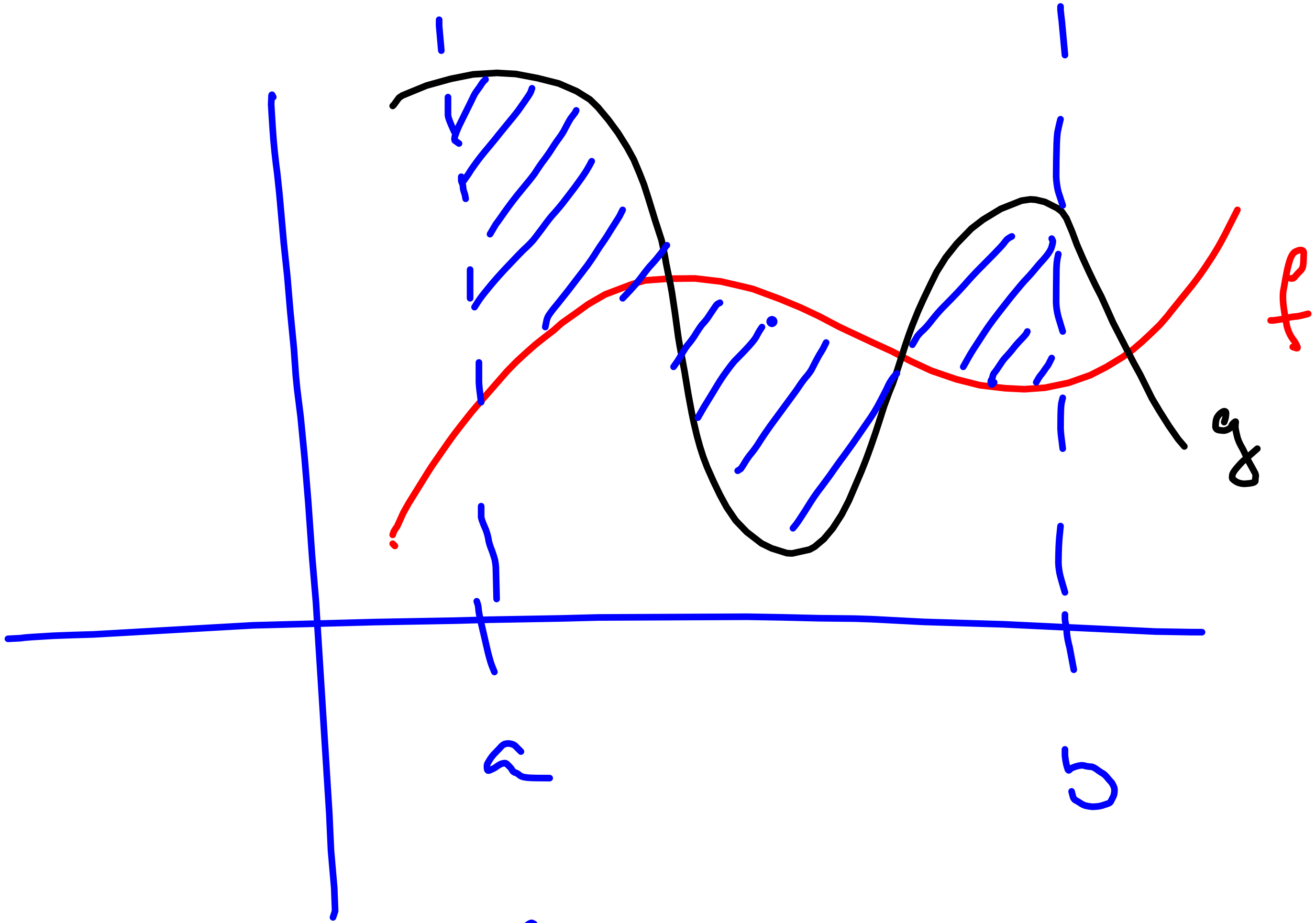
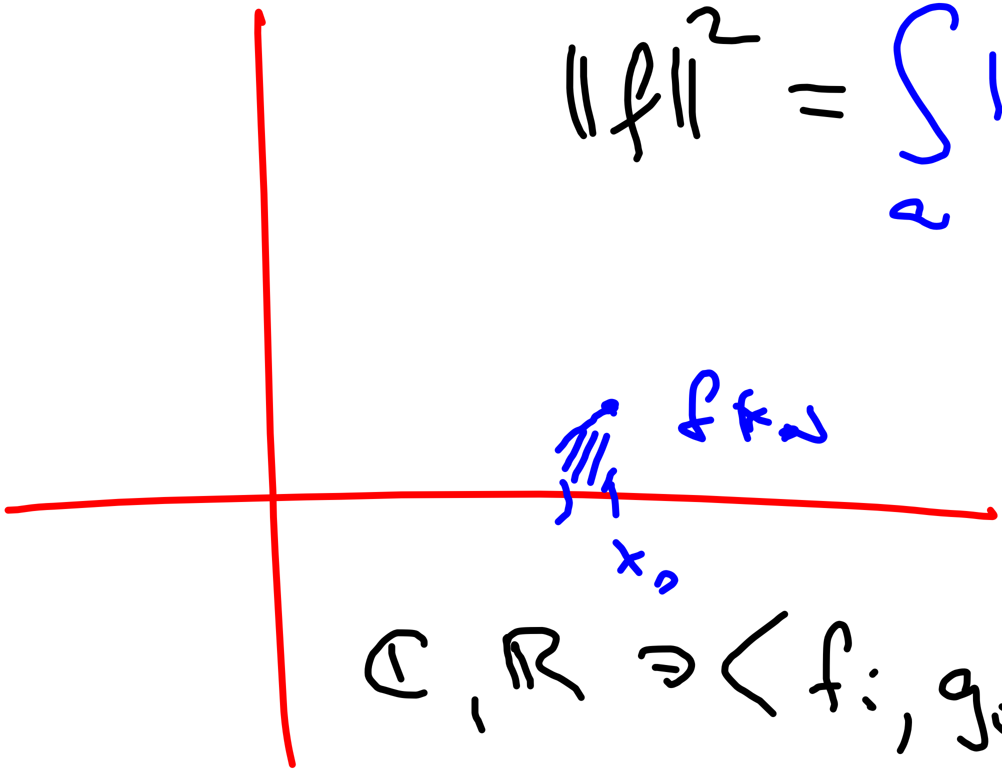


$$\|f-g\|^2 = \int_a^b |f(x)-g(x)|^2\,dx$$

$$\|f\|^2 = \int_a^b |f(x)|^2\,dx > 0$$



$$\mathbb{C}, \mathbb{R} \Rightarrow \langle f_i, g_j \rangle = \int_a^b f_i(x)\, g_j(x)\, dx$$

$$\begin{array}{ccc} \mathcal{F}[a,b] & \supset & \langle f_1, \dots, f_k \rangle = V \\ \cap & \cup & \langle \{f\} \cup V \rangle \\ f & & \end{array}$$

$h_1 = \sqrt{\frac{1}{2}} \qquad h_2 = \sqrt{\frac{3}{2}}\, x \qquad h_3 = \frac{1}{2}\sqrt{\frac{5}{2}}(3x^2 - 1)$

$h(x) \in S(-1, 1]$

$$H = \left(\int_{-1}^{1} h(x)\sqrt{\tfrac{1}{2}}\, dx\right)\sqrt{\tfrac{1}{2}} + \left(\int_{-1}^{1} \sqrt{\tfrac{3}{2}}\, x\, h(x)\, dx\right)\sqrt{\tfrac{3}{2}}\, x$$

$$+ \ \ldots$$

$$\| f(x) - \underset{q}{a} h_1 + b h_2 + c h_3 \|^2$$

$$F(x) = \sum_{n=1}^{\infty} c_n f_n$$

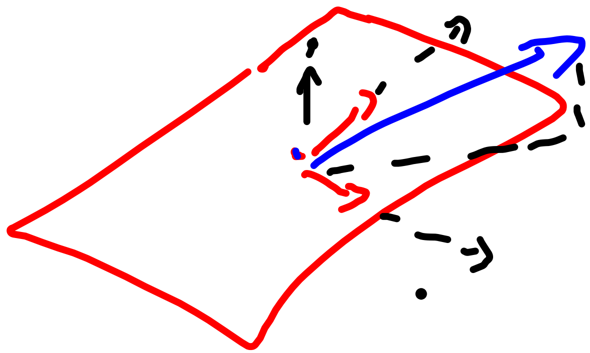
$\langle f_i, f_j \rangle = 0$

$i \neq j$

$\langle f_i, f_i \rangle = \|f_i\|^2$

$\neq 0$

$$c_n = \frac{1}{\|f_n\|^2} \int_a^b g(x) f_n(x)\, dx$$



$$\left\|g-\sum_{n=1}^{k} a_n f_n\right\|^2 = \int_a^b \left(g(x)-\sum_{n=1}^{k} a_n f_n(x)\right)^2 dx$$

$$= \int_a^b g^2(x)\,dx - 2\int_a^b g(x)\sum_{n=1}^{k} a_n f_n(x)\,dx + \int_a^b \left(\sum_{n=1}^{k} a_n f_n(x)\right)^2 dx$$

$$= \|g\|^2 - 2\sum_{n=1}^{k} a_n \underbrace{c_n \int dx}_{\|f\|^2} + \sum_{n=1}^{k} a_n^2 \|f\|^2$$

$$= \|g\|^2 + \sum_{n=1}^{k} \|f_n\|^2 \left(\underbrace{(c_n - a_n)^2}_{\geq 0} - c_n^2 \right)$$

konst

konst

jediný prostor

$\Rightarrow$ $c_n = a_n$ je minimum

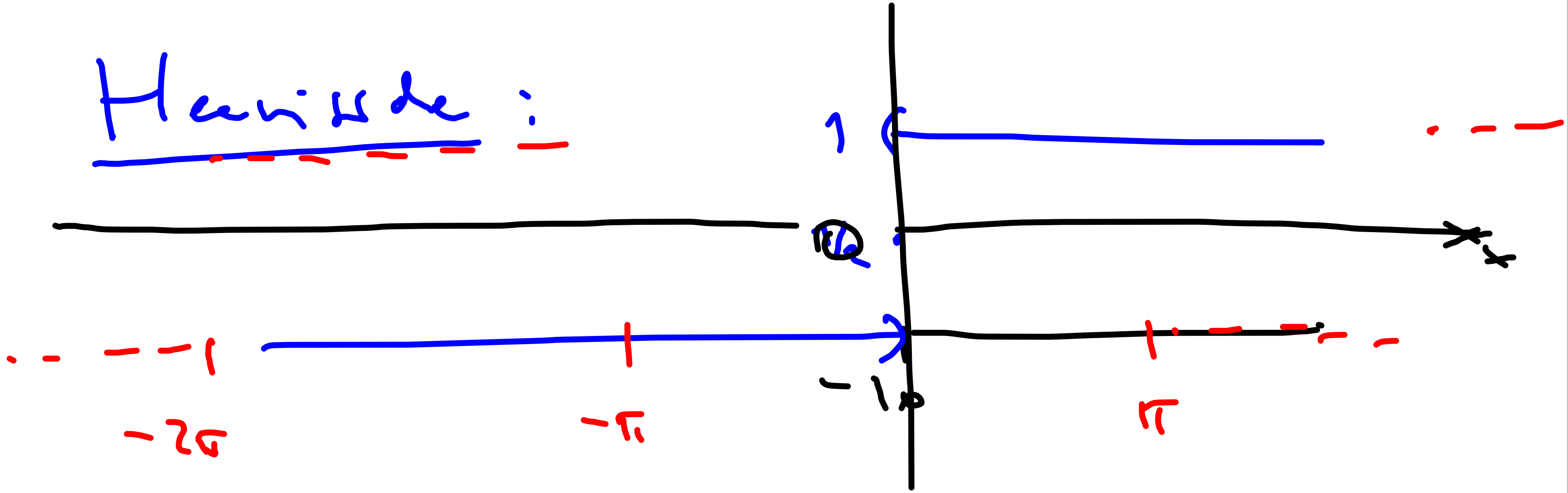
$$\int_{-\pi}^{\pi} 1\,dx = [x]_{-\pi}^{\pi} = \pi + \pi = 2\pi = \|1\|^2$$

$$\int_{-\pi}^{\pi} \sin x\,dx = [-\cos x]_{-\pi}^{\pi} = -1+1 = 0$$

$$\int_{-\pi}^{\pi} \sin^2 x\,dx = \dots = \pi$$

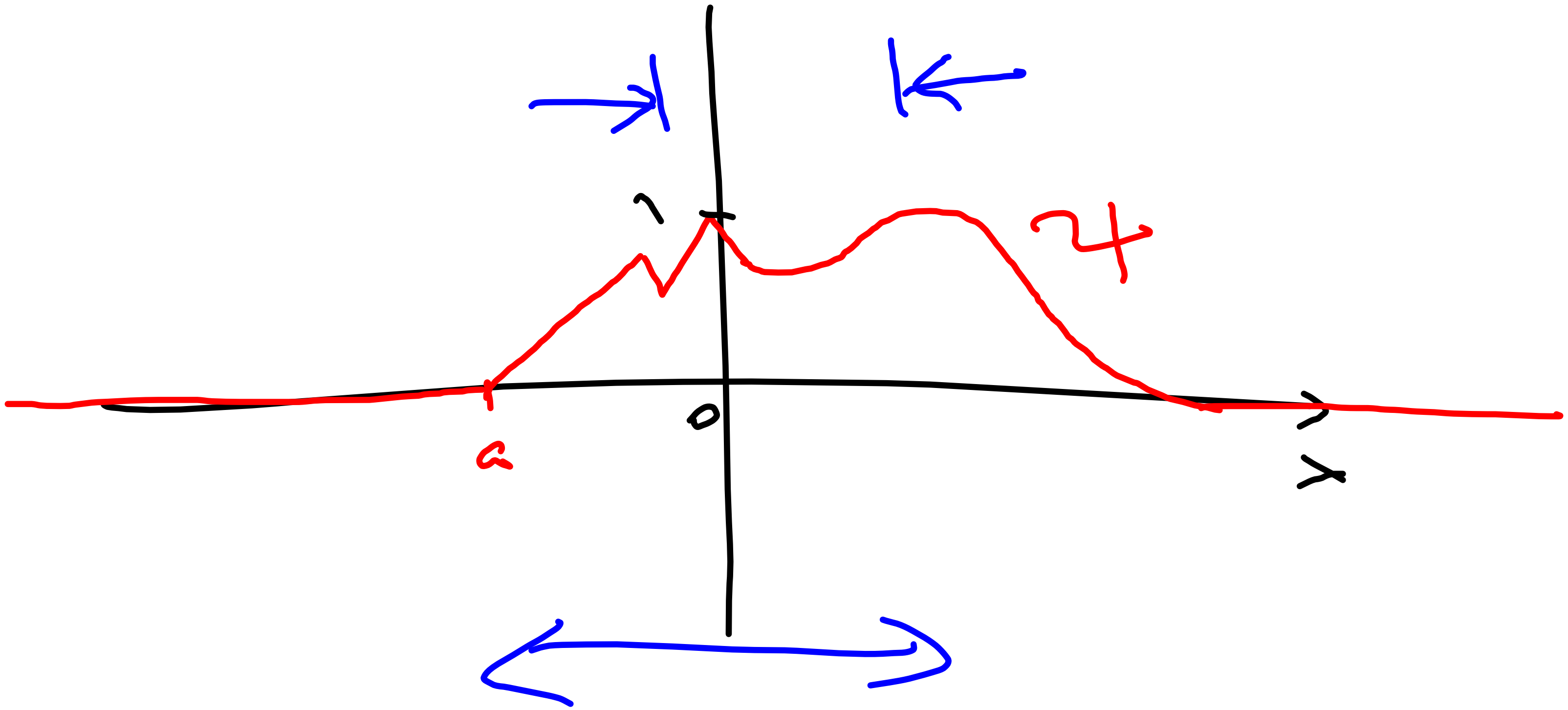
$$\int_{-\pi}^{\pi} \sin x \cos x\,dx = \dots = 0$$

Heaviside :



$$c_n = \frac{1}{\|\sin^2 nx\|^2} \int_{-\pi}^{\pi} g(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \cdot 2 \int_{0}^{\pi} \sin nx \, dx$$


1
0
a
2a
x