$$\sum_{n=0}^{\infty}\left(\frac{1}{(4i)^n}-\frac{1}{5^{n+1}}\right)=$$

$$=\sum_{n=0}^{\infty}\frac{1}{(4i)^n}-\sum_{n=0}^{\infty}\frac{1}{5^{n+1}}=$$

$$=\frac{1}{1-\frac{1}{4i}}-\frac{1}{5}\sum_{n=0}^{\infty}\frac{1}{5^n}=$$

$$=\frac{1}{\frac{4i-1}{4i}}-\frac{1}{5}\,\frac{1}{1-\frac{1}{5}}=\frac{4i}{4i-1}-\frac{1}{4}=$$

$$=\frac{4i(4i+1)}{(4i-1)(4i+1)}-\frac{1}{4}=\frac{-16+4i}{-16-1}-\frac{1}{4}=$$

$$\frac{16}{17} - \frac{1}{4} - \frac{4i}{17} = \frac{47}{68} - \frac{4}{17}i$$

$\sqrt{n} \le n \quad (n \in \mathbb{N})$

1) $$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \ge \sum_{n=1}^{\infty} \frac{1}{n+n} = \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

diverguje

2) $$\sum_{n=1}^{\infty} \overbrace{\sqrt{\frac{n^3+4n}{n^5-5n^2-1}}}^{a_n} \ge \sum_{n=1}^{\infty} \sqrt{\frac{n^3}{n^5}} = \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

$$\sqrt{\frac{n^3+5n}{n^5-5n^2-1}} > \frac{1}{n}$$

$$\frac{n^3+5n}{n^5-5n^2-1} >$$

n^5+5n^3 $\quad 2n^5-10n^2-2$

C $\quad n^5-5n^3-10n^2-2$

pro velká n

$$\rho = \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}}$$

1) $\rho = \frac{1}{2008}$

2) $\lim\limits_{n\to\infty} \sqrt[n]{2008} = 1 \qquad \rho = 1$

3) $\lim\limits_{n\to\infty} \sqrt[n]{\frac{n-1}{n^{2n}}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n-1}}{n^2} \leq \lim\limits_{n\to\infty} \frac{n}{n^2} = 0$

$\rho = \infty$

$$\sum_{n=0}^{\infty} n!\, x^n$$

$$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$$

$$\rho = 0$$

$$\lim_{n\to\infty}(n-4\sqrt{n}) = \lim_{n\to\infty}\frac{(n-4\sqrt{n})(n+4\sqrt{n})}{n+4\sqrt{n}} =$$

$$= \lim_{n\to\infty}\frac{n^2-16n}{n+\sqrt{n}} = \lim_{s\to\infty}\frac{s^4-16s^2}{s^2+s}$$

subst. $\sqrt{n}=s$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} =$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)} =$$

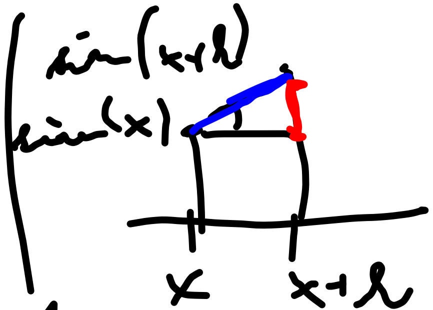
$$= \lim_{x\to 0}\frac{\sin^2 x}{x^2}\cdot\lim_{x\to 0}\frac{1}{1+\cos x} =$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\cdot\frac{1}{2} = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} \overset{l}{\underset{x\to 0}{=}} \frac{\sin x}{2x} = \lim_{x\to 0}\frac{\cos x}{2} = \frac{1}{2}$$

$$\sin'(x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} =$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin(x)}{h}$$



$$= \lim_{h\to 0}\frac{\cos x\sin h}{h} + \underbrace{\sin(x)\lim_{h\to 0}\frac{\cos h-1}{h}}_{0} =$$

$$= \cos x + \sin(x)\lim_{h\to 0}\frac{\cos^2 h-1}{h(\cos h+1)} = \cos x$$

$$\lim_{h\to 0}\frac{\cos h-1}{h} = \lim_{h\to 0}\frac{\cos^2 h-1}{h(\cos h+1)} = \lim_{h\to 0}\frac{\sin^2 h}{h(\cos h+1)} =$$

$$= \overbrace{\lim_{h\to 0}\frac{\sin h}{h}}^{1}\cdot\overbrace{\lim_{h\to 0}\frac{\sin h}{\cos h+1}}^{0}$$

$$\sum_{n=1}^{\infty} x^{n!} = \sum_{n=1}^{\infty} a_n x^n$$

$a_1 = 1$
$a_2 = 1$
$a_3 = 0$
$a_4 = 0$
$a_5 = 0$
$\vdots$

$\lim_{n\to\infty} \sqrt[n]{|a_n|}$ neexistuje

nicméně $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = 1$

$$\rho = \frac{1}{\limsup_{n\to\infty} \sqrt[n]{|a_n|}} = 1$$

řada ~~konverguje~~ pro $x \in (-1,1)$,
diverguje pro $x \in (-\infty,-1) \cup (1,\infty)$
Pro $x = 1$ diverguje $x = -1$ také diverguje

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = \sum_{k=1}^{\infty} a_k x^k$$

0

$a_1 = 0$

$a_2 = 0$

$a_3 = -\frac{1}{3}$

$a_4 = 0$

$a_5 = \frac{1}{5}$

$$\lim_{n \to 0} n \cdot \ln(n) = \lim_{n \to 0} \left(\ln(n) \cdot e^{\ln(n)} \right) = \lim_{n \to 0} \frac{n^2 \cdot \ln(n)}{n} = \lim_{n \to 0} \frac{2n\ln(n) + n}{1} = 2\lim_{n \to 0} (\ldots)$$

$$\lim_{k \to \infty} \sqrt[k]{|a_k|} = \text{neexistuje}$$

$$\limsup_{k \to \infty} \sqrt[k]{|a_k|} = \lim_{k \to \infty} \sqrt[k]{\frac{1}{k}} = 1$$

$$= \lim_{k \to \infty} e^{\ln\left(\frac{1}{k}\right) \cdot \frac{1}{k}}$$

a) $\rho = \dfrac{1}{\limsup\limits_{n\to\infty} a_n} = 1$

$x = 1$: $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{2n+1}$ ... konverguje (podle L. kritéria)

$x = -1$

$\lim\limits_{n\to\infty} |a_n| = 0$

$\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{2n+1}$ .. konverguje

$\sum\limits_{n=1}^{\infty} \binom{2n}{n} x^n$

$a_n = \dfrac{(2n)(2n-1)\dots(2n+1)}{n!}$

$\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} \geq \sqrt[n]{(n+1)!} = \infty$

podílové krit.

$\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \dfrac{\binom{2n+2}{n+1}}{\binom{2n}{n}} = \dfrac{(2n+2)(2n+1)}{n+1} \to \infty$

Řada konverguje pouze pro $x = 0$.