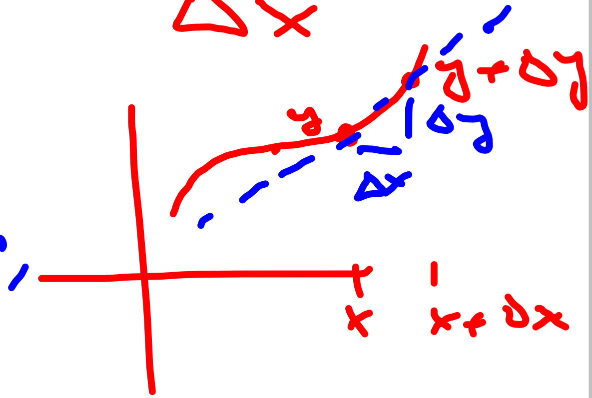
$f: \mathbb{R} \to \mathbb{R}$ (na intervalu) $\quad y = f(x)$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \cong \frac{\Delta y}{\Delta x}$$

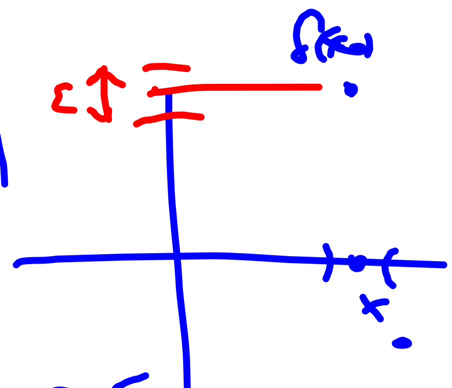


ad 1) kdyby f nebyla spojitá v x_0,

pak ex. $x_n \to x_0$ $|f(x_n) - f(x_0)| > \varepsilon$

pro nějaké kladné ε

$$\Longrightarrow \lim_{x \to x_0} \frac{|f(x) - f(x_0)|}{|x - x_0|} > \lim_{x \to x_0} \frac{\varepsilon}{|x - x_0|} = \infty$$



spojitost $\forall \varepsilon\ \exists \delta$

$\Leftrightarrow$ nespojitost $\exists \varepsilon\ \forall \delta\ \exists x \ldots$

ad 2) $\underbrace{\lim_{x\to x_0} \frac{c\cdot f(x) - c\cdot f(x_0)}{x-x_0}}_{(c\cdot f)'(x_0)} = c\cdot \underbrace{\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}}_{= f'(x_0)}$

ad 3) $(f+g)'(x_0) \overset{?}{=} \frac{\Delta y}{\Delta x} = \frac{\Delta h + \Delta f}{\Delta x}$ $\boxed{y = h(x) + f(x)}$

$= \frac{\Delta h}{\Delta x} + \frac{\Delta f}{\Delta x}$

ad 4) $y = f(x)\cdot g(x)$

$\Delta y = f(x+\Delta x)\cdot g(x+\Delta x) - f(x)\cdot g(x)$

$= \underbrace{f(x+\Delta x)}\underbrace{(g(x+\Delta x) - g(x))} = f(x+\Delta x)\cdot g'(x)$ $\to f(x)$

$+ \underbrace{\underbrace{(f(x+\Delta x) - f(x))}\cdot \underbrace{g(x)}}$ $+ f'(x)\cdot g(x)$

$= f(x)\,g'(x) + f'(x)\cdot g(x)$

$g = h \circ f \qquad z = h(y), \; y = f(x)$

$x \mapsto f(x) = y \mapsto h(f(x))$

$$g' \approx \frac{\Delta z}{\Delta x} = \frac{\Delta z}{\Delta y} \cdot \frac{\Delta y}{\Delta x} \longrightarrow h'(f(x_0)) \cdot f'(x_0)$$

$$\left(\frac{f}{g}\right)' = \left(f \cdot (g^{-1})\right)' \underset{\text{součin}}{=} f' \cdot g^{-1} + f \cdot \left((g^{-1})\right)' \overset{\text{složené f.ce}}{=}$$

$$= f' \cdot g^{-1} - f \cdot g^{-2} \cdot g' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$y = f(x) \Longrightarrow$ inverzní f.ce $x = f^{-1}(y)$

je definována pomocí f

na obrazu Im f

$\underbrace{f,\ f^{-1} \text{ diferencovatelné}} \Longrightarrow$

$$(f \circ f^{-1})(x) = x$$

$$(f \circ f^{-1})'(x) = 1 = f'(\underbrace{f^{-1}(x)}_{y}) \cdot (f^{-1})'(x)$$

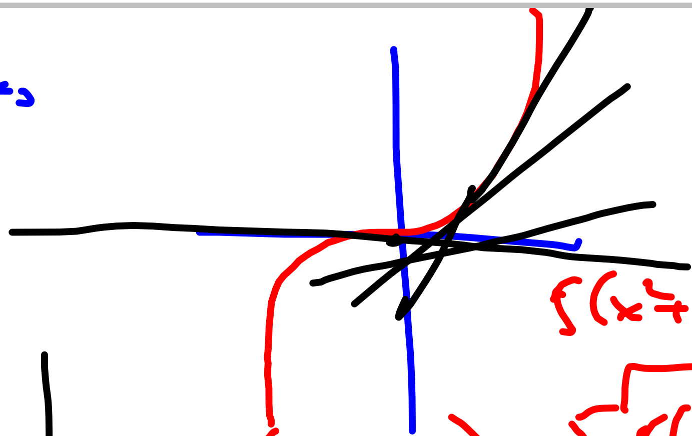
$$f'(y) = \frac{1}{(f^{-1})'(x)}$$

tj. $(f^{-1})'(f(x)) = \dfrac{1}{f'(x)}$

Příklad 1.

$$y = x^3 = f(x)$$

$$x = \sqrt[3]{y}$$

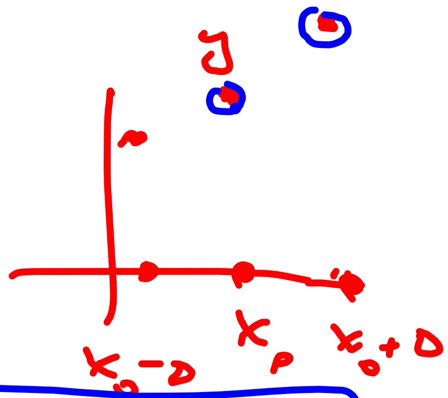


Příklad 2.

$$\left(\frac{x^2+x+1}{x^5}\right)' = \frac{(2x+1)x^5 - 5(x^2+x+1)x^4}{x^{10}}$$

$$= \frac{-3x^6-5x^4-4x^5}{x^{10}} \checkmark$$

$$f^{(k)} = \left(f^{(k-1)}\right)'$$



$$\checkmark \quad \boxed{\frac{-2f(x_0)+f(x_0+\Delta)+f(x_0-\Delta)}{\Delta}}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \leftarrow \text{kořeny}$$

φ má stejné kořeny

$$f(x) = \underbrace{(x-a_1)^{c_1}}_{\varphi} \cdot \ldots \cdot (x-a_q)^{c_q}$$

Je $c_1 = 1$, je $f'(a_1) \neq 0$

$$\underbrace{c_1}_{\substack{= 1 \\ \neq 1}} \underbrace{(x-a_1)^{c_1-1}}_{\substack{=1 \\ =0}} \cdot (x-a_2)^{c_2} \cdot \ldots (x-a_q)^{c_q}$$

$$\left.\begin{aligned} &+ \\ &\vdots \\ &+ \\ &(x-a_1)^{c_1} \cdot \ldots \cdot c_q (x-a_q)^{c_q-1} \end{aligned}\right\} = 0 \;\Big\}\; 0$$

$$(x^{-1})' = -x^{-2} \quad ✓$$

$$[illegible]\,\left[(x^{q})'\right]^{-1} = \left[q\,x^{q-1}\right]^{-1}$$

$$\boxed{(x^{a})' = a\,x^{a-1}}$$

pro $a \in \mathbb{Q}$ jednoduché

Např. $\boxed{a^{x+y} = a^{x} \cdot a^{y}}$ $\quad (a^{x})^{y} = a^{x\cdot y}$

$$a^{x+\Delta x} - a^{x} = a^{x} \cdot (a^{x} - 1)$$

$$f'(0) = 1 \Rightarrow \boxed{(a^{x})' = a^{x}}$$

$$\left(a^{1/n}\right)^n \longrightarrow \left(1+1/n\right)^n$$

$$a \longleftarrow \underbrace{\left(1+1/n\right)^n}_{a_n} \Leftarrow \text{má limitu?}$$

$$(1+b)^n = \sum_{i=0}^{n} \binom{n}{i} 1^i b^{n-i} > 1+nb$$

$$\frac{a_n}{a_{n-1}} = \left(\frac{n^2-1}{n^2}\right)^n \frac{n}{n-1} > 1 \Rightarrow \text{rostoucí}$$

a_n rostoucí $\qquad a_n < b_n$

b_n klesající

$$\left(e^{x\ln a}\right) = \left(e^{\ln a}\right)^x = a^x$$

$$\left(e^{a\ln x}\right) = \left(e^{\ln x}\right)^a = x^a$$

$$\underbrace{e^{a\ln x}}(a\ln x)' = \underbrace{x^a}\cdot a\frac{1}{x} = a x^{a-1}$$