$$e^x = 1 + x + \frac{1}{2}x^2 + \dots + \frac{1}{k!}x^k + \dots$$

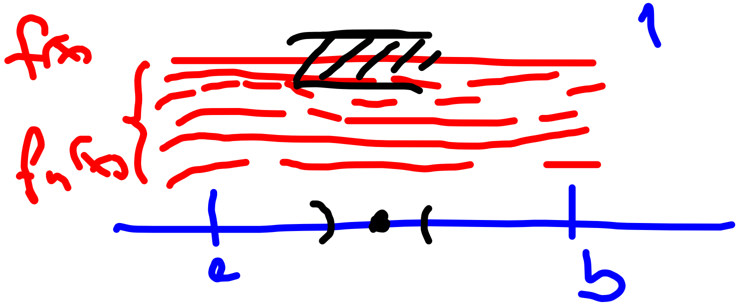
$$S(x) = \sum_{n=0} a_n (x - x_0)^n \quad \dots \text{ poloměr konvergence } \delta$$

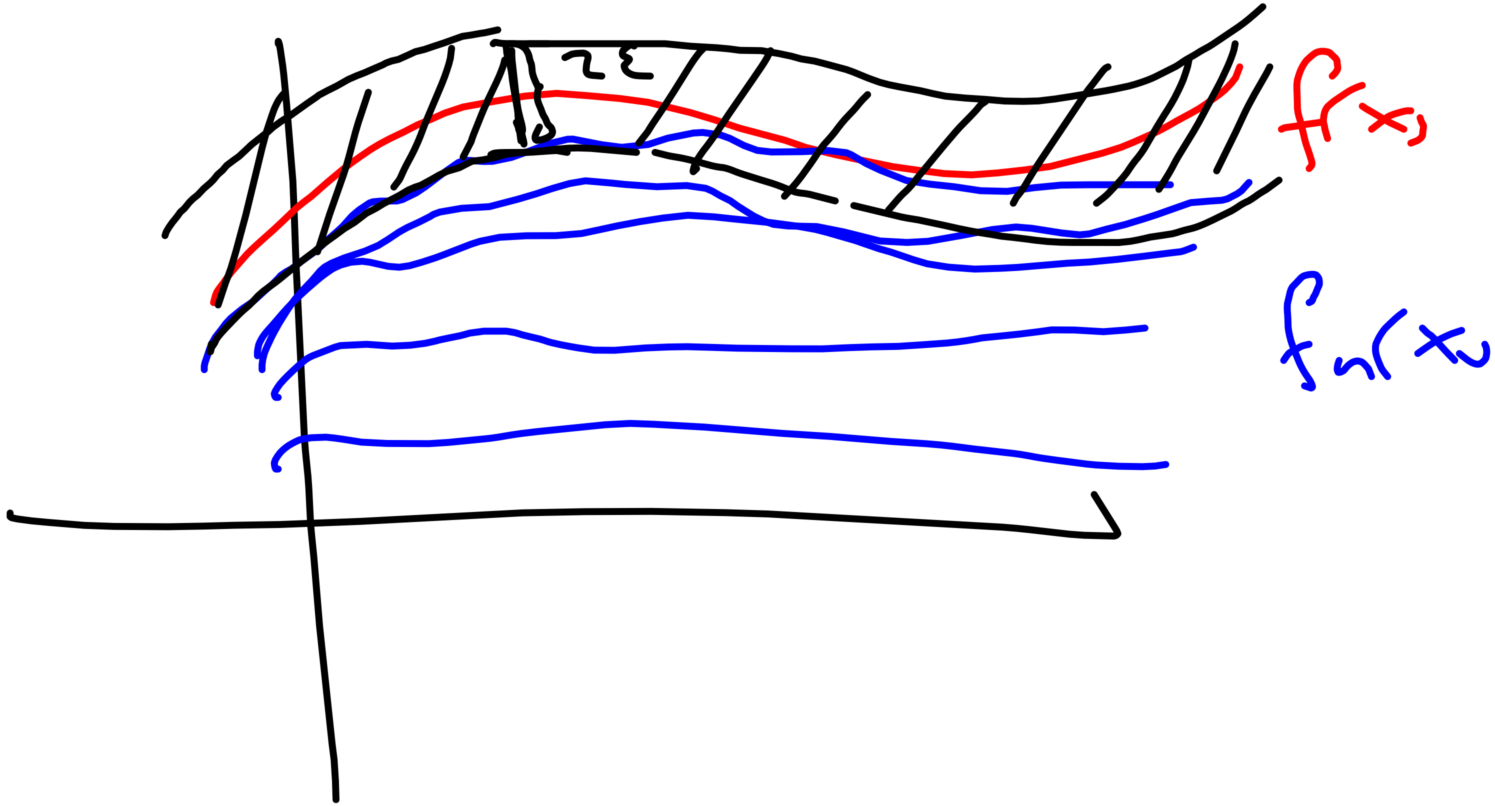
$$S(x) = \lim_{n \to \infty} s_n, \quad s_n = \sum_{i=0}^{n} f_i(x)$$

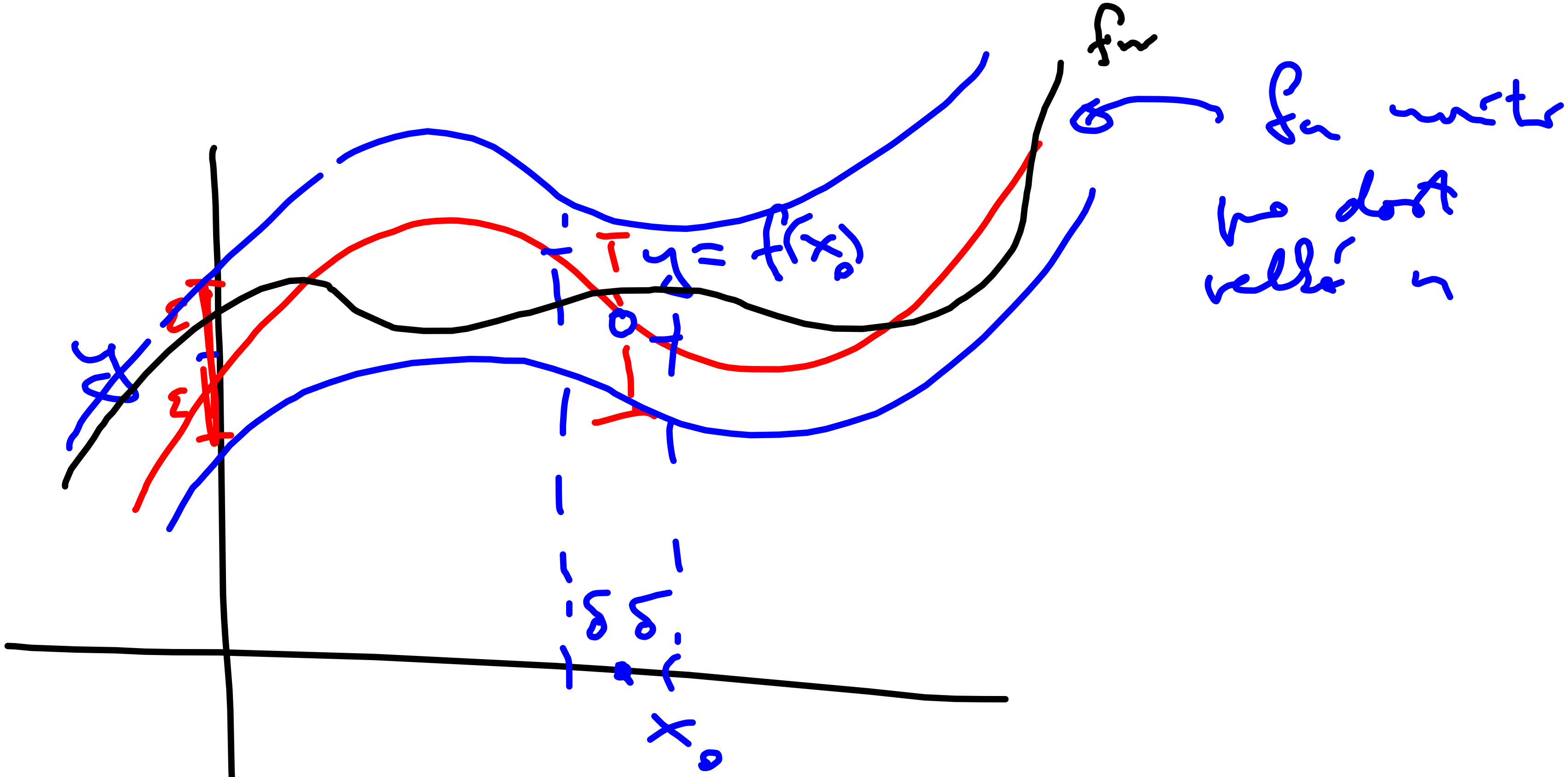
$$\chi(x) = \begin{cases} 0 & \text{pro } x \notin \mathbb{Q} \\ 1 & \text{pro } x \in \mathbb{Q} \end{cases}$$

$$\chi(x) = \lim_{n \to \infty} \chi_n(x)$$

„první n" čísel z $\mathbb{Q}$ 1
jinak 0.




2ε
f(x)
fn(x)



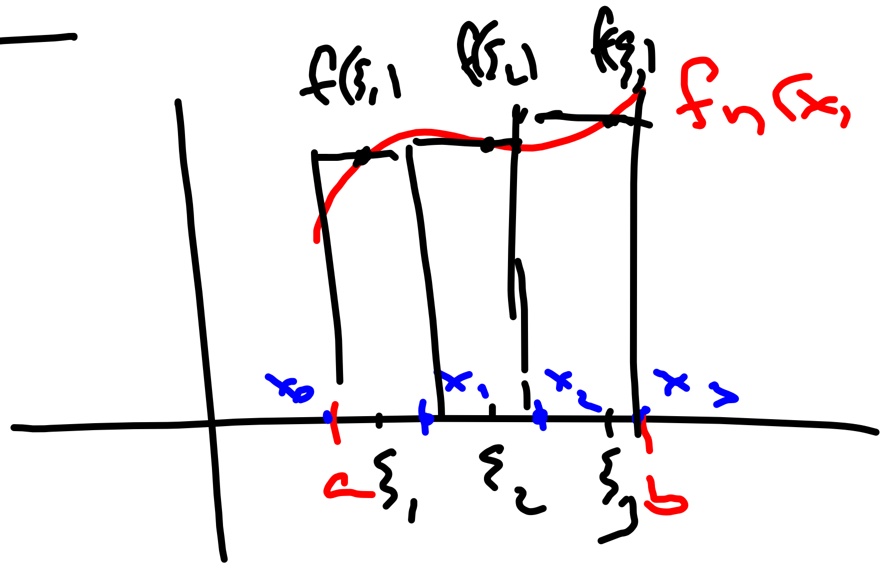
$$|f(x)-f(x_0)| = |f(x)-f_n(x)+f_n(x)-\dots|$$

$$\le |\cdot\ /\ \cdot| + |\cdot\ /\ \cdot| + |\cdot/\cdot|$$

$a_0, a_1, \ldots$

$$\boxed{\forall \varepsilon\ \exists n\ \forall m, k \geq n \quad |a_m - a_k| < \varepsilon}$$

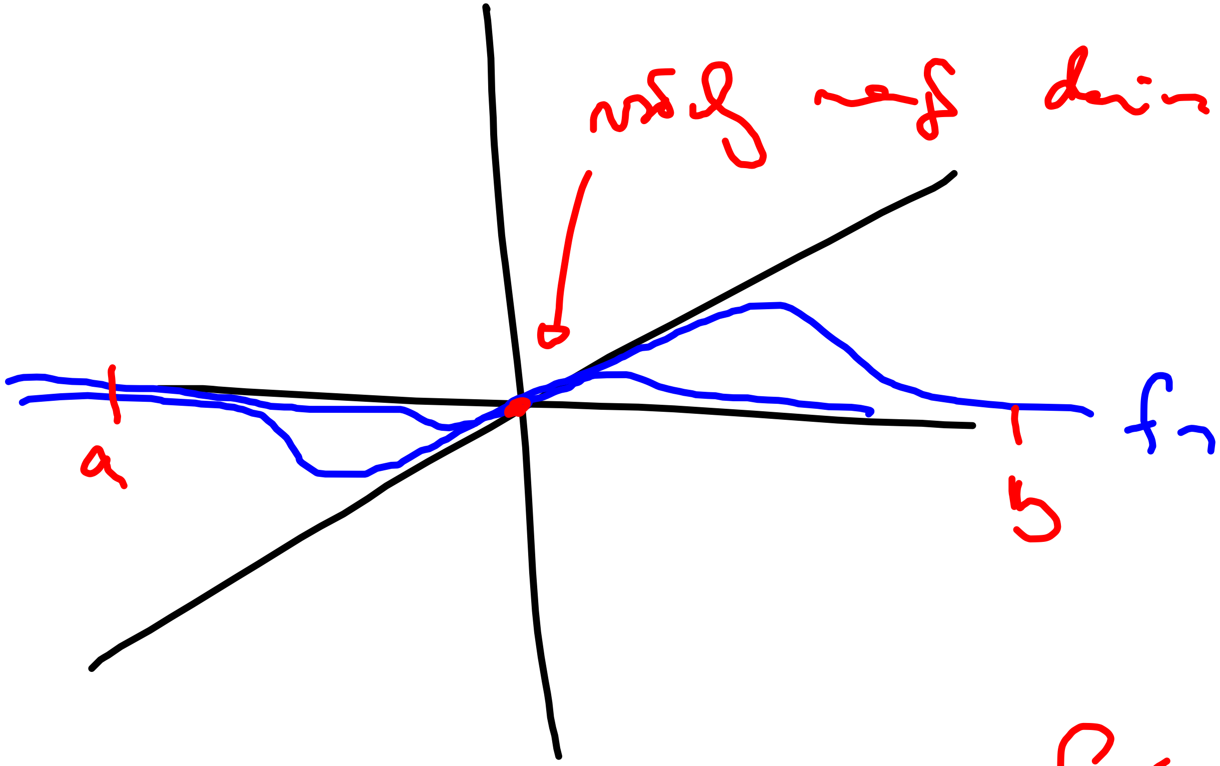
$$f_n(x) \xrightarrow{n \to \infty} f(x) \qquad \forall x \in [a, b]$$



$$|f_n(\xi) - f_m(\xi)|$$

$$< \varepsilon$$

ale g neg derivaci 1 v 0.



$$f_n \to f$$

$$g_n = f_n' \to g$$

$$f_n(x) = \int_a^x g_n(t)\,dt$$

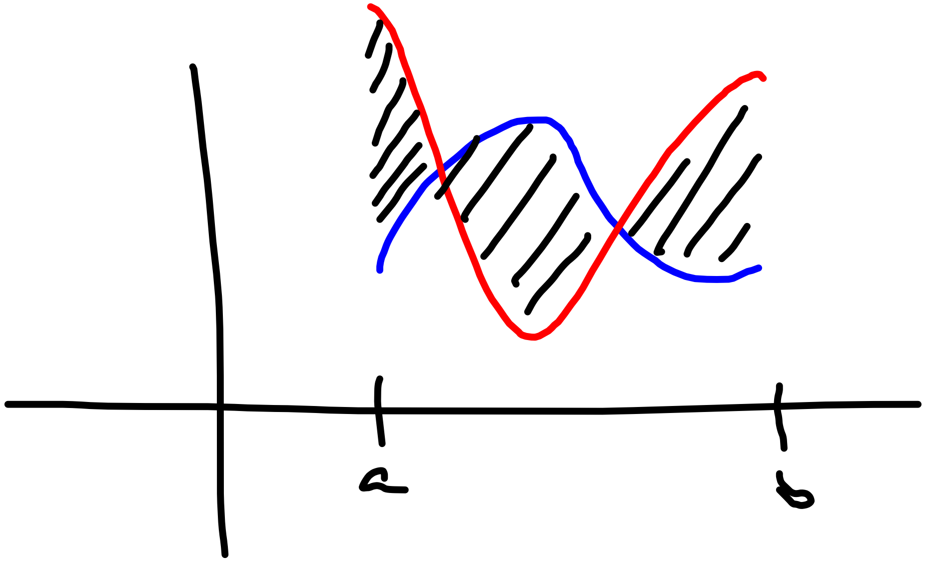
$$\downarrow \qquad \downarrow$$

$$f(x) = \int_a^x g(t)\,dt$$

$I = [a,b]$

$S = \{$spojité fce na $I\}$ (po částech)

vekt. prostor nad $\mathbb{R}$



$$\|f-g\|^2 = \int_a^b |f-g|\,dx$$

$$\langle f,g\rangle = \int_a^b f(x)\,g(x)\,dx$$

vektory

číslo

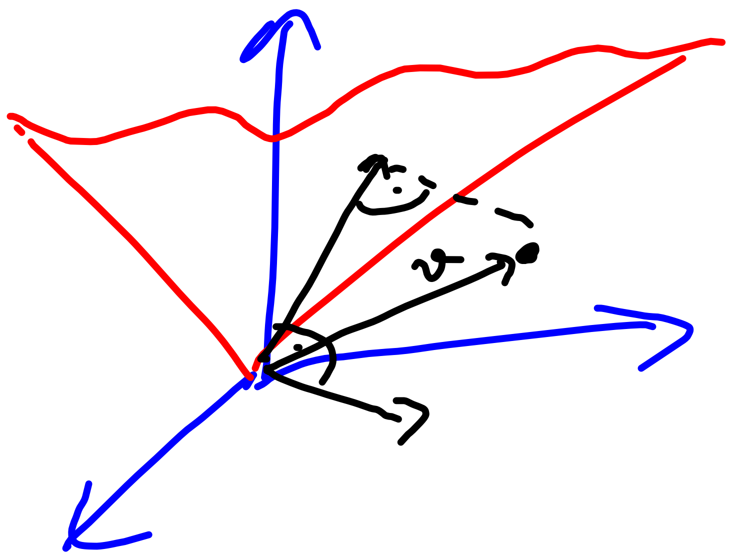
$$\int_a^b (f+\tilde{f})(x)\, g(x)\, dx = \int_a^b \left(f(x) g(x) + \tilde{f}(x) g(x)\right) dx$$

$$= \int_a^b f(x) g(x)\, dx + \int_a^b \tilde{f}(x) g(x)\, dx$$

$$\int_a^b |f(x)|^2\, dx = 0 \iff f \equiv 0$$

$$\Uparrow$$

$$f \in S$$



$$\mathbb{R}^k[x]$$

$$\langle x^k, x^l \rangle = \int_a^b x^{k+l}\, dx$$

$$= \left[\frac{1}{k+l+1} x^{k+l+1}\right]_a^b$$

$$g_2 = f_2 - \frac{\langle f_2, g_1 \rangle}{\| g_1 \|^2} \cdot g_1$$

$\langle f_2, g_1 \rangle = \int_{-1}^{1} x \, dx = \left[\frac{1}{2} x^2 \right]_{-1}^{1} = 0$

$f_2 = x$

$\| g_1 \|^2 = \int_{-1}^{1} 1 \, dx = 2$

$g_1 = 1$

$$\int_{-1}^{1} x^2 \, dx = \left[\frac{1}{3} x^3 \right]_{-1}^{1} = \frac{2}{3}$$