1) $x = e^{\ln x}$ Definiční obor je $(0,\infty)$

$$x^x = \left(e^{\ln x}\right)^x = e^{x\cdot \ln x}$$

$$(x^x)' = \left(e^{\overbrace{x\cdot \ln x}^{y(x)}}\right)' = (x\cdot \ln x)'\left(e^{\overbrace{x\cdot \ln(x)}^{y(x)}}\right) =$$

$$= (\ln x + 1)(x^x)$$

Intervaly monotónie:

$$f'(x) <> 0 \ ?$$

fce je rostoucí na intervalu $(\frac{1}{e},\infty)$, klesající na $(0,\frac{1}{e})$

$$(\ln x + 1)(\cancel{x^x}) > 0 \iff (\ln x + 1) > 0 \iff$$

$$\iff \ln x > -1 \iff$$

$$x > \frac{1}{e}$$

$$x^{x^x} = \left(e^{\ln x}\right)^{x^x} = e^{x^x \cdot \ln x}$$

$$\left(e^{\overbrace{x^x \cdot \ln x}^{y(x)}}\right)' = y'(x) \cdot e^{y(x)} = \left(x^x \cdot \ln x\right)' \cdot x^{x^x} =$$

$$= \left[x^x(\ln x + 1)\ln(x) + x^{x-1}\right] \cdot x^{x^x}$$

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$$x^x$$

$$a^b = e^{b \cdot \ln a}$$

Funkce je složená např. ze dvou polynomů:

a)
$$f(x) = \begin{cases} x^3 + x^2 + x + 1, & \text{pro } x \leq 0 \\ 3x^3 + x^2 + x + 1, & \text{pro } x \geq 0 \end{cases}$$

$$x^{\frac{12}{5}}$$

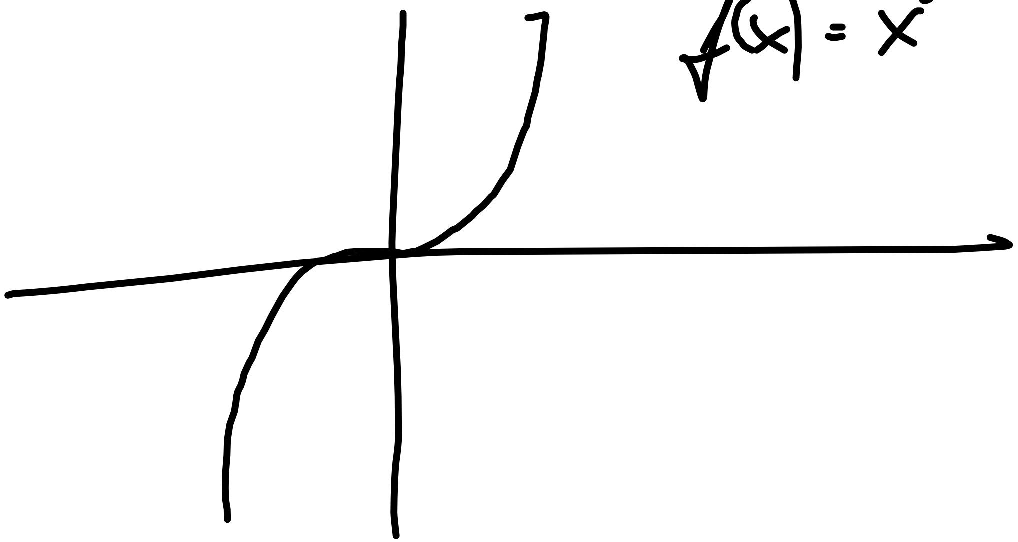
$$\left(x^{\frac{12}{5}}\right)' = \frac{12}{5} x^{\frac{7}{5}}$$

$$\frac{12 \cdot 7}{25} x^{\frac{2}{5}}$$

$$\frac{12 \cdot 7 \cdot 2}{5^3} \cdot \frac{1}{x^{\frac{3}{5}}}$$

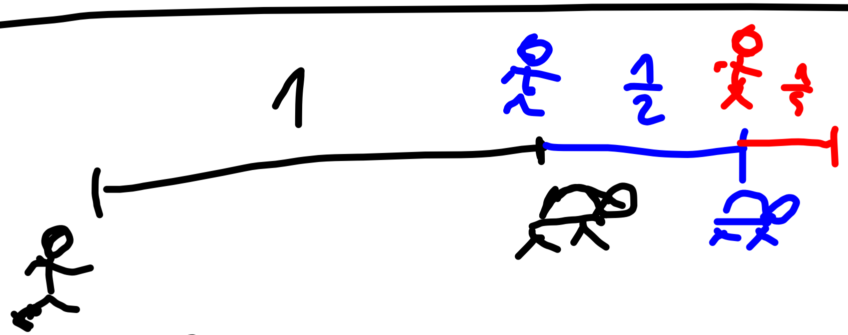
b)

$$f(x) = x^3$$



$$\sum_{n=1}^{\infty} a_n = \lim_{j\to\infty} (a_1+a_2+\dots+a_j) =$$

$$= \lim_{j\to\infty} \sum_{i=1}^{j} a_i$$




$$\sum_{n=0}^{\infty}\frac{1}{2^n} = \lim_{j\to\infty}\left(1+\frac{1}{2}+\dots+\frac{1}{2^j}\right) =$$

$$= \lim_{j\to\infty}\frac{\frac{1}{2^{j+1}}-1}{\frac{1}{2}-1} = \lim_{j\to\infty}\frac{1-\frac{1}{2^{j+1}}}{\frac{1}{2}} = 2$$

$$S = a + aq + aq^2 + \dots + aq^n$$

$$qS = aq + aq^2 + aq^3 + \dots + aq^{n+1}$$

$$S(q-1) = aq^{n+1} - a \Rightarrow S = a\frac{q^{n+1}-1}{q-1}$$

Podílové kritérium:

Jestliže ex. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = q < 1$, pak

řada $\sum_{n=1}^{\infty} a_n$ konverguje

$q > 1$, diverguje

Odmocninové

$\exists \lim_{n\to\infty} \sqrt[n]{|a_n|} = q < 1$

$\sum_{n=1}^{\infty} a_n$ konverguje

$q > 1$ diverguje

Srovnávací:

$\sum_{n=1}^{\infty} |a_n|$ konverguje

$\exists n_0 : \forall n \geq n_0 : |b_n| \leq |a_n| \Rightarrow \sum_{n=1}^{\infty} |b_n|$ konverguje

$$\sum_{i=1}^{\infty} \frac{1}{n} = 1 + \underbrace{\frac{1}{2}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{\geq \frac{1}{2}} + \dots$$

$\sum_{i=1}^{\infty} \frac{1}{n^2}$ konverguje

Podílové kritérium:

k. ř.: $\lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+1} = 1$ !

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+1} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{n^{\frac{1}{n}}} = \lim_{n\to\infty} \frac{1}{e^{\frac{1}{n}\ln n}} = 1$$

$$\lim_{n\to\infty} \frac{1}{n} \ln(n) = \lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \frac{1}{n} = 0$$

---

$\varphi$

$f, g : \mathbb{R} \to \mathbb{R}$, diferencovatelné na okolí bodu $x_0$. Pak

jestliže ex. $\lim\limits_{x \to x_0} \frac{f'(x)}{g'(x)}$, pak ex. $\lim\limits_{x \to x_0} \frac{f(x)}{g(x)}$

a jejich hodnoty jsou si rovny.

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Nutná podmínka pro to, aby $\sum\limits_{n=1}^{\infty} a_n$ konvergovala

je $\lim\limits_{n \to \infty} |a_n| = 0$

1, $\lim\limits_{n \to \infty} a_n = \lim\limits_{n \to \infty} \frac{2^n}{n^2} > 0 \Rightarrow$ řada diverguje

2, podílové kritérium: $\lim\limits_{n \to \infty} \frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n \to \infty} \sqrt{\frac{n}{n+1}} = 1$

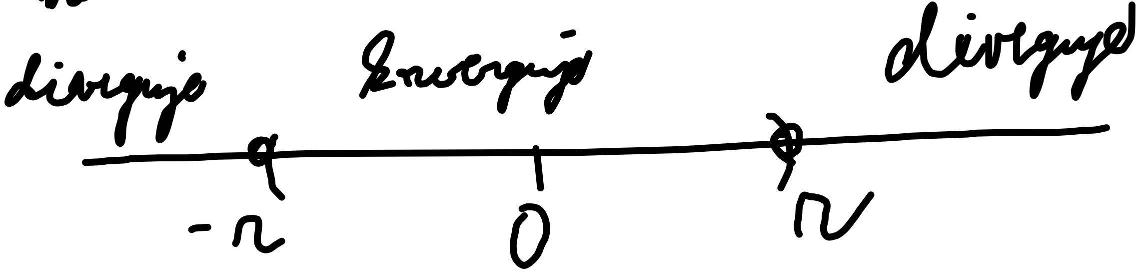
$\forall n \in \mathbb{N}: \sqrt{n} \le n \iff \frac{1}{\sqrt{n}} \ge \frac{1}{n}$

Na základě srovnávacího kritéria s harmonickou řadou diverguje

4) jde o geometrickou řadu s kvocientem $\frac{1}{2+i}$.

$$\left|\frac{1}{2+i}\right| < 1$$

$$\sum_{n=0}^{\infty} a_n x^n$$



Pro jaká x konverguje

$$\sum_{n=1}^{\infty} \sqrt{\frac{1}{n}}\, x^n$$

$$\lim_{n\to\infty} \sqrt[n]{\sqrt{\frac{1}{n}}}\, x < 1 \iff x < \frac{1}{\lim\limits_{n\to\infty} \frac{1}{n^{\frac{1}{2n}}}} = \frac{1}{1} = 1$$

$x = 1$ ... diverguje (srovnání s harm. řadou)

$x = -1$ ... konverguje (Leibn. kritérium)

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(2)

$$R = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{\frac{2^n}{n^2}}} = \frac{1}{\lim\limits_{n\to\infty} \frac{2}{n^{\frac{2}{n}}}} = \frac{1}{2}$$

řada konverguje pro $|x| < \frac{1}{2}$

$$\frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{a_n}} = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{1}} = 1$$

$$r = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}} = \frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{\left|\frac{1}{(2+i)^n}\right|}} = \frac{1}{\left|\frac{1}{(2+i)}\right|} =$$

$$= \sqrt{5}$$