| \begin{matrix} u = 2x & v' = \sin nx \\ u' = 2 & v = -\frac{1}{n} \cos nx \end{matrix} \right| = \frac{1}{n} x^2 \sin nx - \frac{1}{n}\left(-\frac{1}{n} 2x \cos nx - \int -\frac{1}{n} \cdot 2 \cdot \cos nx \, dx\right) =$$

$$= \frac{1}{n} x^2 \sin nx + \frac{1}{n^2} 2x \cos nx - \frac{2}{n^3} \sin nx + c$$

$$* = \frac{2}{\pi} \left[\frac{1}{n} x^2 \sin nx + \frac{1}{n^2} 2x \cos nx - \frac{2}{n^3} \sin nx \right]_0^{\pi} = \frac{2}{\pi} \left[\left(0 + \frac{1}{n^2} 2\pi \overbrace{\cos n\pi}^{(-1)^n} - 0\right) - (0 + 0 - 0) \right] =$$

$$= \frac{2}{\pi} \cdot \frac{1}{n^2} \cdot 2\pi \cdot (-1)^n = \frac{4}{n^2} (-1)^n$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x^2 \cos 0x \, dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \left[\frac{x^3}{3} \right]_0^{\pi} = \frac{2}{\pi} \frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

$$\underline{\underline{F(x) = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx}}$$

$$x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx \quad \text{na } \langle -\pi, \pi \rangle$$

spec. případ pro $x = 0$:

$$0^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \underbrace{\cos n \cdot 0}_{1}$$

$$-\frac{\pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\underline{\underline{\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}}}$$