$$T_k(x) = a_k x^k + \dots + a_1 x + a_0$$

$$S(x) = \sum_{i=0}^{\infty} a_i x^i \quad \leftarrow \text{posl. diferencí}$$

$$a_k = \frac{1}{k!} S^{(k)}(0)$$

$$\left(S(x) = \sum_{i=0}^{\infty} a_i (x-x_0)^i \quad \text{---} \quad S^{(k)}(x_0) \right)$$

$$f(x) = f(a) + f'(a)(x-a) + \dots + \frac{1}{i!} f^{(i)}(a)(x-a)^i + \dots$$

$$\text{pro } \{a_i\} \text{ ex. } f(x), \quad f^{(i)}(a) = a_i$$

$$f(x) = e^{-x^{-2}} \text{ pro } x \neq 0, \quad f(0) = 0$$

$$\xrightarrow{x \to 0} 0 \quad ✓ \qquad f(x) = \frac{1}{e^{1/x^2}}$$

složená fce: $x^2 \longrightarrow 1/x^2 \longrightarrow -1/x^2 \longrightarrow e^{-1/x^2}$ všude $x \neq 0$

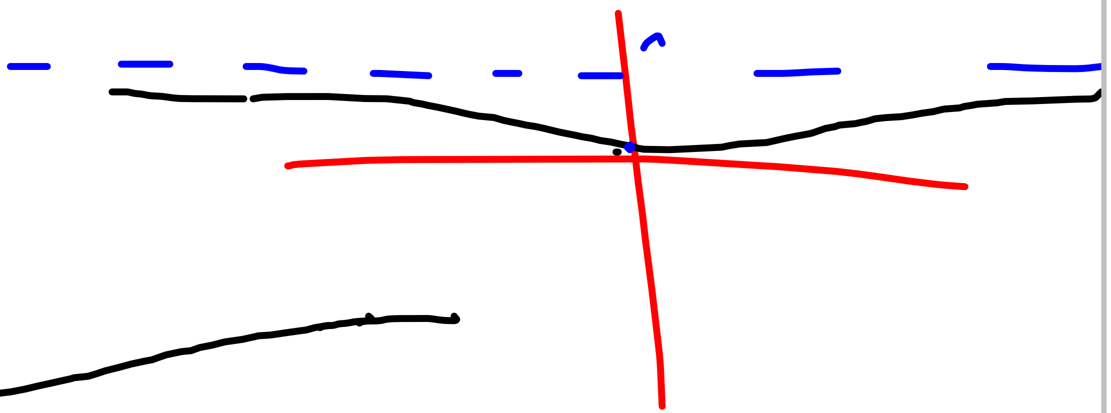
$$f'(x) = 2 \cdot x^{-3} \cdot e^{-1/x^2}$$

$$\frac{e^{-1/x^2}}{x^3} \quad x \to 0 \qquad y = 1/x \qquad \frac{y^3}{e^{y^2}} \quad y \to \infty$$

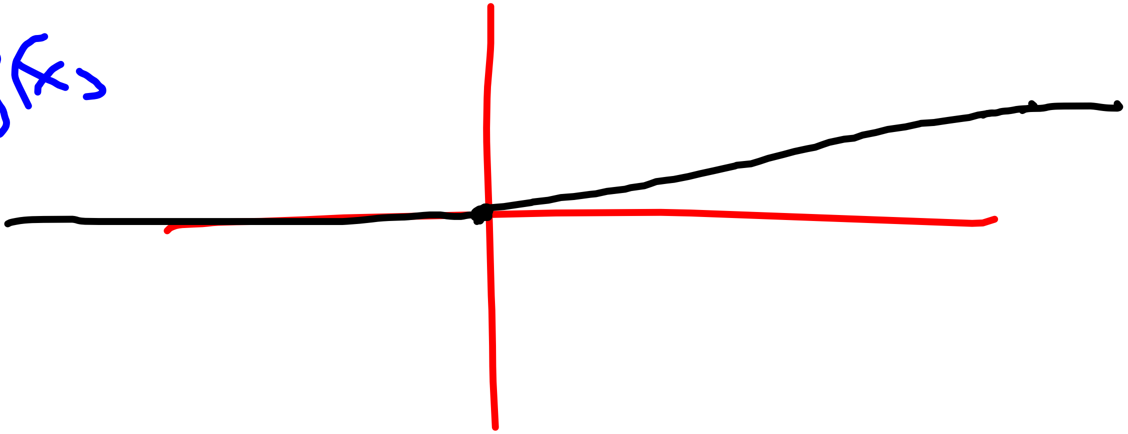
L'Hospital $\quad \dfrac{3y^2}{2y \cdot e^{y^2}} \quad \dots \quad \dfrac{\text{konst}}{P(y)\, e^{y^2}} \to 0$

indukcí $\quad C \cdot \dfrac{e^{-1/x^2}}{P(x)}$ – z toho je $f^{(k)}(x)$

$f(x) = e^{-1/x^2}$



$g(x)$



$$h(x) = \begin{cases} 0 & |x| \geq a \\ e^{\left(\frac{1}{x^2-a^2} + \frac{1}{a^2}\right)} = e^{\frac{1}{x^2-a^2}} \cdot e^{\frac{1}{a^2}} & \end{cases}$$

↑ $a = 1$

$$h(x) = e^{\frac{1}{x^2-1}+1} = e^{\frac{x^2}{x^2-1}}$$

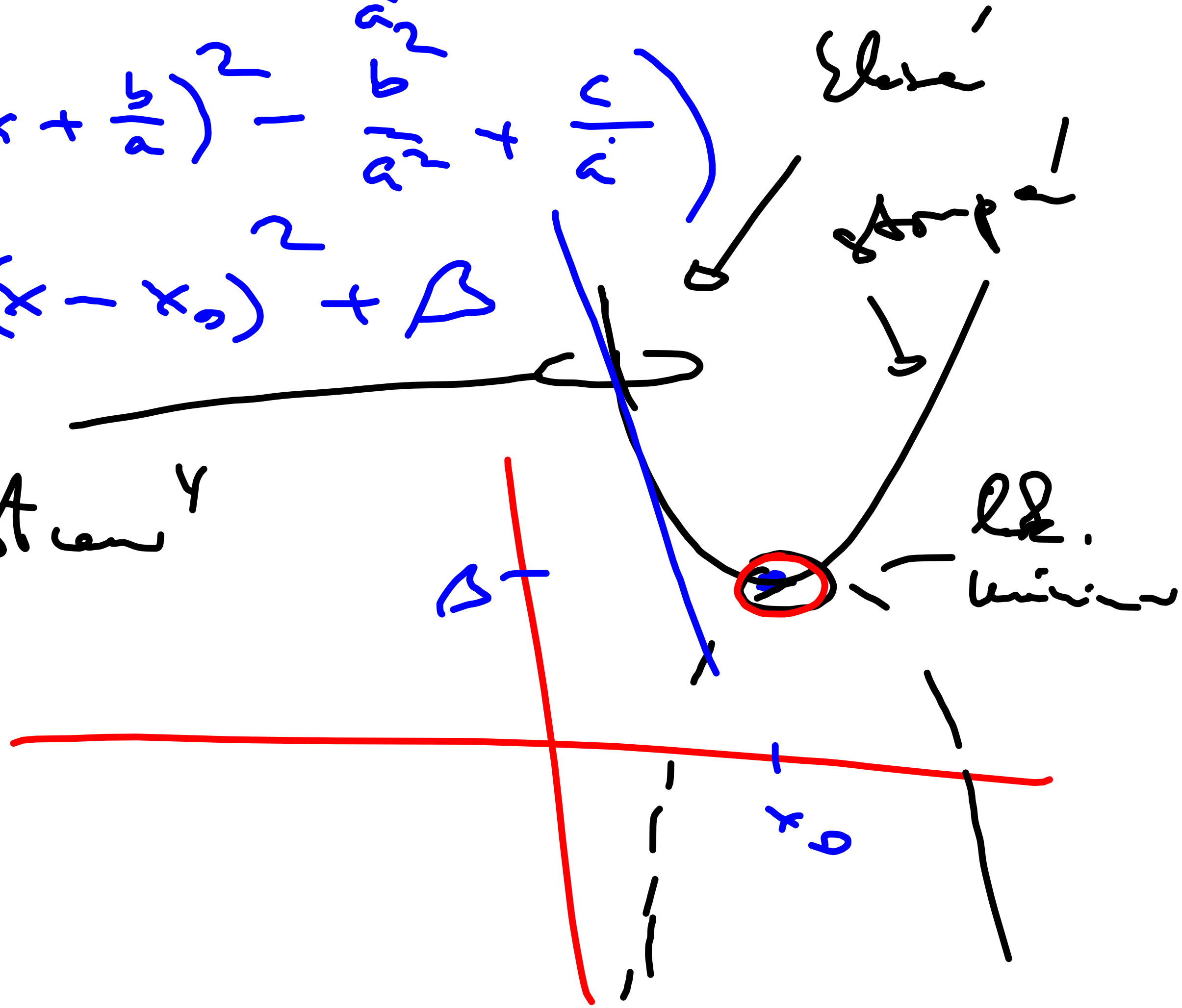
$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + 2\frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left(\left(x + \frac{b}{a}\right)^2 - \frac{b}{a^2} + \frac{c}{a}\right)$$

$$= \alpha\,(x - x_0)^2 + \beta$$

„pořád značí ve stejném stavu"



$$f(x) = f(a) + 0 + \frac{1}{2} f''(c)(x-a)^2$$

– ?

$c \in [a, x]$

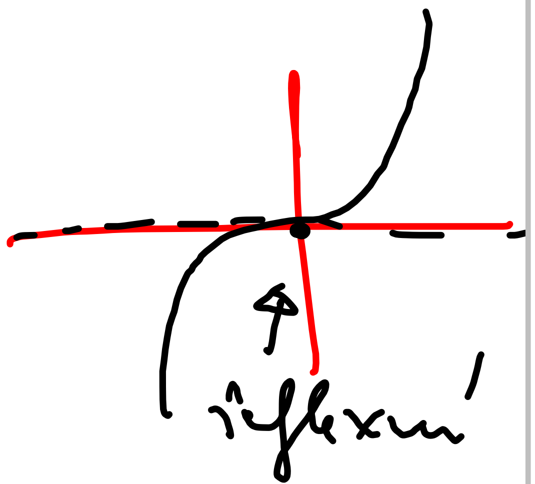
$f''(a) > 0 \Rightarrow f''(c) > 0$ pro x blízké a

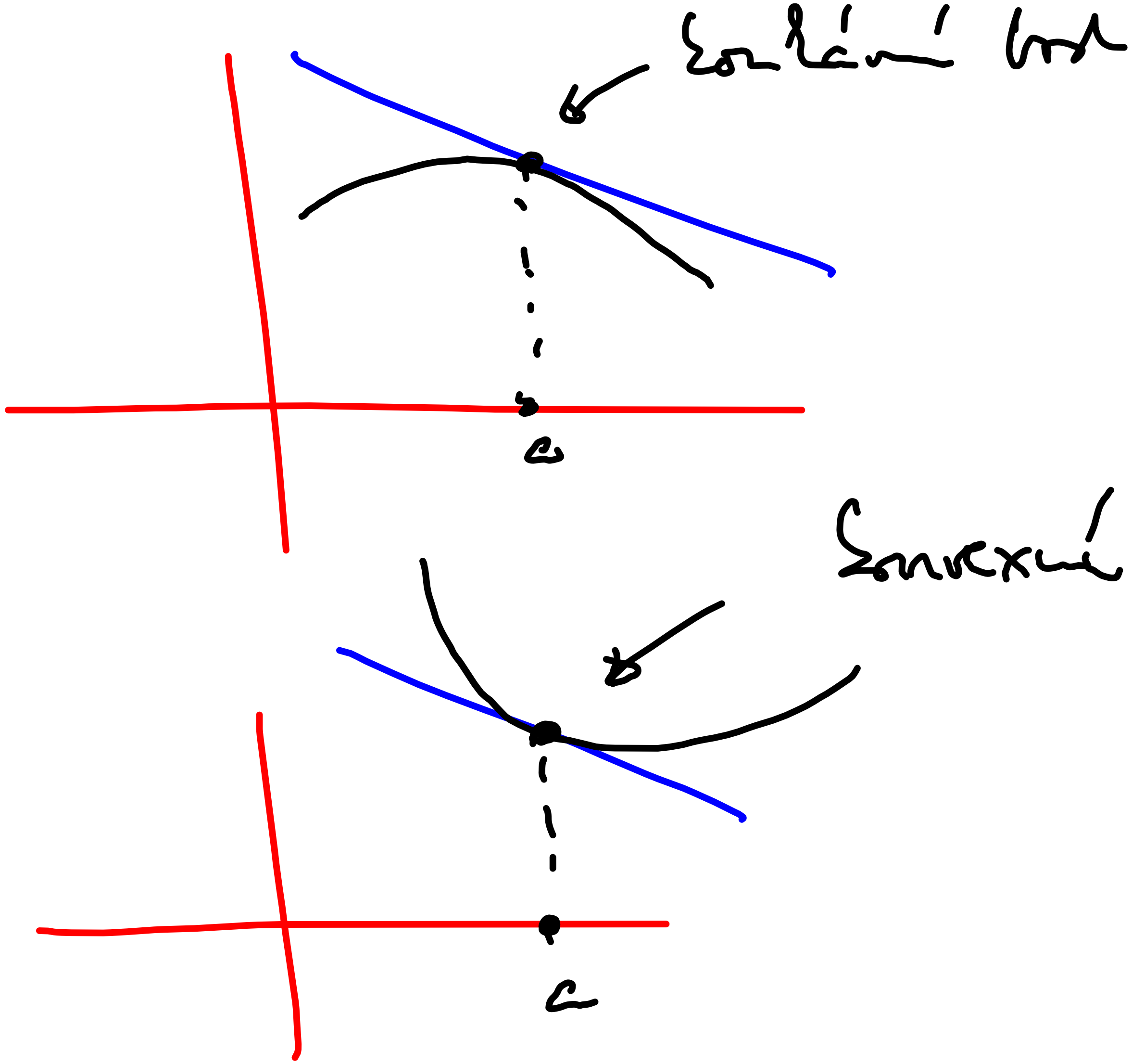
$\Rightarrow f(x) > f(a)$ pro x blízké a

pro liché $k+1$ (k je řád lib. bodu)

nefunguje!

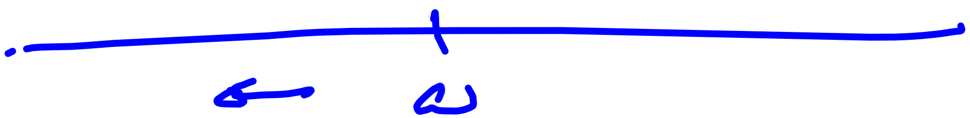
$f(x) = x^3$




konkávní bod
a
konvexní
a

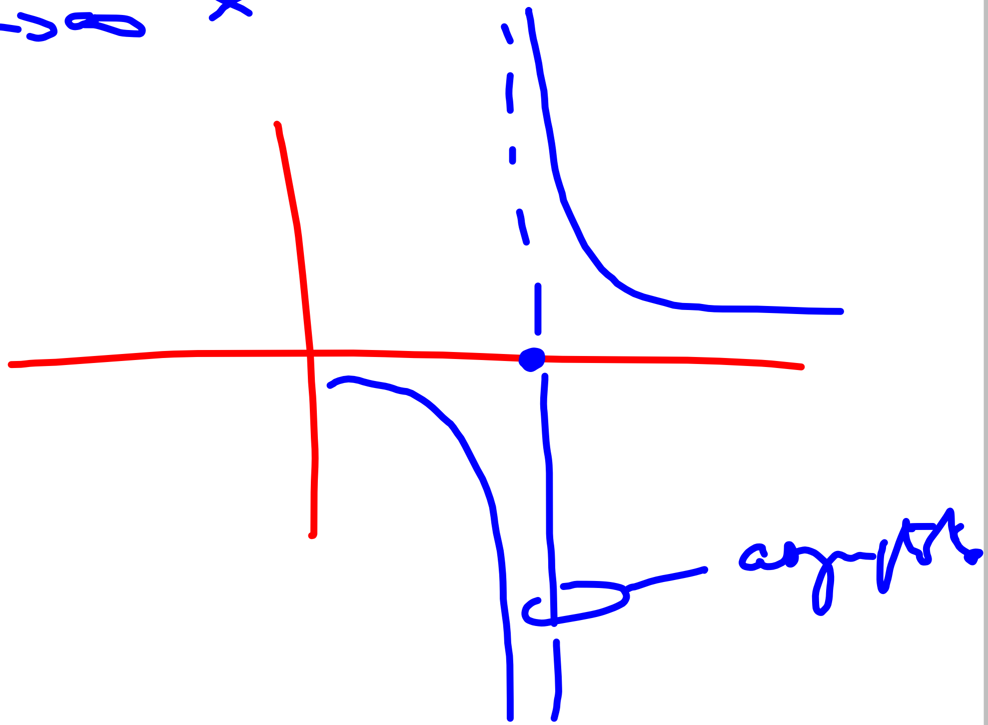
$$f(x) = \underbrace{f(a) + f'(a)(x-a)}_{T_1(x)} + 0 + \frac{1}{6} f'''(c)(x-a)^3$$

↑ [illegible]



$$f(x) - a\cdot x \xrightarrow[x\to\infty]{} b$$

$$\frac{f(x)}{x} - a \longrightarrow \lim_{x\to\infty} \frac{b}{x} = 0$$



$f(x) = x + \frac{1}{x}$ def. obor $(-\infty, 0) \cup (0, \infty)$

$\lim_{x \to 0_-} f(x) = -\infty$ $\lim_{x \to 0_+} f(x) = +\infty$

krit. body: $f'(x) = 1 - x^{-2}$

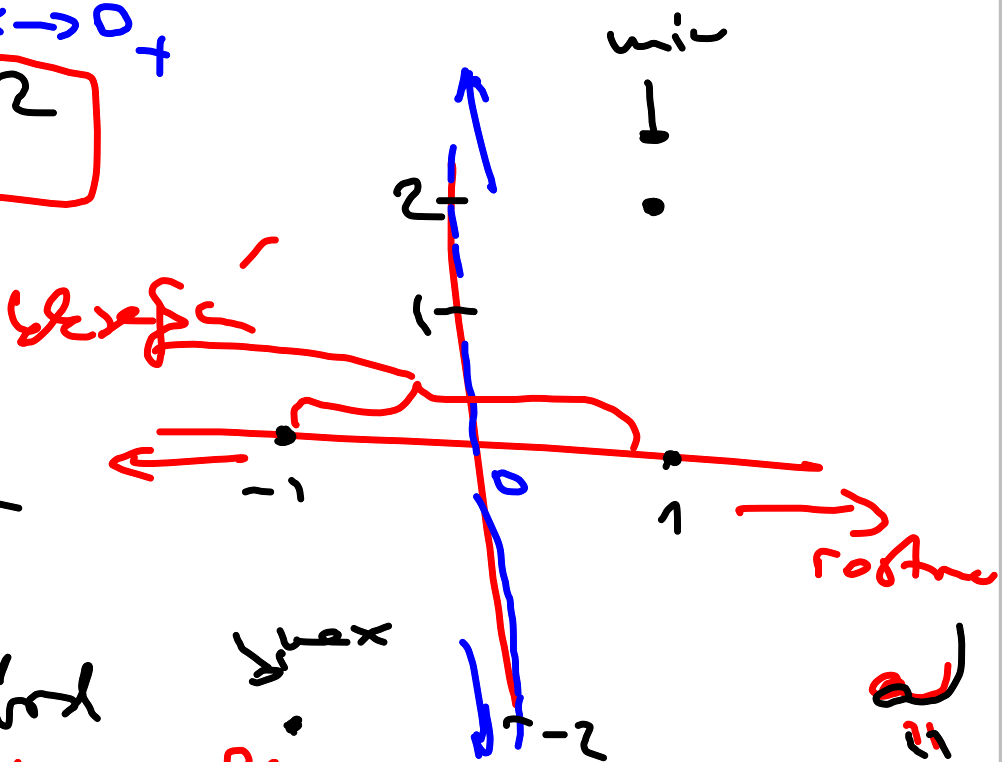
$f'(x) = 0 \iff x = \pm 1$

$f''(x) = +2x^{-3}$

$f''(1) = 2$ $f''(-1) = -2$

$f''(x) \neq 0 \ \forall x \neq 0$

$\Rightarrow$ žádný inflexní bod



$f'(x) < 0$ pro $|x| < 1$

> 0 pro $|x| > 1$

$\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} 1 + \frac{1}{x^2} = 1$

$f(x) - ax = \frac{1}{x} \xrightarrow{x \to \infty} 0$

$\Rightarrow$ asymptota $y = x$

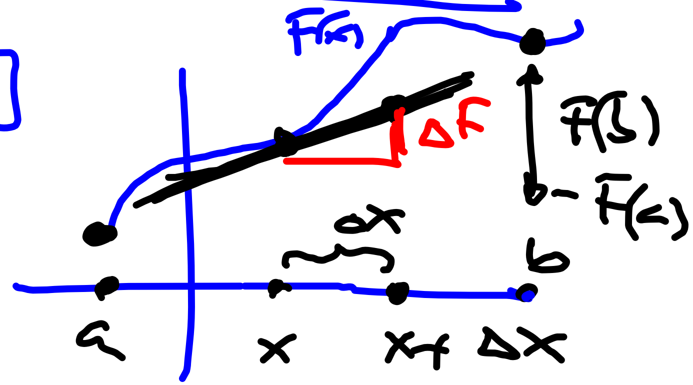
Opačný problém k derivování:

ANTIDERIVACE / INTEGRÁL

=

$F(x)$ funkce na $[a,b]$

$f(x) = F'(x)$

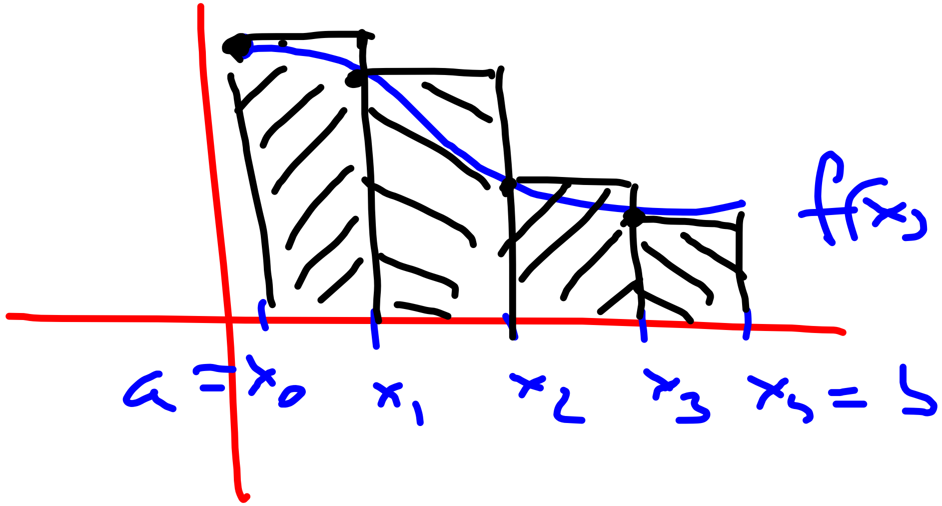


$$F(x+\Delta x) \doteq F(x) + f(x)\cdot \Delta x$$

($f(x)\cdot\Delta x$ = ΔF)

$F(a) = F(x_0)$ $\qquad$ $F(x_n) = F(b)$

$a = x_0, \; x_1, \; \ldots, \; x_{n-1}, \; x_n = b$



$\Rightarrow$ "pravdivý podíl" $\{$plocha pod grafem $f(x)\} = F(b) - F(a)$ pro antiderivaci F funkce f.