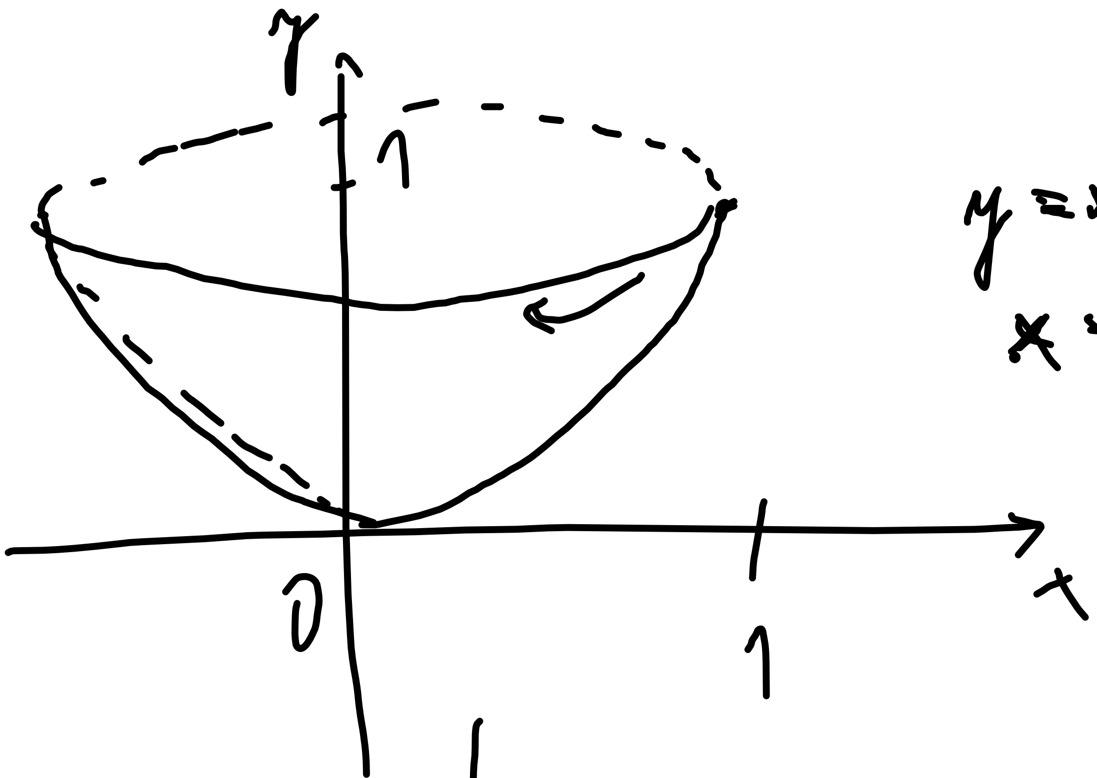
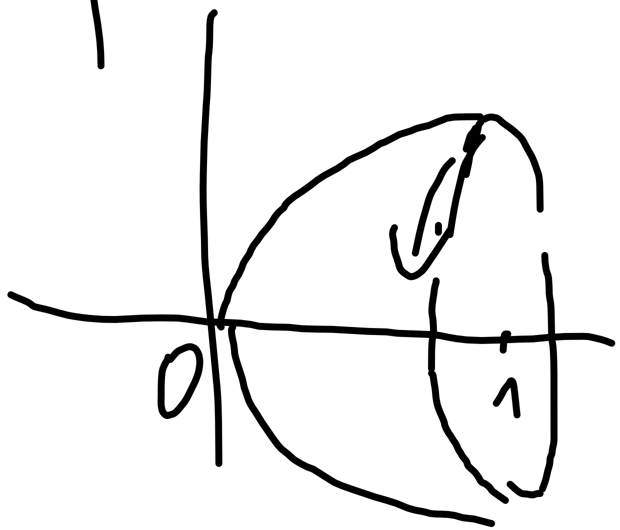



$y = x^2$

$x = \sqrt{y}$




$$\pi \int_0^1 x \, dx = \pi \left[ \frac{x^2}{2} \right]_0^1 = \frac{\pi}{2}$$

$$\int_0^4 \sqrt{1+[f'(x)]^2}\,dx = \int_0^5 \sqrt{\underbrace{1+\tfrac{81}{5}x}_{y}}\,dx =$$

$$f'(x) = \frac{9}{2}\sqrt{x} \qquad dy = \frac{81}{5}dx$$

$$= \frac{2}{3}\cdot\frac{4}{81}\left[\left(1+\frac{81}{5}x\right)^{\frac{3}{2}}\right]_0^5 = \frac{8}{243}\left((82)^{\frac{3}{2}}-1\right)$$

$$\int_0^1 \ln(x)\,dx = \lim_{\delta\to 0^+} \int_\delta^1 \ln(x)\,dx = \lim_{\delta\to 0^+} \left[x\ln x - x\right]_\delta^1 =$$

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$$\int \ln x = x\ln x - \int 1\,dx = x\ln x - x$$

$u' = 1 \quad u = x$

$v = \ln x \quad v' = \frac{1}{x}$

---

$$= -1 - \lim_{\delta\to 0^+}\left[\delta\ln\delta\right] = -1$$

$$\lim_{\delta\to 0^+} \delta\cdot\ln\delta = \lim_{\delta\to 0^+} \frac{\ln\delta}{\frac{1}{\delta}} = \lim_{\delta\to 0^+} \frac{\frac{1}{\cancel{\delta}}}{-\frac{1}{\delta^{\cancel{2}}}} = \underbrace{-\delta}_{0} =$$

$$\int_0^\infty e^{-x}\,dx = \lim_{\delta\to\infty} \int_0^\delta e^{-x}\,dx = \lim_{\delta\to\infty} \left[-e^{-x}\right]_0^\delta =$$

$$= \underbrace{\lim_{\delta\to\infty} -e^{-\delta}}_{0} + 1 = 1$$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 \dots$$

$a = 1$

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f^{(2)}(x) = -\frac{1}{x^2}$

$f^{(3)}(x) = \frac{2}{x^3}$

$f^{(4)}(x) = -\frac{2\cdot 3}{x^4}$

$f^{(n)}(x) = (-1)^{n+1}\frac{(n-1)!}{x^n}$

$$f(x) = \ln(x) =$$

$$= 0 + (x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - $$

$$= \sum_{i=1}^{\infty} (-1)^{i+1}\frac{1}{i}(x-1)^i$$

Pro která x řada konverguje?

Podílovým kritériem

$$\lim_{n\to\infty}\left|\frac{\frac{1}{n+1}(x-1)^{n+1}}{\frac{1}{n}(x-1)^{n}}\right| < 1$$

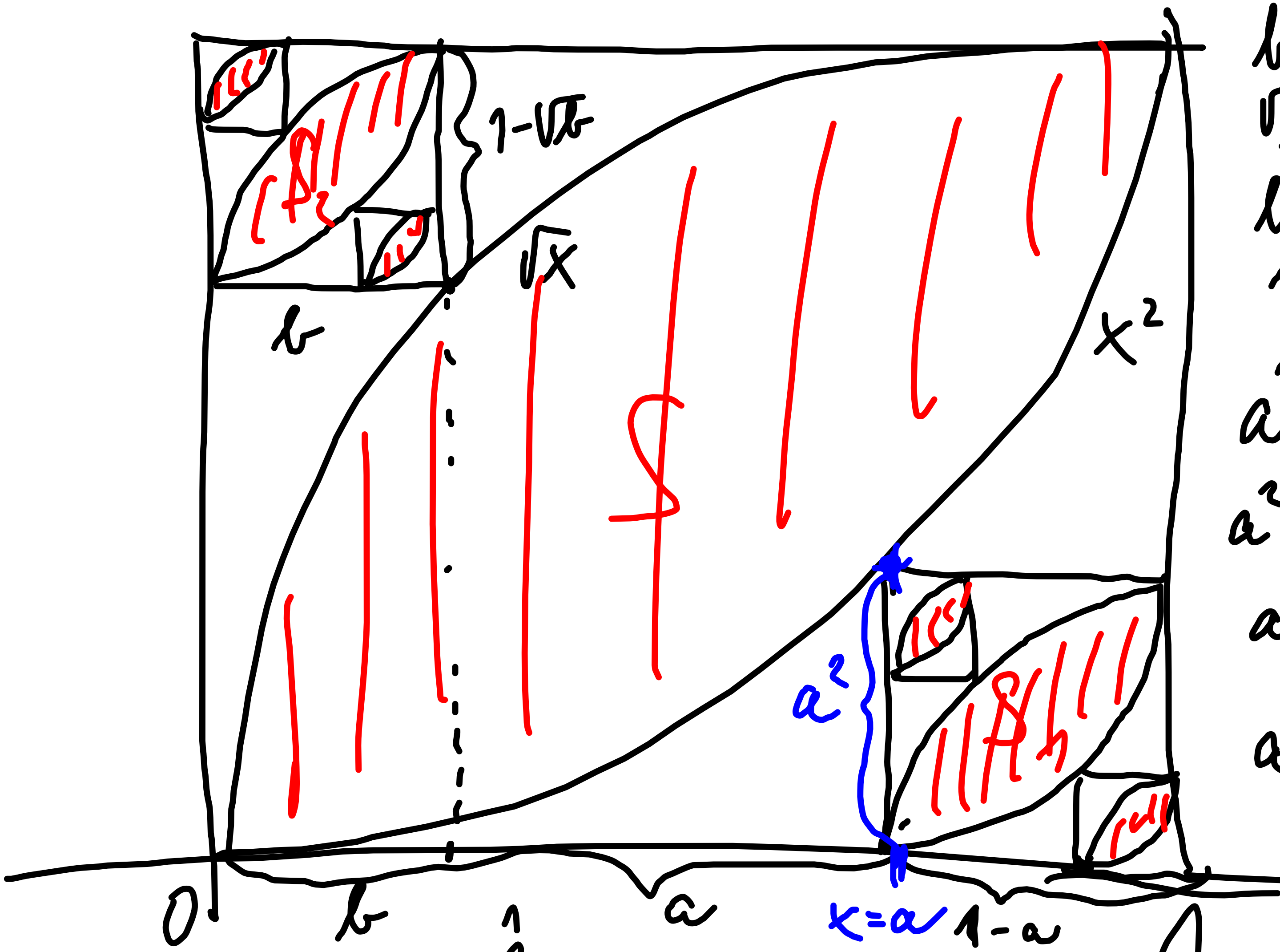
$$\lim_{n\to\infty}\left|\frac{n}{n+1}\right| \, |x-1| < 1$$

$$|x-1| < 1 \Leftrightarrow x \in (0,2)$$

Konvergence v krajních bodech:

$x=0$ : $-\sum_{n=1}^{\infty}\frac{1}{n} \to -\infty$

$x=2$ : $\sum_{n=1}^{\infty}(-1)^{i+1}\frac{1}{i} = \ln(2)$




$b = 1-\sqrt{b}$

$\sqrt{b} = 1-b$

$b = 1-2b+b^2$

$b^2-3b+1=0$

$b_{1,2} = \frac{3\pm\sqrt{5}}{2}$

$a^2 = 1-a$

$a^2+a-1=0$

$a_{1,2} = \frac{-1\pm\sqrt{5}}{2}$

$a = \frac{-1+\sqrt{5}}{2}$

$1-a = \frac{3-\sqrt{5}}{2}$

$$S = \int_0^1 (\sqrt{x}-x^2)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$$

$$\frac{S_1}{S_0} = \left(\frac{3-\sqrt{5}}{2}\right)^2 = \frac{9-6\sqrt{5}+5}{5} = \frac{1}{2}\left(7-3\sqrt{5}\right)$$

$$\frac{S_1+S_2}{S_0} = \left(7-3\sqrt{5}\right)$$

$$\sum_{i=0}^{\infty} S_i = \frac{1}{3} + \frac{1}{3}\left(7-3\sqrt{5}\right) + \frac{1}{3}\left(7-3\sqrt{5}\right)^2 + \dots$$

$$= \frac{1}{3}\sum_{i=0}^{\infty}\left(7-3\sqrt{5}\right)^i =$$

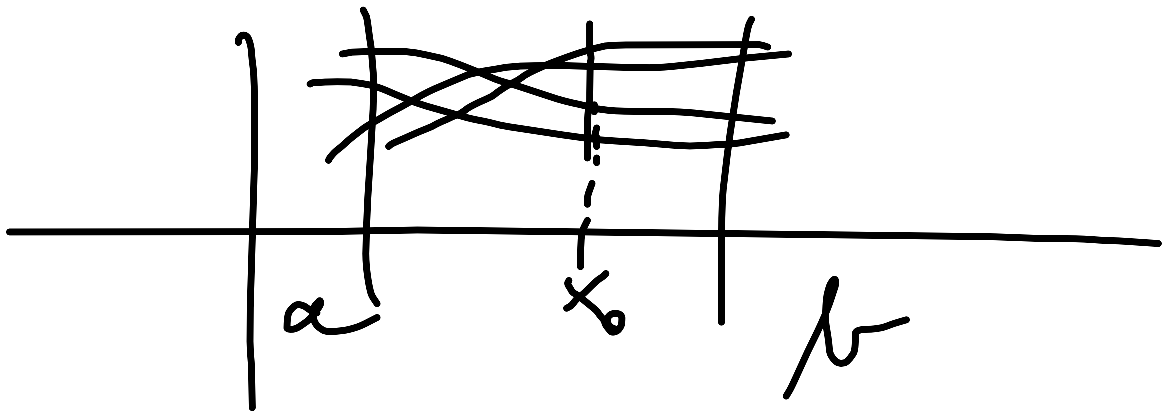
$$= \frac{1}{3}\,\frac{1}{1-7+3\sqrt{5}} = \frac{1}{3}\left(\frac{1}{3\sqrt{5}-6}\right) = \frac{1}{9}\,\frac{1}{\sqrt{5}-2} =$$

$$= \frac{1}{9}\,\frac{\sqrt{5}+2}{5-4} = \frac{\sqrt{5}+2}{9}$$

$$\int_a^b \underbrace{\sum_{n=1}^{\infty} f_n}_{S} = \sum_{n=1}^{\infty} \int_a^b f_n$$

$f_n \to f$ na $\langle a, b\rangle$

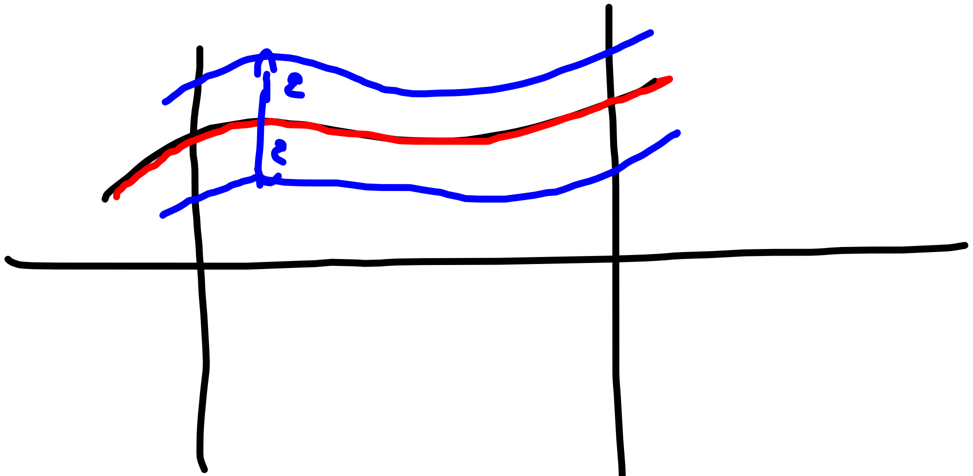
$\lim\limits_{n\to\infty} f_n(x) = f(x)$ pro lib. $x \in \langle a, b\rangle$



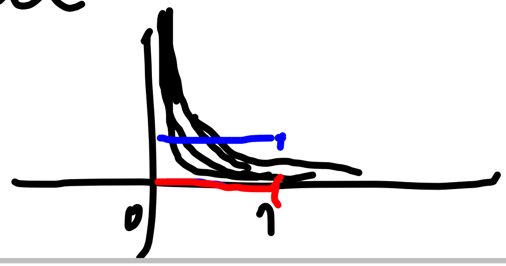
$f_n \rightrightarrows f$ na $\langle a,b\rangle$

jestliže $(\forall \varepsilon>0)(\exists n_0)(\forall n>n_0)(\forall x\in\langle a,b\rangle)$

$$|f_n(x)-f(x)|<\varepsilon$$




$f_n = \frac{1}{nx}$ na $(0,1)$




$f_n \to 0$

$$\sum_{n=1}^{\infty} \frac{1}{n\,3^{n-1}}$$

$$\int_3^{\infty} \frac{1}{x^{n+1}} = \lim_{\delta\to\infty} \left[-\frac{1}{n\,x^{n}}\right]_3^{\delta} = \underbrace{\lim_{\delta\to\infty}\left[-\frac{1}{n\,\delta^{n}}\right]}_{0} + \frac{1}{n\,3^{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n\,3^{n-1}} = 3\sum_{n=1}^{\infty} \frac{1}{n\,3^{n}} = 3\sum_{n=1}^{\infty} \int_3^{\infty} \frac{1}{x^{n+1}}\,dx$$

$$= 3\int_3^{\infty} \left(\underbrace{\sum_{n=1}^{\infty} \frac{1}{x^{n+1}}}_{\text{konverguje stejnoměrně na } (3,\infty)}\right) dx$$

$|f_n(x)| \leq a_n$ na $\langle a, b\rangle$ a $\sum_{n=1}^{\infty} a_n$ konverguje

platí $f_n \rightrightarrows f$ na $\langle a, b\rangle$

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$f_n(x) = \frac{1}{x^{n+1}}$ a $|f_n(x)| \leq \frac{1}{3^{n+1}}$ na $\langle 3, \infty)$

$\sum_{n=1}^{\infty} \frac{1}{3^{n+1}}$ konverguje $\Rightarrow$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{x^{n+1}}$ konverguje stejnoměrně na $\langle 3, \infty)$

dokonce $\sum_{n=1}^{\infty} \frac{1}{x^{n+1}} = \frac{1}{x^2}\left(1 + \frac{1}{x} + \frac{1}{x^2} + \dots\right) =$

$= \frac{1}{x^2} \frac{1}{1 - \frac{1}{x}} = \frac{1}{x(x-1)}$

$$\int_3^\infty \left(\sum_{n=1}^\infty \frac{1}{x^{n+1}}\right) dx = \int_3^\infty \frac{1}{x(x-1)}\, dx = \int_3^\infty \left(\frac{1}{x-1} - \frac{1}{x}\right) dx =$$

$$= \lim_{\delta \to \infty} \int_3^\delta \left(\frac{1}{x-1} - \frac{1}{x}\right) dx = \lim_{\delta \to \infty} \Big[\ln|x-1| - \ln|x|\Big]_3^\delta =$$

$$= \lim_{\delta \to \infty} \big(\ln|\delta - 1| - \ln \delta\big) + \ln(3) - \ln(2),$$

$$\lim_{\delta \to \infty} \ln\left(\frac{(\delta - 1)}{\delta}\right) = 0$$

$$\sum_{n=1}^\infty \frac{1}{n 3^{n-1}} = 3\big(\ln(3) - \ln(2)\big)$$

$$\int_{\ln(2)}^{\ln(3)} \sum_{n=1}^{\infty} n e^{-nx} = \sum_{n=1}^{\infty} \int_{\ln(2)}^{\ln(3)} n \cdot e^{-nx}$$

na $\langle \ln(2), \ln(3)\rangle$ je $f_n = n e^{-nx}$ klesající,

je tedy $\underset{\displaystyle n2^{-n}}{\underset{\shortparallel}{f_n(\ln(2))}} \geq f_n(x)$, pro $x \in \langle \ln(2), \ln(3)\rangle$

$\sum_{n=1}^{\infty} n 2^{-n}$ konverguje

(podle podílového kritéria

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)2^{-n-1}}{n 2^{-n}} \right| = \frac{1}{2} \lim_{n\to\infty} \left| \frac{n+1}{n} \right| = \frac{1}{2} < 1$$

$$\sum_{n=1}^{\infty} \left[-e^{-nx}\right]_{\ln(2)}^{\ln(3)} = \sum_{n=1}^{\infty}\left(-3^{-n}+2^{-n}\right) =$$

$$= -\frac{1}{3}\left(\frac{1}{1-\frac{1}{3}}\right) + \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right) = -\frac{1}{2} + 1 = \frac{1}{2}$$