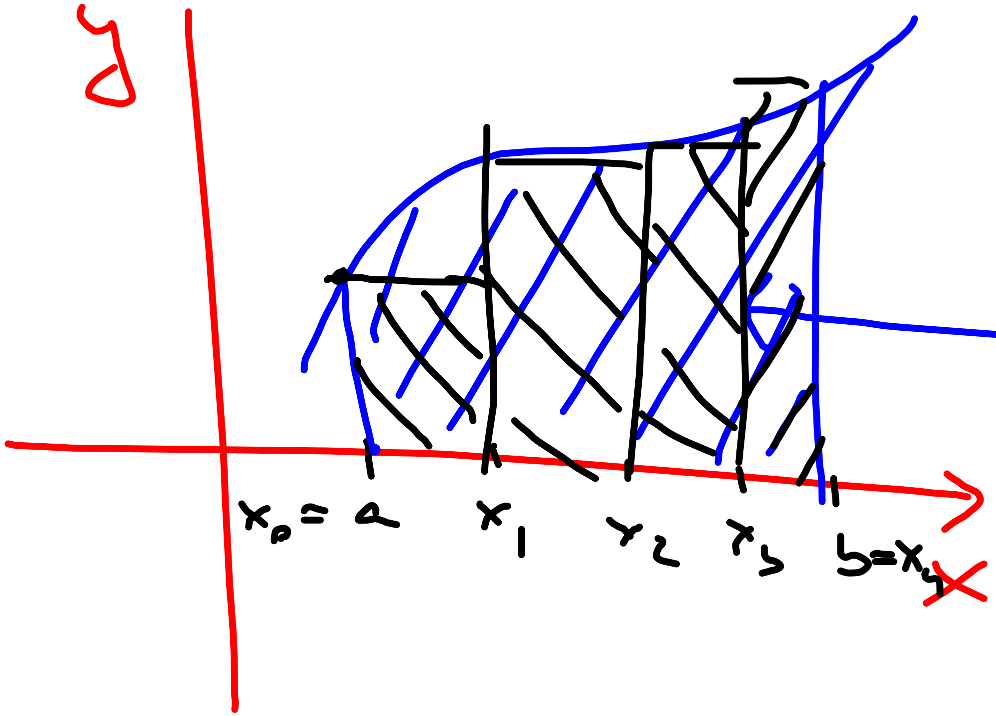


$y = f(x)$

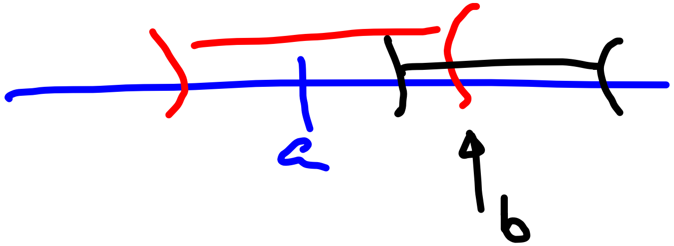
plocha

$\approx F(b) - F(a)$

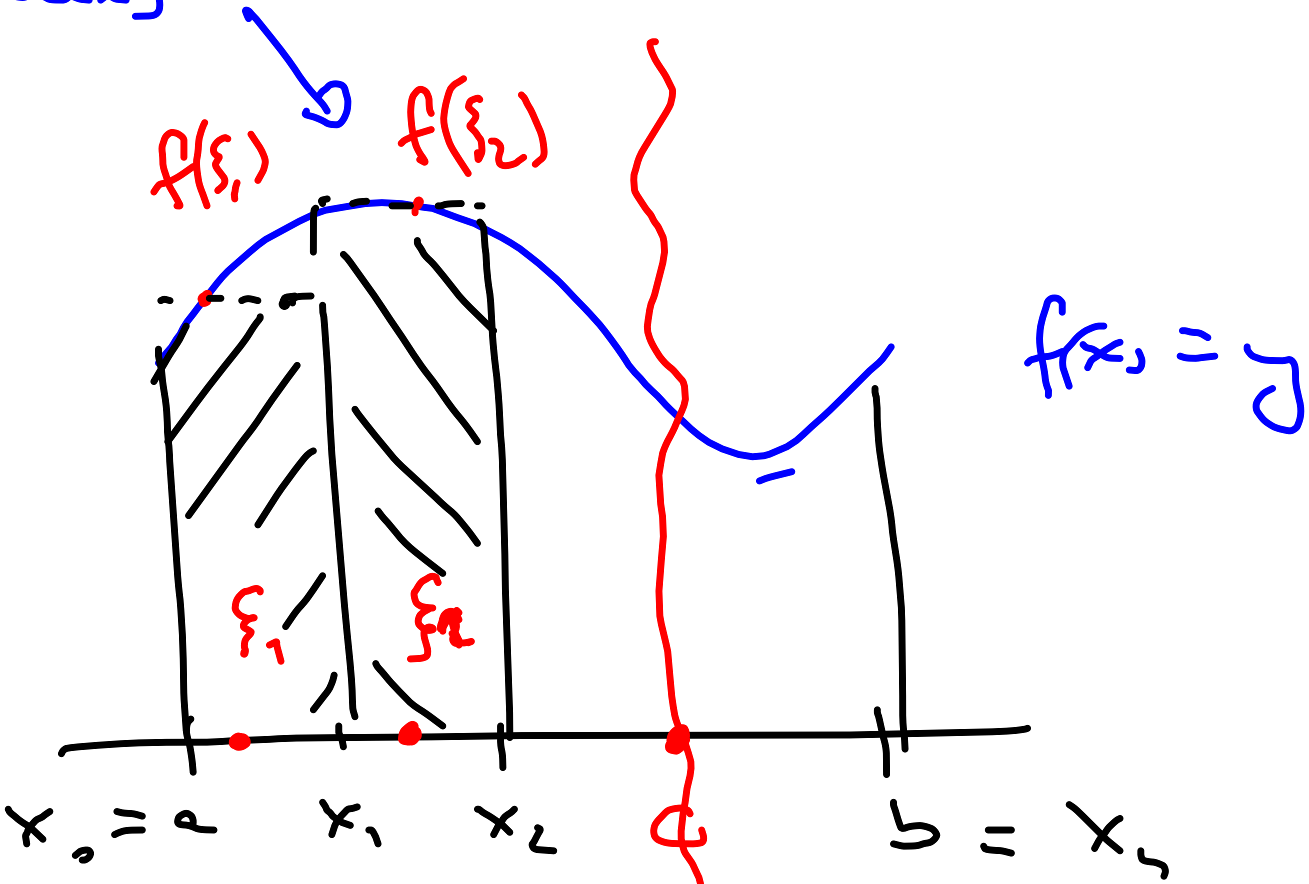
$$f(x_i) = F'(x_i) \approx \frac{F(x_{i+1}) - F(x_i)}{x_{i+1} - x_i}$$

$$F(b) - F(a) \doteq \sum_i f(x_i)(x_{i+1} - x_i)$$

$$F(x) - G(x) = F(a) - G(a)$$



$f(\xi_2) \in [\min, \max]$



$$\int_a^c + \int_c^b$$

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

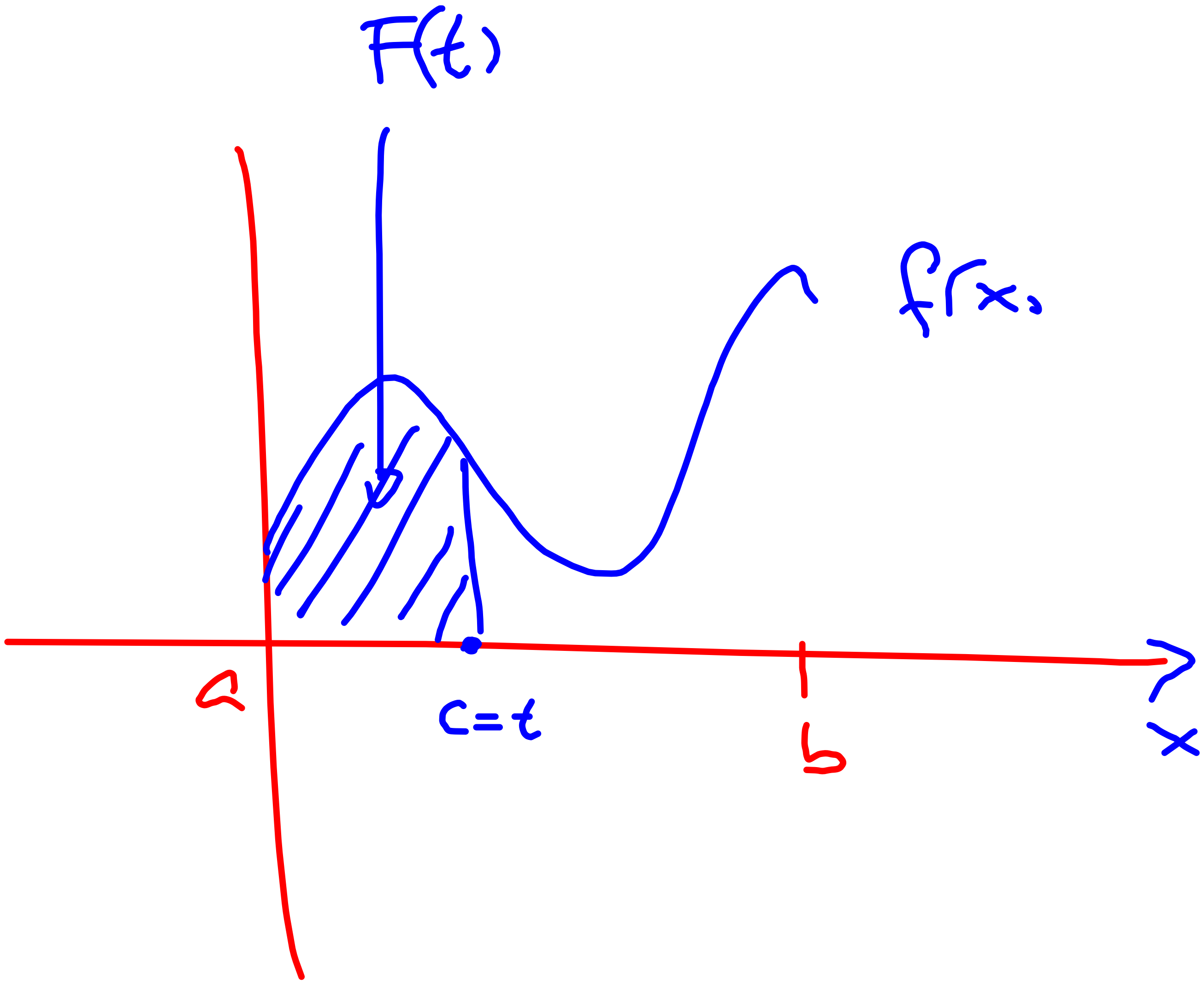
$$f_1(\xi_i) + f_2(\xi_i)$$

$$\|$$

$$S_{\Xi,\xi} = \sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1})$$

$$= \sum_{x_i \le c} \cdot/\cdot + \sum_{x_i > c} \cdot/\cdot$$

$$\int_a^c f(x)\,dx \qquad \int_c^b f(x)\,dx$$


F(t)
f(x)
a
c=t
b
x

<u>Důkaz</u>: $S_{\Xi,\xi} \longrightarrow \begin{cases} f(\xi_i) = \text{"max"} \\ f(\xi_i) = \text{"min"} \end{cases}$

$$\Rightarrow S_{\Xi,\inf} = \sum_{i=1}^{n} \left(\inf_{x_{i-1} \le \xi \le x_i} f(\xi) \right) \cdot (x_i - x_{i-1})$$

$$S_{\Xi,\sup} = \sum_{i=1}^{n} \sup_{\text{''}} f(\xi) \quad \text{''}$$

$$S_{\Xi,\inf} \le S_{\Xi,\xi} \le S_{\Xi,\sup}$$

$\int^b f_4 \, dx$ ex. $\Leftrightarrow S_{\Xi,\inf} = S_{\Xi,\sup}$

<u>"po normě</u> <u>fin f"</u>

Tvrzení: f nezáporné $\Rightarrow$ ex.

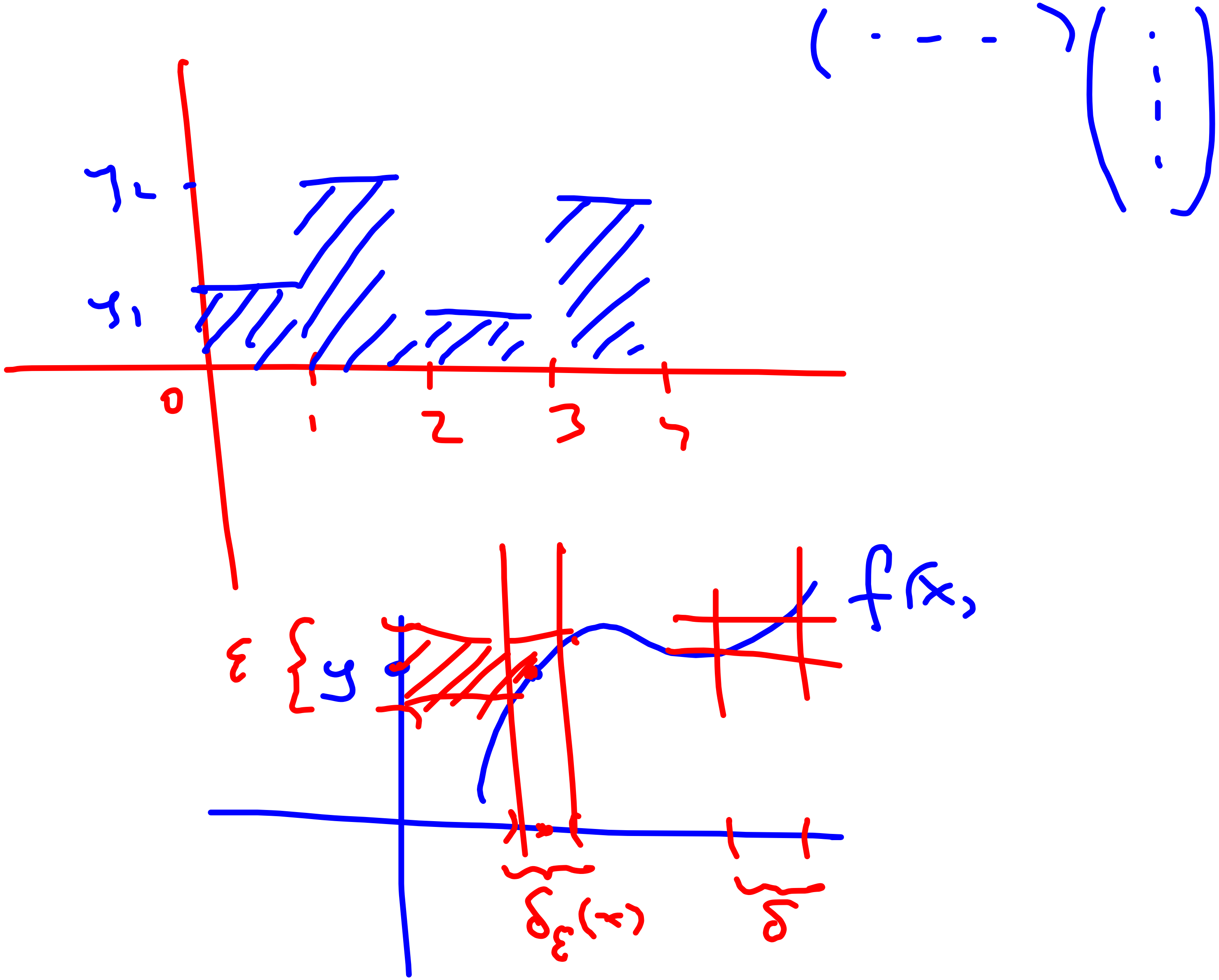
$$S_{sup} = \inf_{\Xi} S_{\Xi;\sup}$$

$$S_{inf} = \sup_{\Xi} S_{\Xi;\inf}$$

$\Rightarrow$ ex. po částech spojitá

odhady $\rightsquigarrow$ $S_{sup} = S_{inf} \Rightarrow \int_a^b f(x)\,dx$ ex.

$$\left|\frac{1}{\Delta t}\cdot\left(\int_a^{t+\Delta t} f(x)\,dx - \int_a^{t} f(x)\,dx\right) - f(t)\right| \xrightarrow{\Delta t \to 0} 0$$

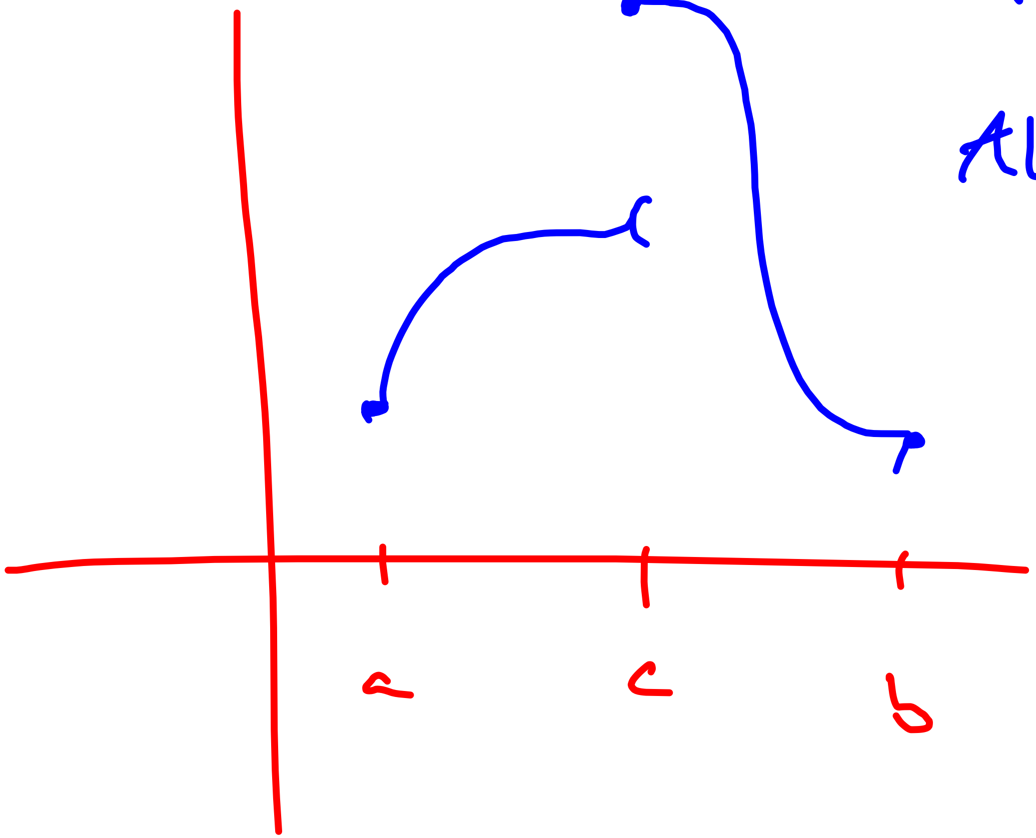

y₂
y₁
0
1
2
3
4
f(x)
ε
y
δε(x)
δ

$\int_a f(t)$ není definovaná
v c

$$F(t) = \int_a^t f(x)\, dx$$

ALE JE

SPOJITÁ



$$\int_a^b \alpha \, dx = [\alpha \cdot x]_a^b = \alpha (b-a)$$

$$\int_a^b (a_0 + a_1 x + \dots + a_n x^n)\, dx = \int_a^b a_0 \, dx + \dots$$

$$= \left[\sum_{i=0}^{n} \frac{a_i}{i+1} x^{i+1} \right]_a^b$$

$$(\ln(\cos x))' = \frac{1}{\cos x} \cdot (-\sin x)$$

$$(\operatorname{arctg} x)' = \frac{1}{\left(\frac{\sin y}{\cos y}\right)'} =$$

$$\int \frac{x\,dx}{x^2+1} = \frac{1}{2}\int \frac{(x^2+1)'\,dx}{x^2+1}$$

$$= \ln(x^2+1) + C$$

$$I = \int \underset{F}{\underset{\parallel}{x}} \cdot \underset{G'}{\underset{\vee}{\sin x}}\,dx = -\underset{F}{\underset{\parallel}{x}} \cdot \underset{G}{\underset{\vee}{\cos x}} + \int 1 \cdot \cos x \cdot dx$$

$$= \underline{-x\cos x + \sin x + C}$$

$$\underline{I = \int \frac{1}{\sqrt{1-x^2}}\,dx} \qquad \underbrace{\left\{ x = \sin t \right\}}$$

$$= \int \frac{1}{\underbrace{\sqrt{1-\sin^2 t}}_{\cos t}} \cdot \cos(t)\,dt$$

$$= \int dt = t + C \qquad \left\{ t = \arcsin x \right|$$

$$= \arcsin x + C$$

$$\int \underbrace{\cos^{n-1} x}_{f(x)} \cdot \underbrace{\cos x}_{G'(x)} \, dx = \sin x \cdot \cos^{n-1} x$$

$$+ (n-1) \int \cos^{n-2} x \cdot \underbrace{\sin^2 x}_{1 - \cos^2 x} \, dx$$

$$\int \frac{f(x)}{g(x)} \, dx$$ pro polynomy $f(x), g(x)$

dělení se zbytkem $\Rightarrow \int \text{polynom} + \int \varphi$

φ: $\deg f < \deg g$

$$\int \frac{dx}{x+1} = \int \frac{dy}{y} = \ln|y| = \ln|x+1|$$

$$y = x+1$$

$$dy = dx$$

$$4x+2 = A(x+2) + B(x+1)$$

$$\Rightarrow \left.\begin{array}{l} 2 = 2A + B \\ 4 = A + B \end{array}\right\} \Rightarrow \begin{array}{l} 2 = -A \\ 6 = B \end{array}$$

$$\frac{x-4}{(x+1)(x-2)^2} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$\Rightarrow$ řešení (Rozklad na parciální zlomky)

$$\frac{B_1 x + C_1}{(x-a)^2 + b^2} + \dots + \frac{B_s x + C_s}{((x-a)^2 - b^2)^s}$$