$$\underbrace{e^x = \,?}$$

$$\boxed{m = n/x}$$

$(x \in \mathbb{R})$

$$e = \lim_{n\to\infty} \underbrace{\left(1+\frac{1}{n}\right)^n}_{a_n}$$

$\uparrow$ rostoucí

$$e^x = \lim_{n\to\infty} \underbrace{\left(1+\frac{x}{n}\right)^n}_{\uparrow\, u_n}$$

$\leftarrow$ rozepíšeme

$$u_n = 1 + n\frac{x}{n} + \ldots$$

$$u_n \simeq v_n = \sum_{i=0}^{n} \frac{1}{i!} x^i$$

$$\Big\downarrow m\to\infty$$

$$\sum_{j=0}^{\infty} \frac{1}{j!} x^j$$

$$\sum_{j=0}^{\infty} c_n \qquad c_{j+1}/c_j = \frac{\frac{x^{j+1}}{(j+1)!}}{\frac{x^j}{j!}} = \frac{x}{j+1}$$

$$1+q+\dots+q^k = \frac{1-q^k}{1-q}$$

$k \to \infty \downarrow$ $\quad \Leftrightarrow$ -prvok $q^k \to 0$

$$\sum_{k=0}^{\infty} q^k = \frac{1}{1-q} \qquad \Updownarrow \quad |q|<1$$

$$\Rightarrow \quad s_n \to \sum_{i=0}^{\infty} \frac{1}{i!} x^i$$

po velké N:

$$\mu_N \simeq \mu'_n \quad \text{kde} \quad n \ll N \quad a \quad n \to \infty$$

$$\nu_n = \mu'_n < \mu_N < \nu_N \longrightarrow \sum_{i=n}^{\infty} \frac{x^i}{i!}$$

$$\downarrow$$

$$\sum_{i=0} \frac{x^i}{i!}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = ?$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \dots = ?$$

<u>Důkaz</u> $\quad S + T = ? \quad \sum_{j=0}^{\infty} (a_j + b_j)$

$$\lim_{k\to\infty} s_k + \lim_{k\to\infty} t_k = \lim_{k\to\infty} (s_k + t_k)$$

$$(a_0 + a_1 + \dots) \cdot (b_0 + b_1 + \dots) =$$

$$= a_0 b_0 + (a_0 b_1 + b_0 a_1) + (a_0 b_2 + a_1 b_1 + a_2 b_0)$$

$$+ \dots$$

$$\frac{a_{n+1}}{a_n} \in (q - \varepsilon, q + \varepsilon) \qquad n > N_\varepsilon$$

$$|q| < 1$$

$$a_{n+1} < q' a_n < \dots \qquad |q + \varepsilon| < 1$$

$$q'$$

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$S(x) = a_0 + a_1 x + a_2 x^2 + \dots$$

$$\sqrt[n]{a_n x^n} = \left(\sqrt[n]{a_n}\right) \cdot x \xrightarrow[n \to \infty]{} \rho \cdot x$$

$$|\rho \cdot x| < 1 \Rightarrow \text{konverguje}$$

$$> 1 \Rightarrow \text{diverguje}$$

$$\frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} = \frac{x}{n+1} \xrightarrow{n\to\infty} 0$$

$$S'(x) = \sum_{n=0}^{\infty} n \cdot a_n x^{n-1} = \sum_{n=0}^{\infty} a_n x^n$$

$$\| \quad S(x)$$

$$\boxed{a_0 = 1}$$

$$1 \cdot a_1 x^0 = a_0 x^0$$

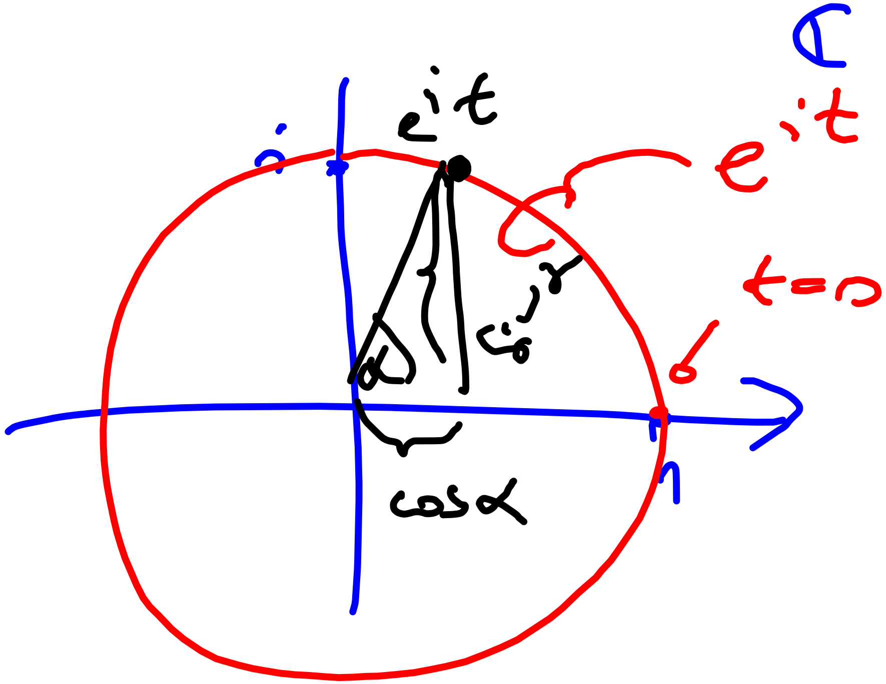
$$2 \cdot a_2 x^1 = a_1 x^1$$

$$\vdots$$

$$\boxed{(n+1)\, a_{n+1} = a_n}$$

$$a_{n+1} = \frac{a_n}{n+1}$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$



$$\overline{(e^{it})} = e^{-it}$$

$$|e^{it}|^2 = e^{it}\cdot e^{-it} = e^0 = 1$$

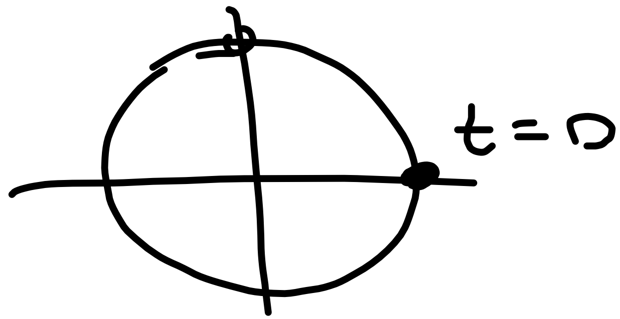
$$\Downarrow \quad \sin^2 + \cos^2 = 1$$

protože $e^{it} = \cos t + i \sin t$

$$(e^x)' = e^x$$

$$(e^{it})' = i e^{it}$$

$$\underbrace{\cos' t}\, + i\, \underbrace{\sin' t} = i(\underbrace{\cos t} + i\, \underbrace{\sin t})$$



$$e^{it} = \cos t + i \sin t$$

$$t = 0 \Rightarrow 1 \qquad 0$$

to platí tehdy, že e^{it_0} ryze imag

$$\frac{1}{4i}\left(e^{2it}-e^{-2it}\right)$$

$$(\sin 2t)' = 2\cos 2t$$

$$(\sin t\cdot\cos t)' = \sin' t\cdot\cos t + \sin t\cdot\cos' t$$

$$= \cos^2 t - \sin^2 t$$

$$e^{i(t+s)} = e^{it}\cdot e^{is} = (\cos t + i\sin t)\cdot$$

$$\cdot(\cos s + i\sin s) =$$

$$= \underbrace{(\cos t\cos s - \sin t\sin s)}_{= \cos(s+t)} + i\underbrace{(\sin t\cos s + \cos t\cdot\sin s)}_{= \sin(s+t)}$$