$$\int \frac{1}{\cos^3(x)}\,dx = \frac{\operatorname{tg}(x)}{\cos(x)} - \int \frac{\sin^2(x)}{\cos^3(x)}\,dx =$$

$$\left[\begin{array}{ll} v' = \frac{1}{\cos^2 x} & v = \operatorname{tg}(x) \\ u = \frac{1}{\cos(x)} & u' = \frac{\sin(x)}{\cos^2(x)} \end{array}\right]$$

$$= \frac{\operatorname{tg}(x)}{\cos(x)} - \int \frac{1-\cos^2(x)}{\cos^3(x)}\,dx = \frac{\operatorname{tg}(x)}{\cos(x)} - \int \frac{1}{\cos^3(x)}\,dx + \int \frac{1}{\cos(x)}\,dx$$

$$\int \frac{1}{\cos^3(x)}\,dx = \frac{1}{2}\left(\frac{\operatorname{tg}(x)}{\cos x} + \int \frac{1}{\cos(x)}\,dx\right)$$

$$\int \frac{1}{\cos(x)}\,dx = \int \frac{\overbrace{\frac{1}{\cos(x)}\left(\frac{1}{\cos(x)} + \operatorname{tg}(x)\right)}^{u'}}{\underbrace{\left(\frac{1}{\cos(x)} + \operatorname{tg}(x)\right)}_{u}}\,dx =$$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \ln\left|\frac{1}{\cos(x)} + \operatorname{tg}(x)\right| + C$$

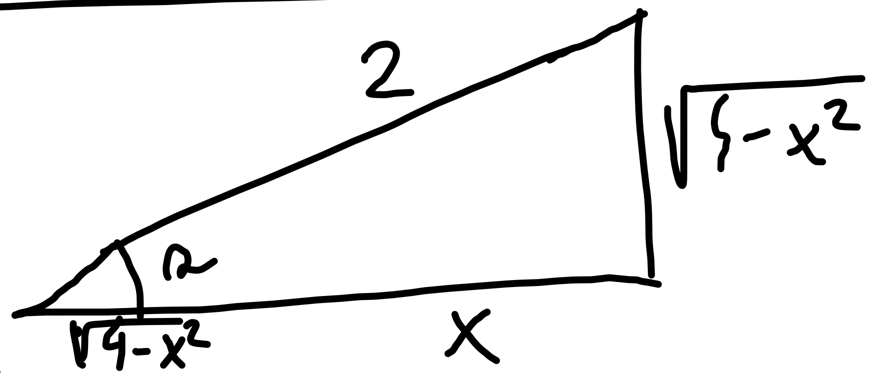
$$\int \frac{1}{\sin^2 x \cos^2(x)}\,dx =$$

$$\left[\begin{array}{l} u = \operatorname{tg} x \\ du = \dfrac{1}{\cos^2(x)}\,dx \end{array}\right] \qquad \left[\begin{array}{l} \operatorname{tg}(x) = \dfrac{\sin(x)}{\cos(x)} \\ \operatorname{tg}^2(x) = \dfrac{\sin^2(x)}{1-\sin^2(x)} \Rightarrow \\ \Rightarrow \sin^2(x) = \dfrac{\operatorname{tg}^2(x)}{1+\operatorname{tg}^2(x)} = \dfrac{u^2}{1+u^2} \end{array}\right]$$

$$= \int \frac{u^2+1}{u^2}\,du = \int 1 + \frac{1}{u^2}\,du = u - u^{-1} + C =$$

$$= \operatorname{tg}(x) - \operatorname{cotg}(x) + C$$

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx$$



$$\cos z = \frac{x}{2}$$

$$-\sin z\,dz = \frac{1}{2}\,dx = \frac{\sqrt{4-x^2}}{2}$$

$$\int \frac{4\cos^2 z}{\sqrt{4-4\cos^2 z}} \cdot (-2\sin z)\,dz = -4\int \cos^2 z\,dz$$

$$\int \cos^2 x\,dx = \cos x \sin x - \int -\sin^2(x)\,dx =$$

$$\left[\begin{array}{ll} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array}\right]$$

$$= \cos x \sin x + \int \sin^2 x\,dx =$$

$$= \cos x \sin x + \int 1 - \cos^2 x\,dx \Rightarrow$$

$$\Rightarrow \int \cos^2 x\,dx = \frac{1}{2}\left(x + \cos x \sin x\right) + C$$

$$\int e^{2x}\sin(2x)\,dx = \frac{1}{2}\int \underbrace{e^y}_{v'}\,\underbrace{\sin y}_{u}\,dy =$$

$$2x = y \qquad = \frac{1}{2}\left[e^y \sin y - \int \underbrace{e^y}_{v'}\,\underbrace{\cos y}_{u}\,dy\right]$$

$$\frac{1}{2}\left[e^y \sin y - \left(e^y \cos y - \int e^y(-\sin y)\,dx\right)\right] \Rightarrow$$

$$= \int e^y \sin y\,dy = \frac{1}{2}\left(e^y \sin y - e^y \cos y\right) + C$$

$$\int e^{2x} \sin(2x)\,dx = \frac{1}{4}\left(e^{2x}\sin 2x - e^{2x}\cos 2x\right) + C$$

$$x^4 - x^3 - 2x^2 + x + 1 : (x^3 - x^2 + x - 6) = x$$
$$x^4 - x^3 + x^2 - 6x$$
$$-3x^2 + 7x + 1$$

$$\int x\,dx - \int \frac{3x^2 - 7x - 1}{x^3 - x^2 + x - 6}\,dx = \int \frac{3x^2 - 7x - 1}{(x-2)(x^2+x+3)} =$$

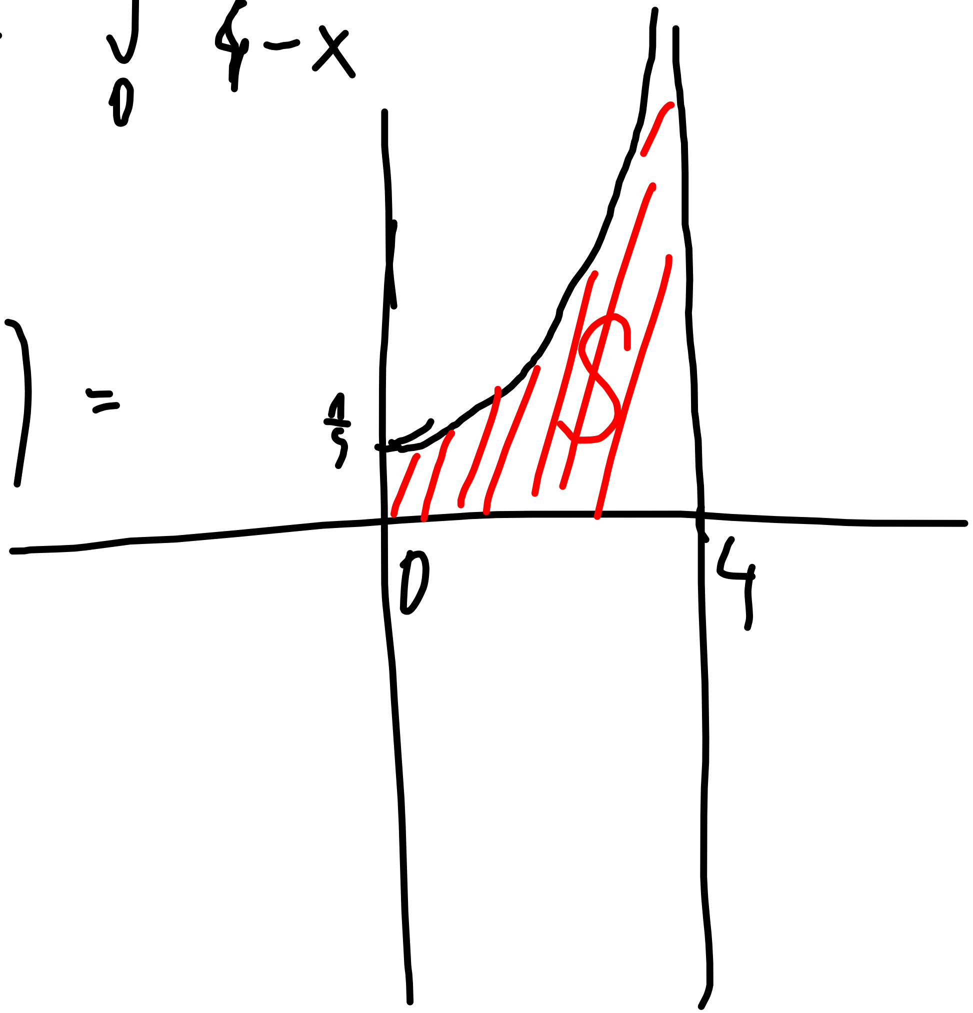
$$= \int \frac{A}{(x-2)}\,dx + \int \frac{Bx + D}{x^2 + x + 3}\,dx$$

$$\int_0^4 \frac{dx}{4-x} = \lim_{\delta \to 4^-} \int_0^\delta \frac{dx}{4-x} =$$

$$= \lim_{\delta \to 4^-} \Big[-\ln|4-x| \Big]_0^\delta =$$

$$= \lim_{\delta \to 4^-} \Big(\underbrace{-\ln|4-\delta|}_{\to -\infty} + \ln 4 \Big) =$$

$$= \infty$$



$$\int_0^1 x\ln x\,dx = \lim_{\delta\to 0^+}\int_\delta^1 x\ln x\,dx$$

$$= \lim_{\delta\to 0^+}\left[\frac{x^2}{2}\ln(x) - \frac{x^2}{4}\right]_\delta^1 =$$

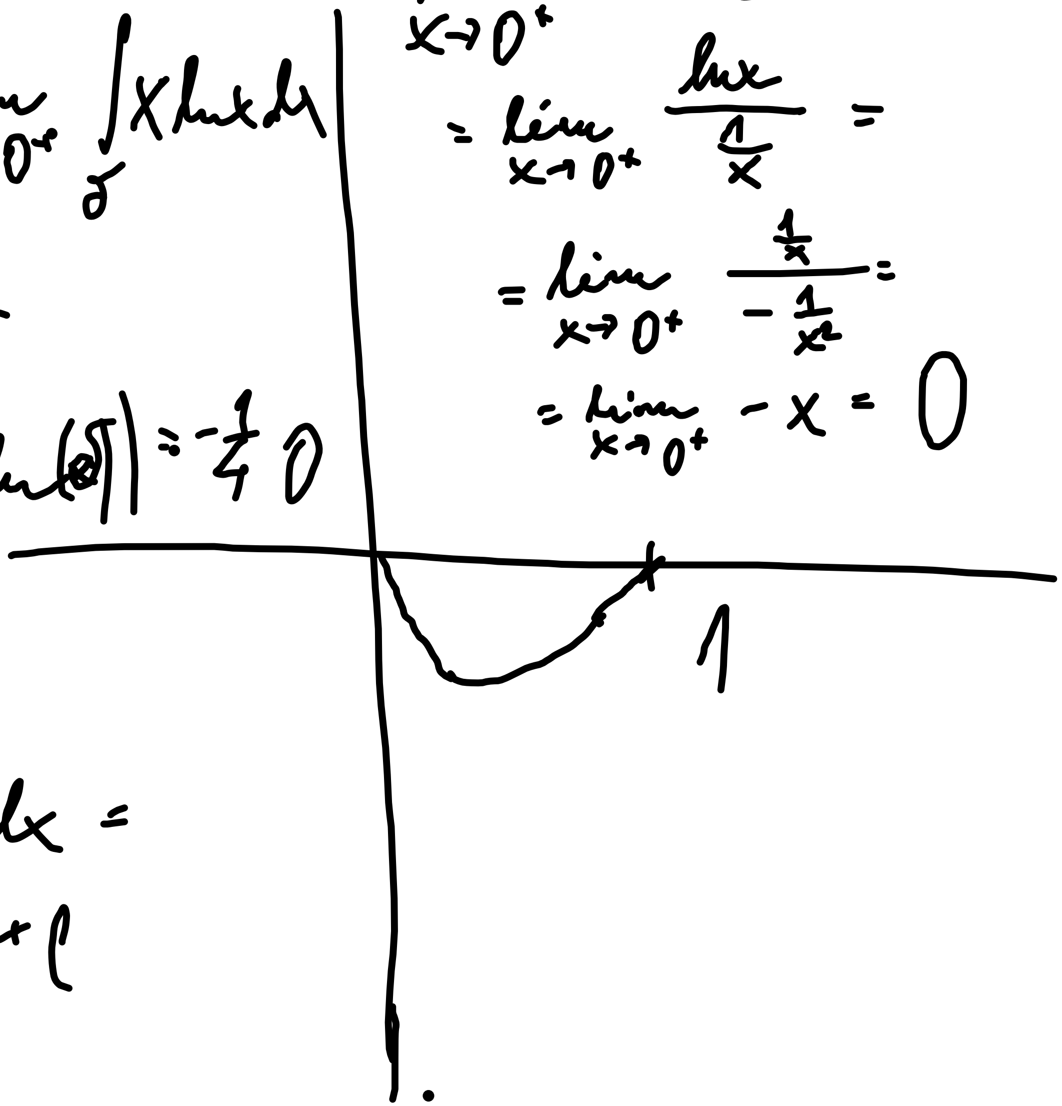
$$= -\frac{1}{4} + \lim_{\delta\to 0^+}\left(\frac{\delta^2}{4} - \frac{\delta^2}{2}\ln(\delta)\right) = -\frac{1}{4}$$

$$\lim_{\delta\to 0^+}\delta^2\cdot\ln(\delta) = 0$$

$$\int \underbrace{x}_{u'}\,\underbrace{\ln x}_{v} = \frac{x^2}{2}\ln x - \int\frac{x}{2}\,dx =$$

$$= \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C$$

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -x = 0$$

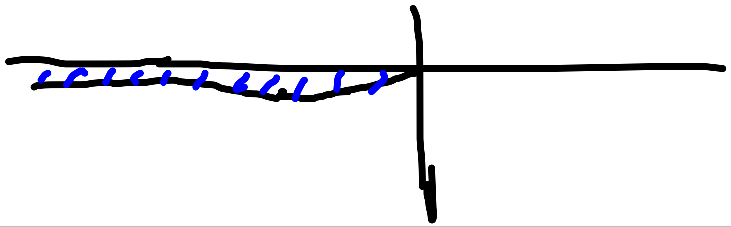


$$\int_{-\infty}^{0} x\cdot e^{x}\,dx = \lim_{\delta\to-\infty}\int_{\delta}^{0} x\cdot e^{x}\,dx =$$

$$\int \underbrace{x}_{u}\cdot\underbrace{e^{x}}_{v'} = xe^{x} - \int e^{x}\,dx = xe^{x} - e^{x}$$

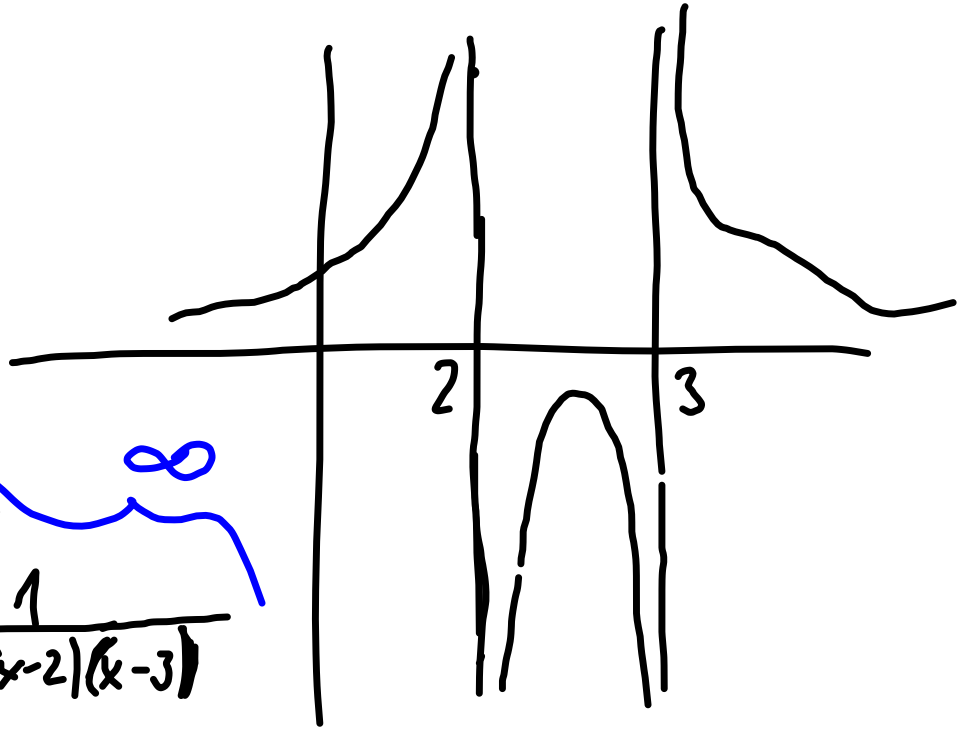
$$\lim_{\delta\to-\infty}\left[xe^{x} - e^{x}\right]_{\delta}^{0} = \lim_{\delta\to-\infty}\left(e^{\delta} - \delta\cdot e^{\delta}\right) - 1 = -1$$

$$\boxed{\lim_{\delta\to-\infty} e^{\delta}(1-\delta) = \lim_{\delta\to-\infty}\frac{1-\delta}{\frac{1}{e^{\delta}}} = \lim_{\delta\to-\infty}\frac{-1}{-e^{-\delta}} = \lim_{\delta\to-\infty} e^{+\delta} = 0}$$



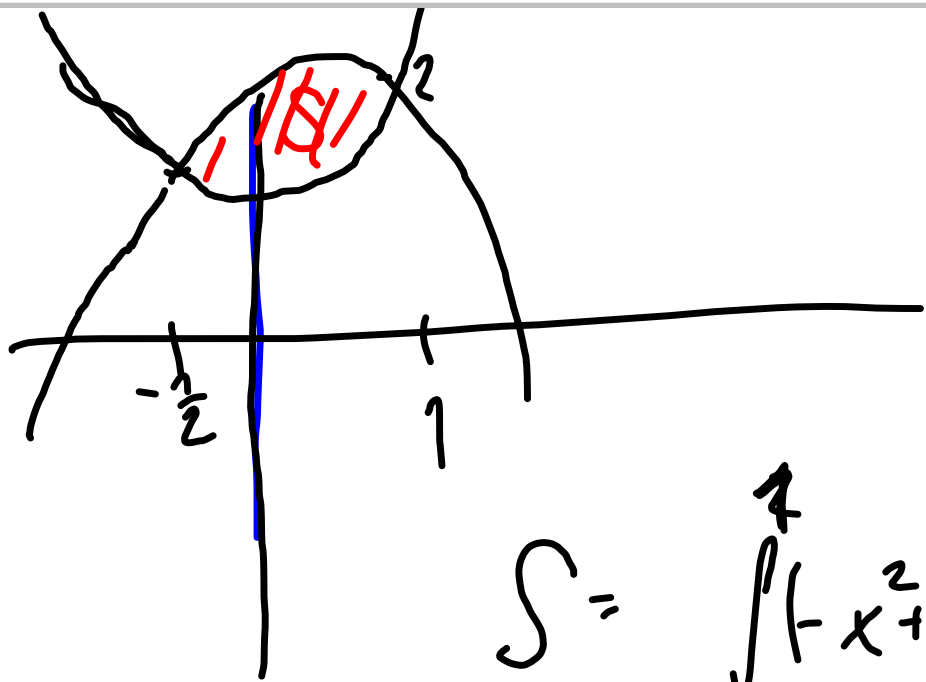
$$\int_0^\infty \frac{1}{x^2-5x+6}\,dx =$$

$$= \underbrace{\int_0^2 \frac{1}{(x-2)(x-3)}\,dx}_{\infty} + \underbrace{\int_2^3 \frac{1}{(x-2)(x-3)}\,dx}_{-\infty} + \overbrace{\int_3^\infty \frac{1}{(x-2)(x-3)}}^{\infty}$$



$$\frac{1}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} = \frac{1}{(x-3)} - \frac{1}{(x-2)}$$

$$\int \frac{dx}{(x-2)(x-3)} = \ln|x-3| - \ln(x-2)$$



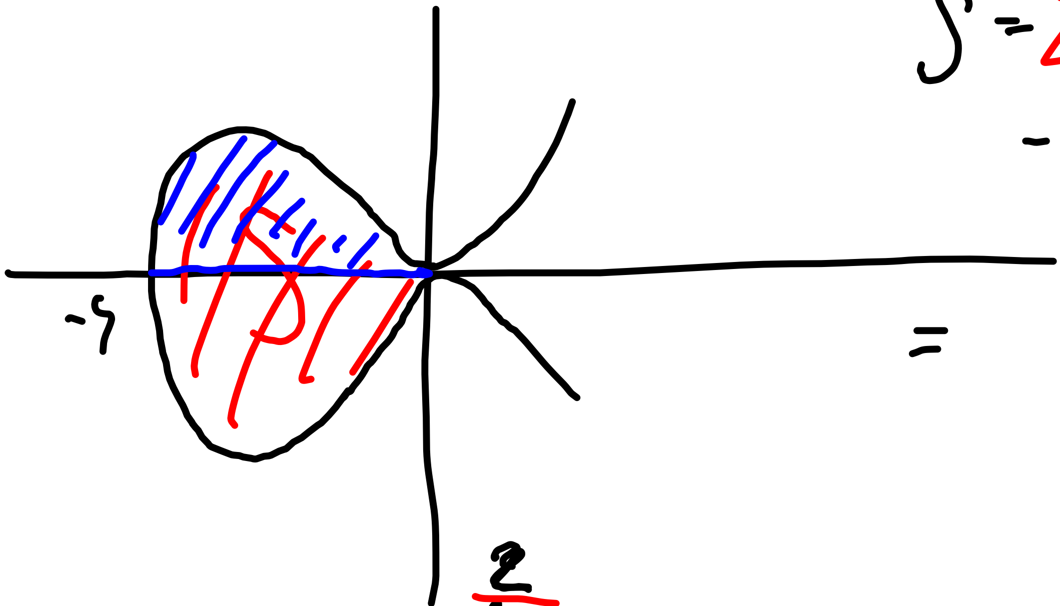
$$x^2+1 = -x^2+x+2$$

$$\Rightarrow \quad x = -\frac{1}{2}$$

$$x = 1$$

$$S = \int_{-\frac{1}{2}}^{1} \left|-x^2+x+2-x^2-1\right| dx = \int_{-\frac{1}{2}}^{1} -2x^2+x+1\, dx$$

$$= \left[-\frac{2}{3}x^3+\frac{x^2}{2}+x\right]_{-\frac{1}{2}}^{1} = -\frac{2}{3}+\frac{1}{2}+1-\frac{1}{12}-\frac{1}{8}+\frac{1}{2}$$



$$S = 2\cdot\int_{-5}^{0} x^2 \sqrt{(4+x)}\,dx =$$

$$(4+x) = r^2 \Rightarrow$$

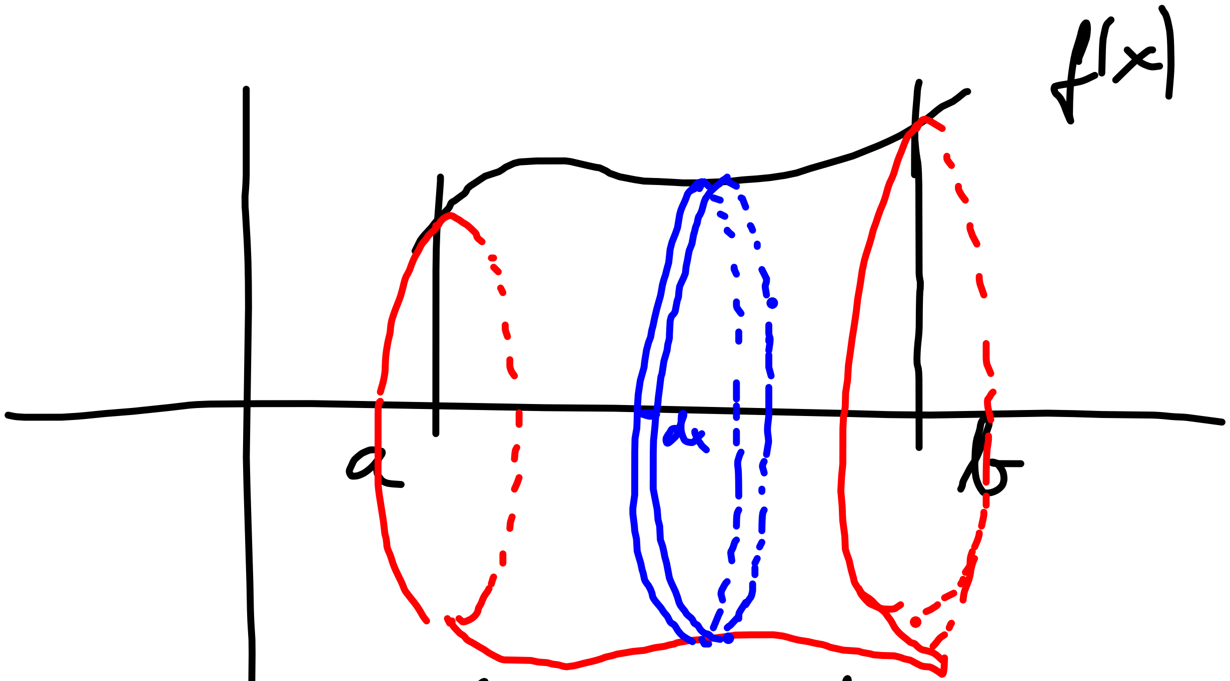
$$\Rightarrow x = r^2 - 4$$

$$x^2 = r^4 - 8r^2 + 16$$

$$dx = 2r\,dr$$

$$S = 2\cdot\int_{0}^{2} (r^4 - 8r^2 + 16)\cdot r \cdot 2r\,dr =$$

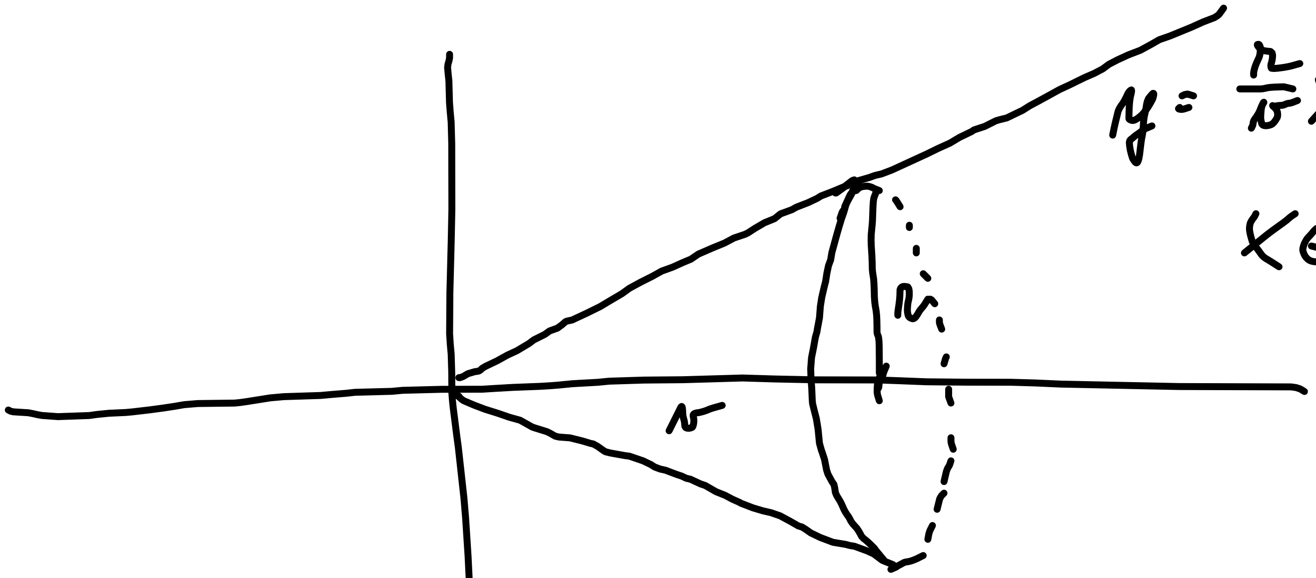
$$= 2\cdot\left[\frac{2r^7}{7} - \frac{16}{5}r^5 + \frac{32}{3}r^3\right]_0^2$$



$$\int_a^b dS = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx =$$

$$= 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

Kužel: $f(x) =$



$y = \frac{r}{v}x$

$x \in \langle 0, v \rangle$

$$S = 2\pi \int_0^v \frac{r}{v} x \cdot \sqrt{1 + \frac{r^2}{v^2}}\, dx =$$

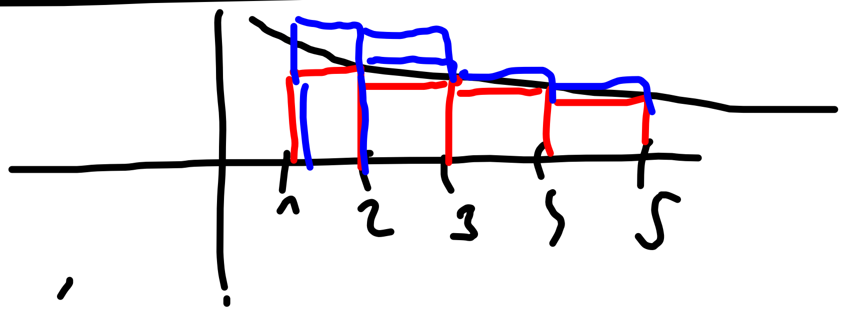
$$= 2\pi \frac{r}{v} \sqrt{1 + \frac{r^2}{v^2}} \int_0^v x\, dx = 2\pi \frac{r}{v} \sqrt{1 + \frac{r^2}{v^2}} \cdot \frac{v^2}{2} =$$

$$= \pi r \sqrt{r^2 + v^2}$$

$$V = \pi \int_0^v f^2(x)\,dx = \pi \int_0^v \frac{r^2}{v^2} x^2\,dx =$$

$$= \pi \frac{r^2}{v^2} \int_0^v x^2\,dx = \pi \frac{r^2}{v^2} \frac{v^3}{3} = \frac{\pi r^2 v}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$



f nezáporná, nerostoucí:

$$\sum_{n=1}^{\infty} f(n) \text{ konverguje} \iff \int_1^{\infty} f(x)\,dx \text{ konečný}$$

$$\int_2^\infty \frac{1}{x\ln(x)}\,dx = \int_{\ln(2)}^{\infty} \frac{1}{u}\,du =$$

$u = \ln(x)$

$$\lim_{\delta\to\infty} \left[\ln(u)\right]_{\ln(2)}^{\delta} = \infty$$