1, $D(f): \mathbb{R} - \{2\}$

$$f'(x) = \frac{(2x-7)(x-2) - (x^2-7x+12)}{(x-2)^2} =$$

$$= \frac{x^2 - 4x + 2}{(x-2)^2}$$

$$f(x) = \sin^2(x)$$

$$f'(x) = 2\sin(x)\cos(x) = \sin(2x)$$

$$f''(x) = 2\cos(2x)$$

$$f^{(3)}(x) = -4\sin(2x)$$

$$f^{(4)}(x) = -8\cos(2x)$$

$$f^{(5)}(x) = 16\sin(2x)$$

$$f'\left(\frac{\pi}{4}\right) = 1$$

$$f^{(2k)}\left(\frac{\pi}{4}\right) = 0$$

$$f^{(2k+1)} = (-1)^k 2^{2k}$$

$$f(x) = f\left(\frac{\pi}{4}\right) + \sum_{i=1}^{\infty} \frac{f^{(i)}\left(\frac{\pi}{4}\right)}{i!}\left(x - \frac{\pi}{4}\right)^i$$

$$= \frac{1}{2} + \sum_{i=1}^{\infty} \frac{(-1)^i 2^{2i}}{(2i+1)!}\left(x - \frac{\pi}{4}\right)^{2i+1}$$

$$\lim_{n\to\infty} \left| \frac{\frac{2^{2i+2}}{(2i+3)!}\left(x-\frac{\pi}{4}\right)^{2i+3}}{\frac{2^{2i}}{(2i+1)!}\left(x-\frac{\pi}{4}\right)^{2i+1}} \right| = \lim_{n\to\infty} \left| \frac{4}{(2i+3)(2i+2)}\left(x - \frac{\pi}{4}\right)^2 \right| = 0 < 1$$

$$\int_0^1 a^2x + a\sqrt{1-x^2} + 1\,dx = \frac{a^2}{2} + a\frac{\pi}{4} + 1 \Rightarrow$$

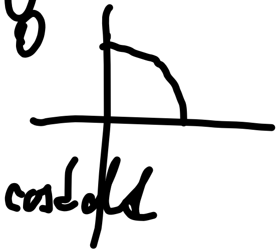
extrém

v $f(a) = 0$

$$= \left[\frac{a^2x^2}{2}\right]_0^1 + a\int_0^1 \sqrt{1-x^2}\,dx + [x]_0^1$$

$a + \frac{\pi}{4} = 0 \Rightarrow a = -\frac{\pi}{4}$

$$\int_0^1 \sqrt{1-x^2}\,dx =$$



$y = \sqrt{1-x^2} \Leftrightarrow y^2 = 1-x^2$

$y^2 + x^2 = 1$

$x = \sin t,\ dx = \cos t\,dt$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 t}\,\cos t\,dt = \int_0^{\frac{\pi}{2}} \cos^2 t\,dt$$

$$\int \cos^2 t = \cos t \sin t + \int \sin^2 t\,dt =$$

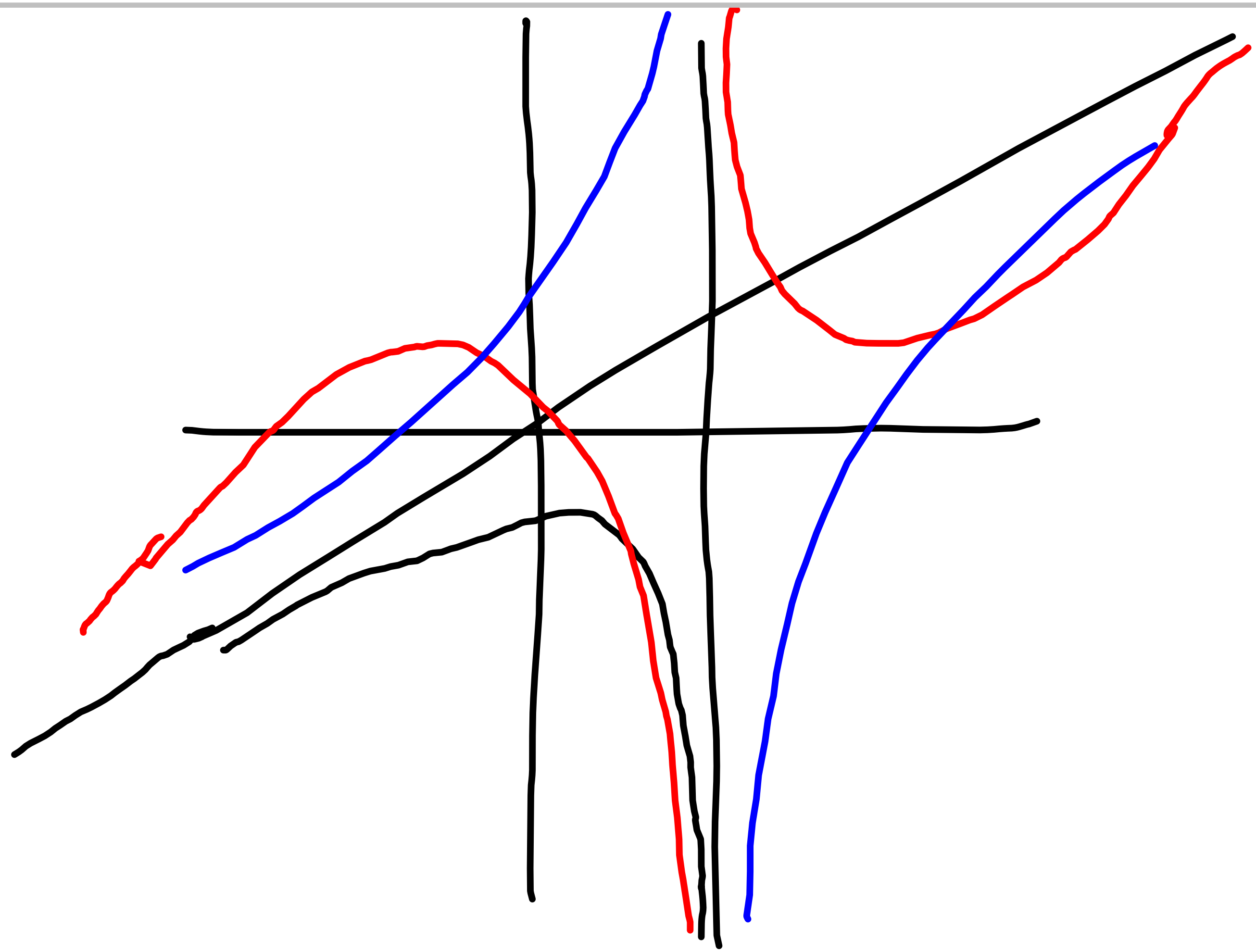
$u' = \cos t \quad u = \sin t$

$v = \cos t \quad v' = -\sin t$

$$= \cos t \sin t + \int 1 - \cos^2 t\,dt$$

$$= \cos t \sin t + t - \int \cos^2 t\,dt$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \cos^2 t\,dt = \frac{1}{2}\left[t + \cos t \sin t\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$



$$\int_0^{\frac{\pi}{4}} a^2x + ax^2\sin(x)\,dx =$$

$$= \frac{a^2\pi^2}{32} + a\int_0^{\frac{\pi}{4}} x^2\sin x = \frac{a^2\pi^2}{32} + a\left[-\frac{\pi^2\sqrt{2}}{32} + \frac{\pi\sqrt{2}}{4} + \sqrt{2} - 2\right]$$

$$\int x^2\sin x = -x^2\cos x + \int 2x\cos x\,dx =$$

$u' = \sin x \quad u = -\cos x$

$v = x^2 \quad v' = 2x$

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x\right] =$$

$$= -x^2\cos x + 2x\sin x + 2\cos x$$

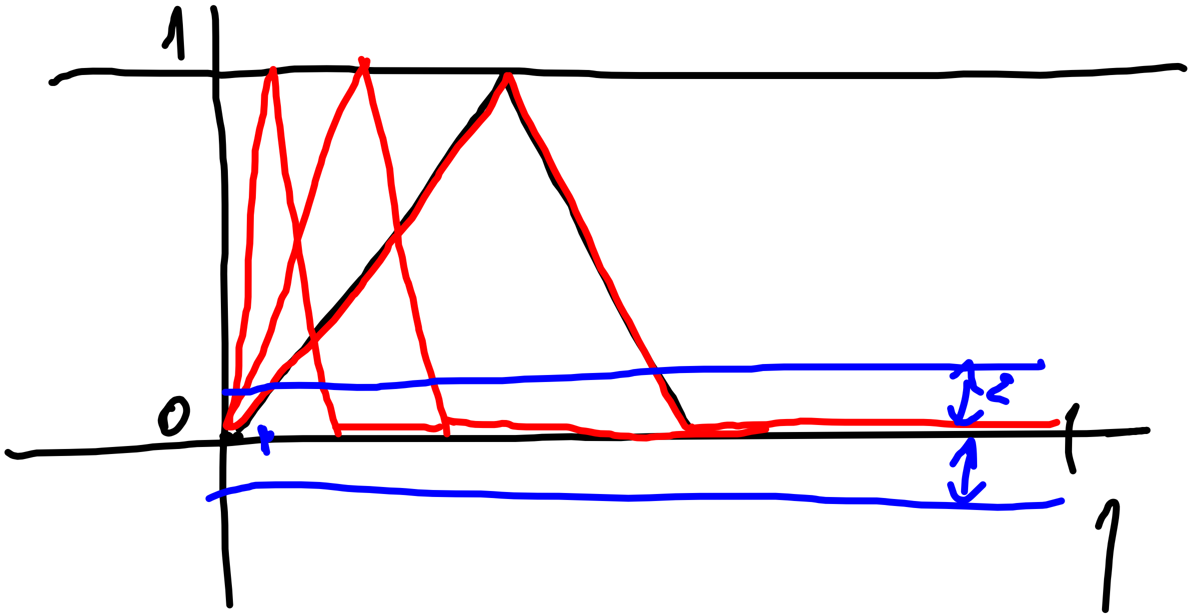
$$f'(x) = -3e^{-3x}$$

$$f^{(2)}(x) = 9e^{-3x}$$

$$\vdots$$

$$f^{(k)}(x) = (-1)^k 3^k e^{-3x} \qquad f^{(k)}(0) = (-1)^k 3^k$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n!} x^n$$


1
0
1

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

$$\int \frac{1}{e^x}\,dx = \int e^{-x} = -e^{-x}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty}\int_0^\infty \frac{\cos\left(\frac{x}{n}\right)}{\left(1+\frac{x}{n}\right)^n}\,dx =$$

$$= \int_0^\infty \lim_{n\to\infty}\frac{\cos\left(\frac{x}{n}\right)}{\left(1+\frac{x}{n}\right)^n} = \int_0^\infty \frac{1}{e^x}\,dx$$

$$= \lim_{\delta\to\infty}\int_0^\delta e^{-x}\,dx = \lim_{\delta\to\infty}\left[-e^{-x}\right]_0^\delta =$$

$$= -\lim_{\delta\to\infty} e^{-\delta} + 1 = 1$$

$$\int_a^b f(x)\cdot g(x)\,dx$$

$$l_1 = \sin(x)$$

$$l_2 := \sin(x) + k\cdot\cos(x) \quad , k \in R$$

k urăme tak, aby $l_1 \cdot l_2 = 0$

$$\int_0^{\frac{\pi}{2}} \sin(x)\left(\sin(x) + k\cdot\cos(x)\right) =$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x) + k\int_0^{\frac{\pi}{2}} \sin(x)\cos(x)$$

$$\int \sin^2 x = -\cos x \sin x + \int \cos^2(x) = -\cos x \sin x + x - \int \sin^2 x \Rightarrow$$

$$\Rightarrow \int \sin^2 x = \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right)$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x = \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin(2x)\, dx = -\frac{1}{4}\left[\cos(2x)\right]_0^{\frac{\pi}{2}} =$$

$$= -\frac{1}{4}\left[-1-1\right] = \frac{1}{2}$$

$$\frac{\pi}{4} + \ell \cdot \frac{1}{2} = 0 \Rightarrow \ell = -\frac{\pi}{2}$$

$$\Rightarrow \ell_2 = \sin(x) - \frac{\pi}{2}\cos(x)$$

$$\|e_1\|^2 = \int_0^{\frac{\pi}{2}} \sin^2 x = \frac{\pi}{4}$$

$$f_1 = \frac{2}{\sqrt{\pi}} \sin(x)$$

$$\|e_2\|^2 = \int_0^{\frac{\pi}{2}} \left(\sin x - \frac{\pi}{2}\cos x\right)^2 dx =$$

$$= \int_0^{\frac{\pi}{2}} \sin^2 x - 2\int_0^{\frac{\pi}{2}} \frac{\pi}{2} \sin x \cos x + \frac{\pi^2}{4}\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$= \frac{\pi}{4} - \pi \cdot \frac{1}{2} + \frac{\pi^2}{4}\left(\frac{\pi}{4}\right) = \frac{\pi^3}{16} - \frac{\pi}{4} = \frac{\pi^3 - 4\pi}{16}$$

$$f_2 = \frac{4}{\sqrt{\pi^3 - 4\pi}} \left(\sin(x) - \frac{\pi}{2} \cdot \cos x\right)$$

[prime] fce x do $\langle \sin(x), \cos(x) \rangle$ je

vektor $\dfrac{x \cdot \sin(x)}{\|\sin(x)\|^2} \cdot \sin(x) + \dfrac{x \cdot \left(\sin(x) - \frac{\pi}{2}\cos(x)\right)}{\|e_2\|^2} \cdot \left(\sin(x) - \frac{\pi}{2}\cos(x)\right)$

$$= (x \cdot f_1) \cdot f_1 + (x \cdot f_2) \cdot f_2$$

$$x \cdot \sin x = \int_0^{\frac{\pi}{2}} x \cdot \sin x = \left[-x\cos x + \int \cos x \right]_0^{\frac{\pi}{2}} =$$

$$= \left[-x\cos x + \sin x \right]_0^{\frac{\pi}{2}} = 1$$

$$x \cdot \left(\sin x - \frac{\pi}{2}\cos x\right) = \int_0^{\frac{\pi}{2}} x \sin x \, dx - \frac{\pi}{2} \int_0^{\frac{\pi}{2}} x \cos x \, dx =$$

$$= 1 - \frac{\pi}{2} \left[x \cdot \sin x - \int \sin x \right]_0^{\frac{\pi}{2}} = 1 - \frac{\pi}{2} \left[x \cdot \sin x + \cos x \right]_0^{\frac{\pi}{2}}$$

$$= 1 - \frac{\pi}{2}\left(\frac{\pi}{2} - 1\right) = -\frac{\pi^2}{4} + \frac{\pi}{2} + 1$$

· Kolmý průmět fce x do $\langle \cos(x), \sin(x) \rangle$ na $\langle 0, \frac{\pi}{2} \rangle$,

$$\frac{1}{\frac{\pi}{4}} \cdot \sin(x) + \frac{-\frac{\pi^2}{4} + \frac{\pi}{2} + 1}{\frac{\pi^3 - 4\pi}{16}} \cdot \left(\sin(x) - \frac{\pi}{2}\cos x\right) =$$

$$= \frac{4}{\pi} \cdot \sin(x) + \frac{-4\pi^2 + 8\pi + 16}{\pi^3 - 4\pi} \cdot \left(\sin(x) - \frac{\pi}{2}\cos x\right)$$

$$\text{Vzdálenost} = \left\| x - \frac{4}{\pi} \cdot \sin(x) - \frac{-4\pi^2 + 8\pi + 16}{\pi^3 - 4\pi}\left(\sin(x) - \frac{\pi}{2}\cos x\right)\right\|$$