$\sqrt{2} = a$ Spor ↯

$2 = a^2 \stackrel{?}{=} p^2/q^2 \rightsquigarrow a^2 q^2 = p^2$

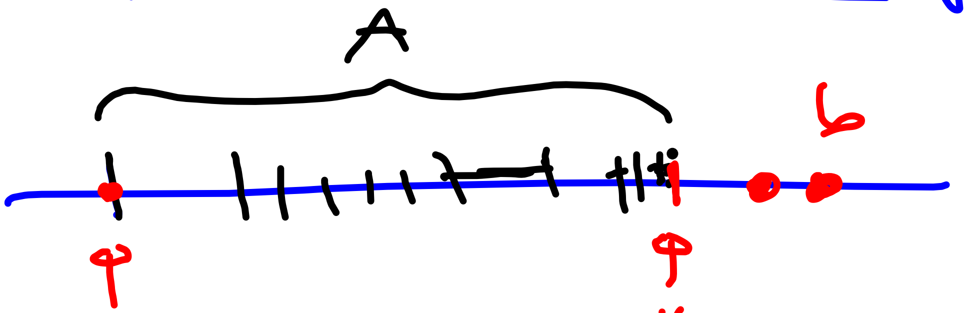
$a^2 q^2 = 2\cdot q^2$

uspořádání!

NENÍ na $\mathbb{C}$ (úplné)

	(R1) – (R9)	(R10) – (R12)	R13
$\mathbb{Q}$	✓	✓	—
$\mathbb{R}$	✓	✓	✓
$\mathbb{C}$	✓	—	—

Supremum × Infimum



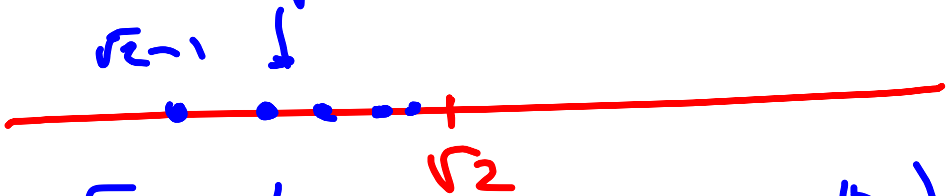
$x \geq b \Rightarrow x \geq a$
$\forall a \in A$

infimum

"nejmenší horní závora"
= supremum

Pro $\mathbb{Q}$ neplatí, že každá ohraničená $A \subset \mathbb{Q}$ má supremum!

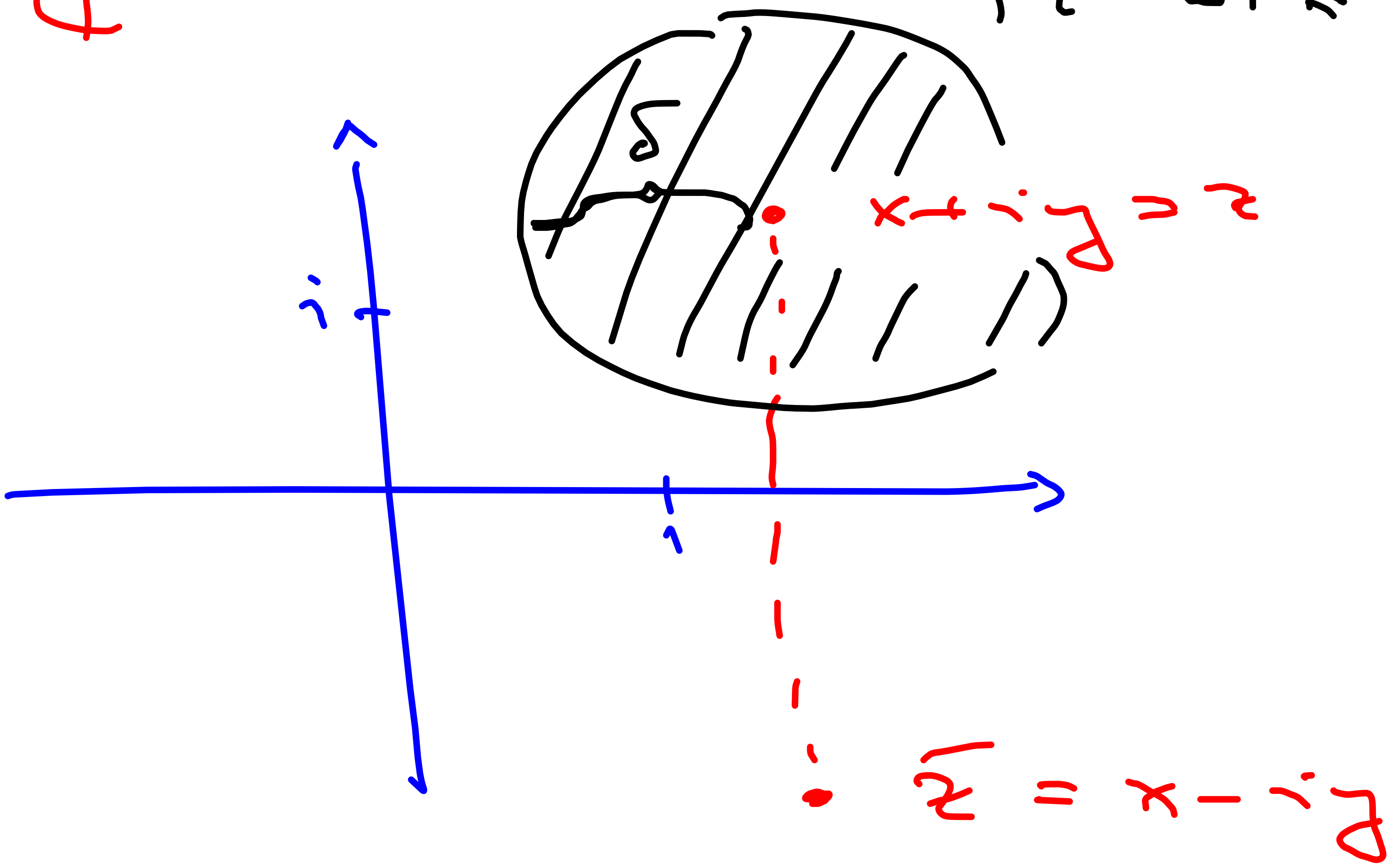
aproximace z $\mathbb{Q}$



$\sqrt{2} - 1/n$ pro $n \in \mathbb{N} \setminus \{0\}$

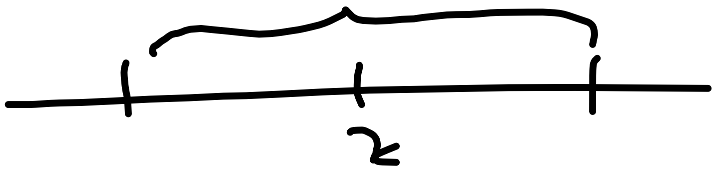
– $\mathbb{C}$

$|z - w| \leq \delta$

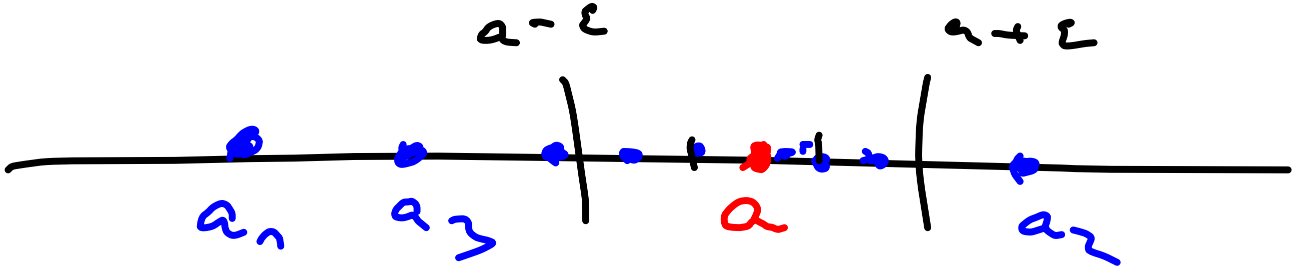


$\Rightarrow$ $|z|$ absolutní hodnota

– $\mathbb{R}$



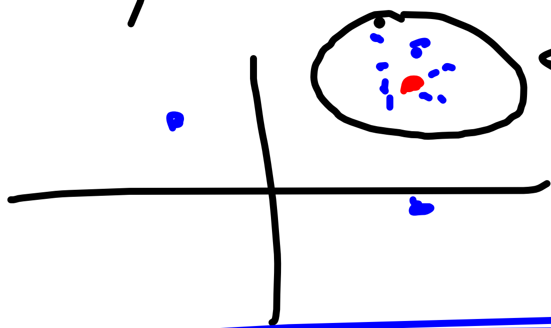
$|z - w| \leq \delta$



$\varepsilon > 0$

$\varepsilon \in \mathbb{R}$

$\forall \varepsilon$, $\forall a_i$ až na konečně mnoho uvnitř



$|z - a| \leq \varepsilon$

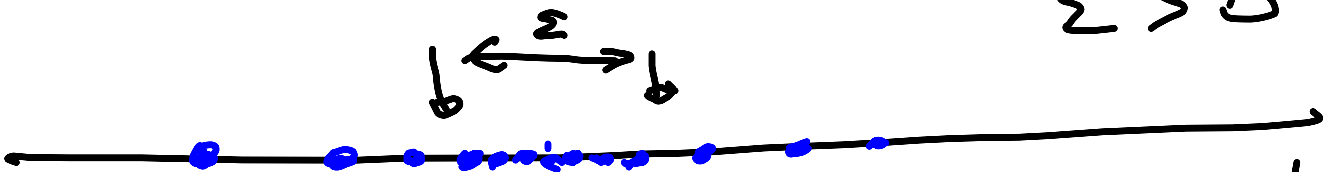
$\mathbb{C}$

KONVERGENCE

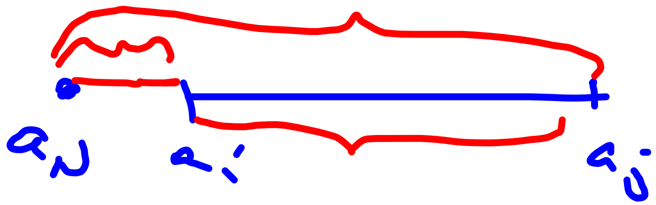
Cauchyovská posloupnost:

$\varepsilon \in \mathbb{R}$

$\varepsilon > 0$



$|a_i - a_j| < \varepsilon \quad \forall\, i, j \geq n_\varepsilon \in \mathbb{N}$



Hromadné body:

$a_1, a_2, \ldots \in A \subset \mathbb{Q}, \mathbb{R}, \mathbb{C}$

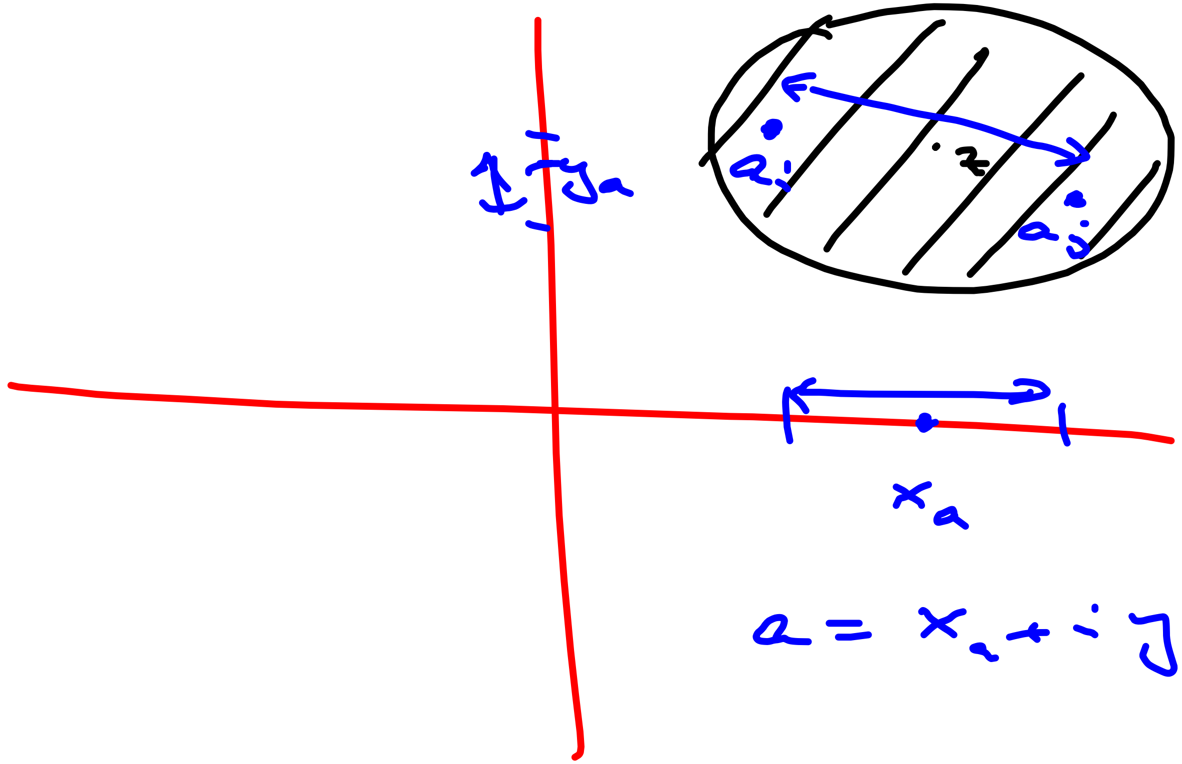
a_i konvergují k $a \in \mathbb{Q}, \mathbb{C}, \mathbb{R}$

$\Rightarrow a$ hromadný bod množiny A

např. $A = \{\sqrt{2} - 1/n,\ n \in \mathbb{N} - \{0\}\}$

$\sqrt{2}$ je hromadný bod A v $\mathbb{R}$

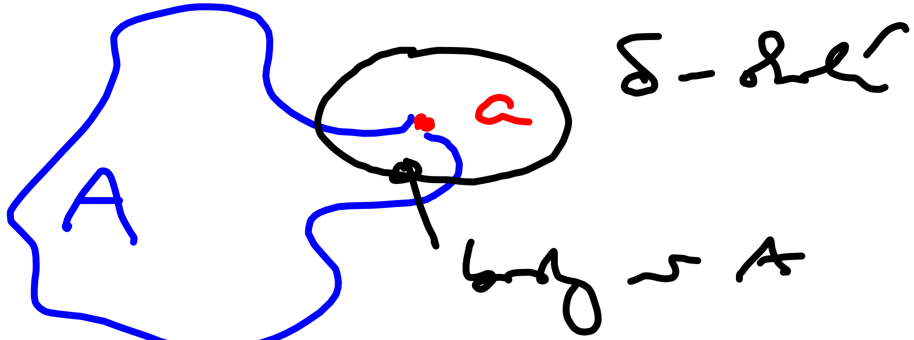
v $\mathbb{C}$



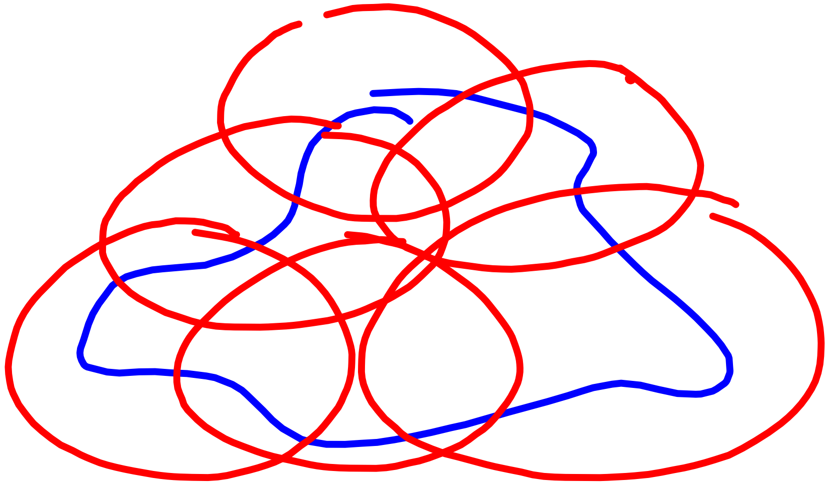
$|z - w| \le 2$

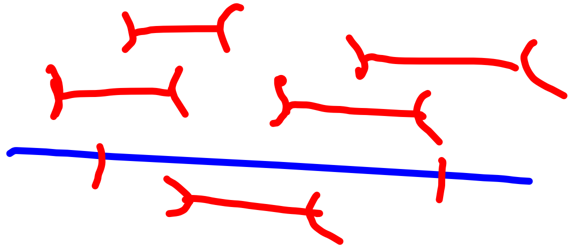
$a = x_a + i\, y_a \leftarrow$ limita

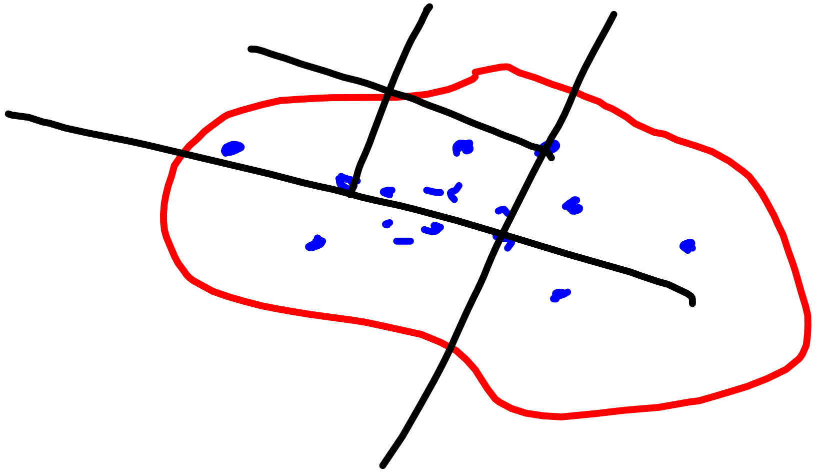
posloupnost v $\mathbb{C}$ konverguje $\Leftrightarrow$ když re a_i a im a_i konvergují v $\mathbb{R}$.

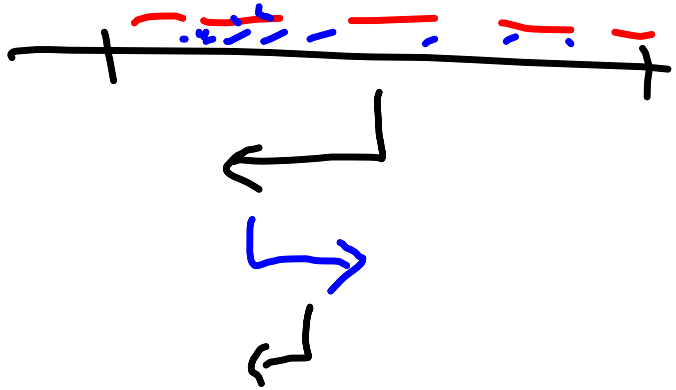


← Hraniční a









$\to$ vybrané posl.

Cauchyovské

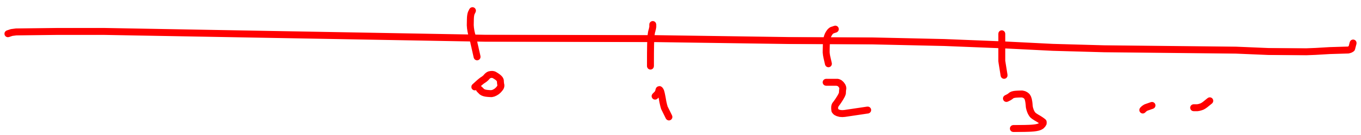
$\Rightarrow$ limita v $\mathbb{R}$

$A \subset \mathbb{R}$ $f: A \to \begin{cases} \mathbb{R} \\ \mathbb{C} \end{cases}$

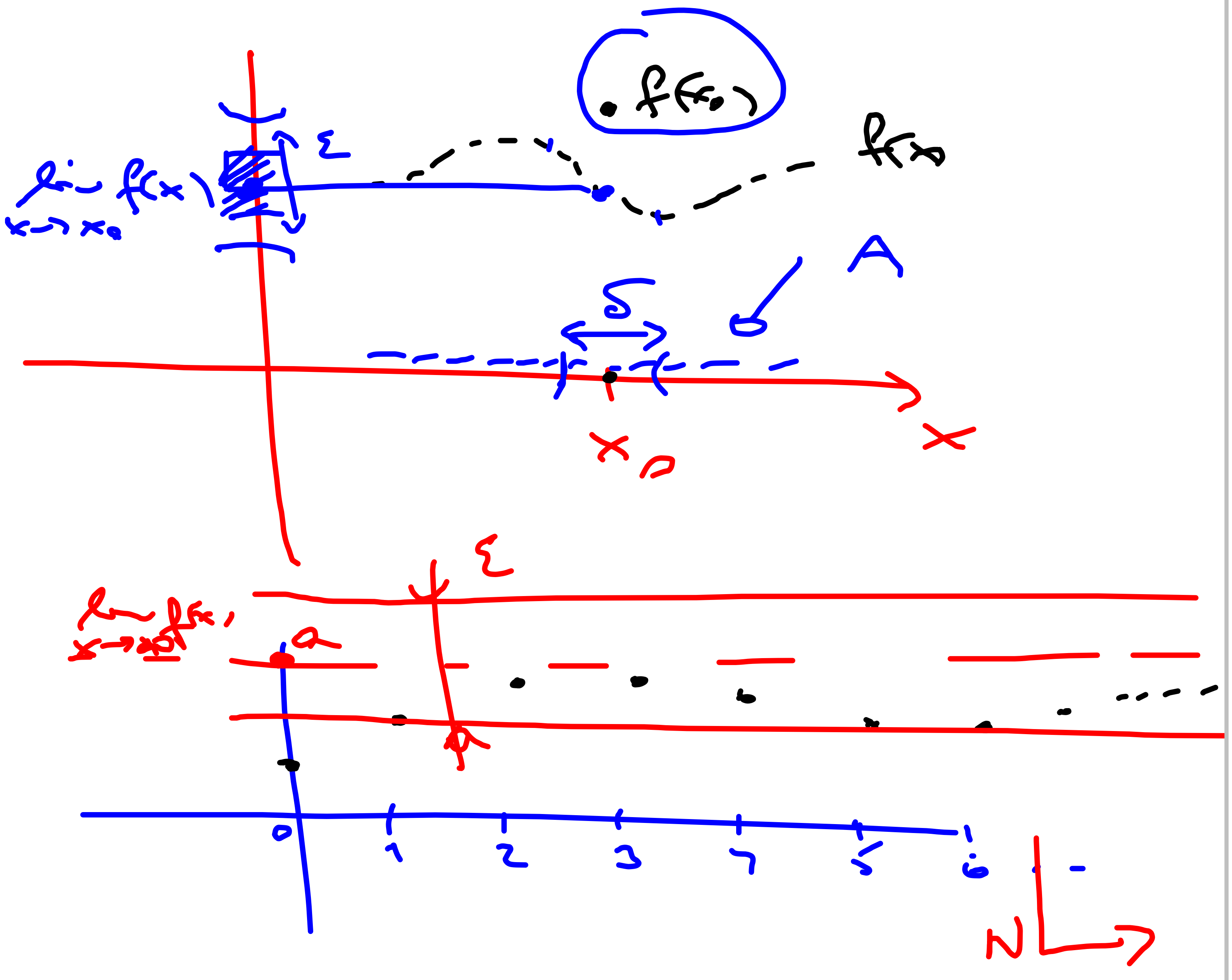
speciálně $A = \mathbb{N}$ dává opět posloupnosti:

$f(0), f(1), \ldots$

$a_0, a_1, \ldots, a_i, \ldots$ $i \to \infty$



$\lim_{x \to \infty} f(x)$ $\leftarrow$ limita posloupnosti


lim f(x)
x→x0
ε
f(x0)
f(x)
A
δ
x0
x
lim f(x)
x→x0
a
ε
0
1
2
3
4
5
6
N

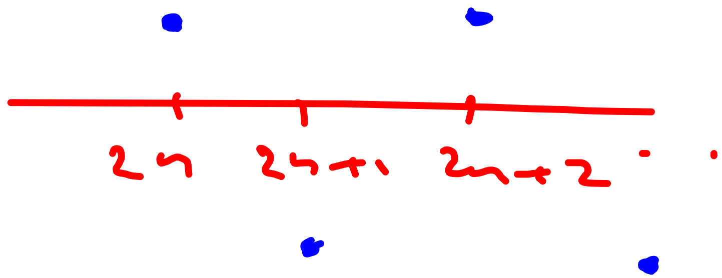
<u>Příklad 1</u> $\lim_{n\to\infty} \frac{1}{n} = 0$

ε-okolí 0 je $(-\varepsilon, \varepsilon)$ a pro $N > \frac{1}{\varepsilon}$ platí

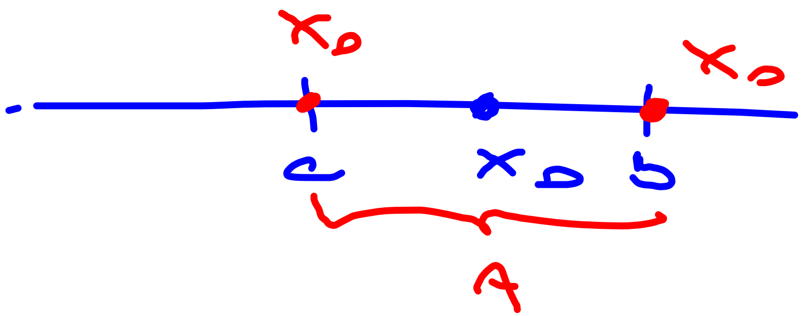
$\frac{1}{n} < \varepsilon$ pro $n > N$

$\lim_{n\to\infty} a_n$ neexistuje !

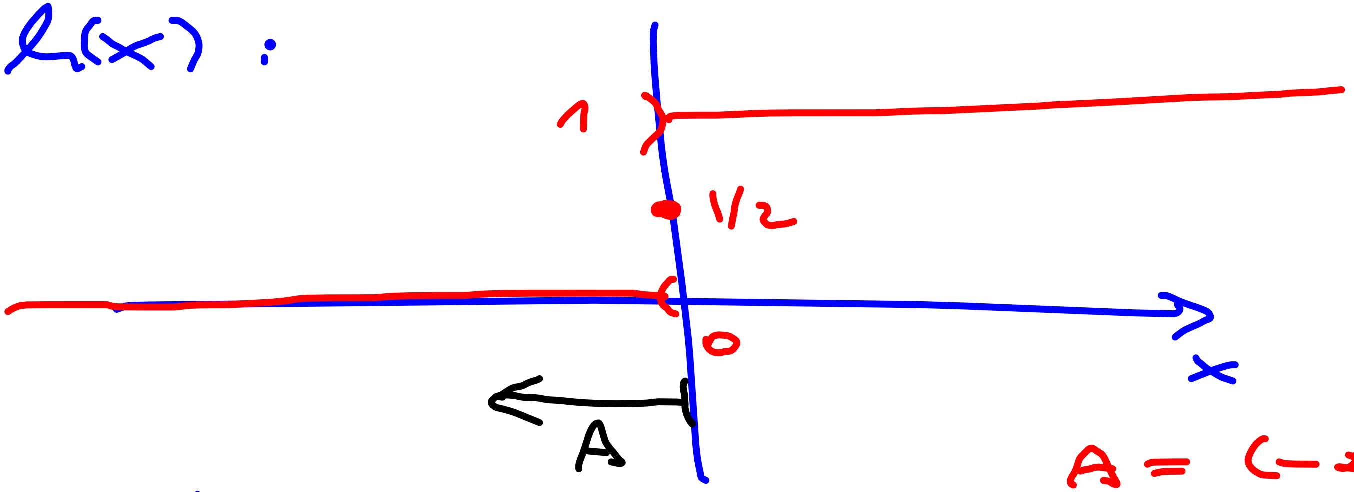
$a_n = sg(n)$



<u>Příklad 2</u>



$h(x)$:



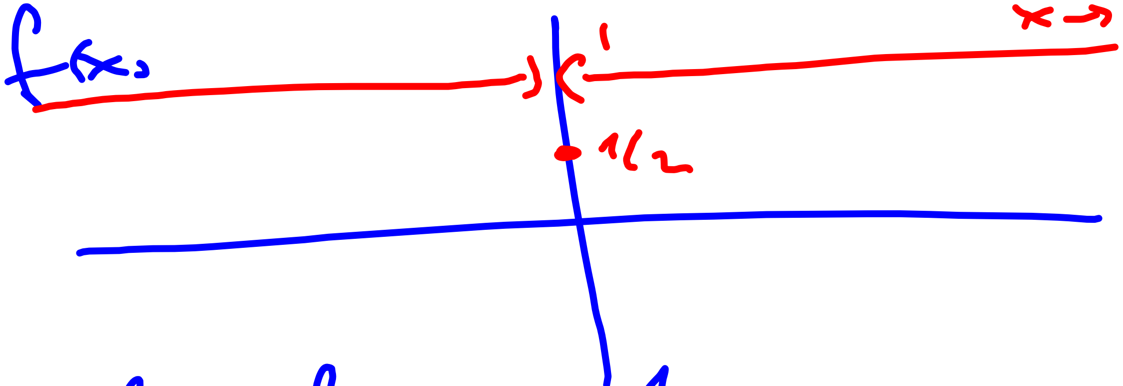
$\lim_{x \to 0} h(x)$ neex.

$A = (-\infty, 0]$

$\lim_{x \to 0^-} h(x) = 0$

$\lim_{x \to 0^+} h(x) = 1$

$f(x)$



$\lim_{x \to 0} f(x) = 1$

Příklad 5

$$\lim_{x \to x_0} f(x) = f(x_0)$$

$$f(x) = a_n x^n + \dots + a_0$$

$$f(x+\Delta x) = a_n (x+\Delta x)^n + a_{n-1}(x+\Delta x)^{n-1} + \dots$$

$$= a_n x^n + a_{n-1} x^{n-1} + \dots a_0$$

konečné

$$+ \Delta x \left(n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots a_1 \right)$$

$$+ \Delta x^2 (\dots)$$

$$\vdots$$

$$+ a_n \Delta x^n$$

násobíme Δx { zmenšíme hodně (přesněji limita) }

PEVNÉ