$(A, \leq) \qquad \leq \; \subseteq A \times A$

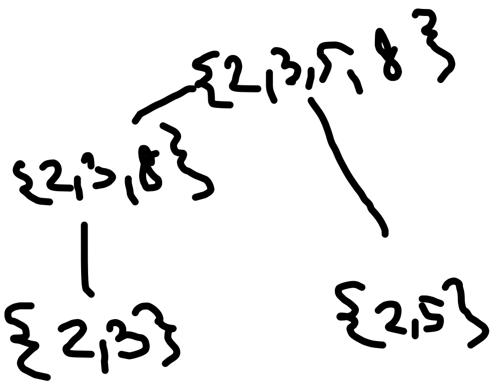
R, AS, T

ad 1. (d) není konečná

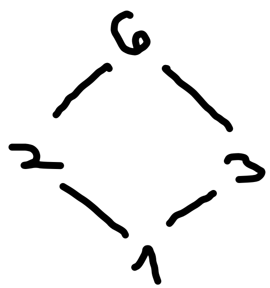
$(x_1, x_2, x_3, \ldots, x_n) = (x_1, (x_2, \ldots, x_n))$

(e) nejsou nesoudělné

ad 2. inkluzí ... částečné




$(D_6, |)$



nejmenší $a$: $\forall x \in A: a \leq x$

největší $a$: $\forall x \in A: x \leq a$

minimální $a$: $\forall x \in A$: neplatí $x < a$

max. $a$: $\forall x \in A$: neplatí $x > a$

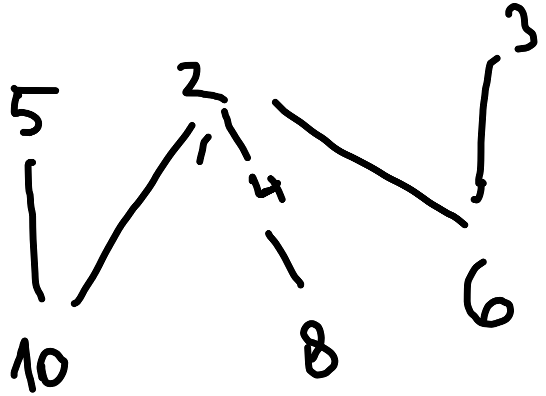
$$< \;:=\; \leq \setminus \underbrace{\Delta_A}_{\{(x,x);\, x \in A\}}$$

ad 3. $a$ nejmenší, $b$ nejmenší

$a \leq b,\ b \leq a \Rightarrow a = b$

ad 4.

$\{2,3,4,5,6,8\} \ldots a \leq b \iff b \mid a$




max. $2, 3, 5$

min. $10, 8, 6$

nejv. a nejm. nejsou (kdybychom přidali 1, byl by největší) ad 5. $A = \{x \in \mathbb{Q} ; x^3 < 3\}$

$x < \sqrt[3]{3}$

$(-\infty, \sqrt[3]{3})$

$\sqrt[3]{3}$

a) horní ANO
, dolní NE

$\sqrt[3]{3} \notin \mathbb{Q}$

$\sqrt[3]{3} = \frac{m}{n}$ $(m,n)=1$ $m,n \in \mathbb{Z}$

$3n^3 = m^3$

$\Downarrow$ $3 | m^3 \Rightarrow 3 | m$

$\Rightarrow 27 | m^3$

$\Rightarrow m^3 = 27 m_1^3$

$\Rightarrow 3n^3 = 27 m_1^3$

$n^3 = 9 m_1^3$

$\Rightarrow 3 | n^3 \Rightarrow 3 | n$

SPOR

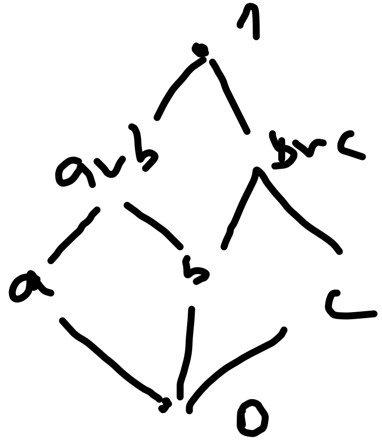
b) supremum ... nejmenší horní závora
supremum v $\mathbb{Q}$ NE, v $\mathbb{R}$ ANO
infimum NE

Potom $\sqrt[k]{m} \notin \mathbb{Q} \setminus \mathbb{N}$

Bool. alg.  kon. množina $M$

$$\mathcal{P}(M) = 2^M = \{A;\ A \subseteq M\}$$

$(2^M; \cap, \cup, ')$, kde $A' = M \setminus A$




1.a) $A \wedge 0 = 0$

$$A \wedge 0 \overset{(6)}{=} A \wedge (A \wedge A') \overset{(1)}{=}$$

$$= (A \wedge A) \wedge A' \overset{(c)}{=} A \wedge A' \overset{(6)}{=} 0$$

c) $A \overset{(5)}{=} A \wedge 1 \overset{(6)}{=} A \wedge (A \vee A') \overset{(3)}{=} (A \wedge A) \vee (A \wedge A') =$

$$\overset{(6)}{=} (A \wedge A) \vee 0 \overset{(4)}{=} A \wedge A$$

e) $(A')' \overset{?}{=} A$

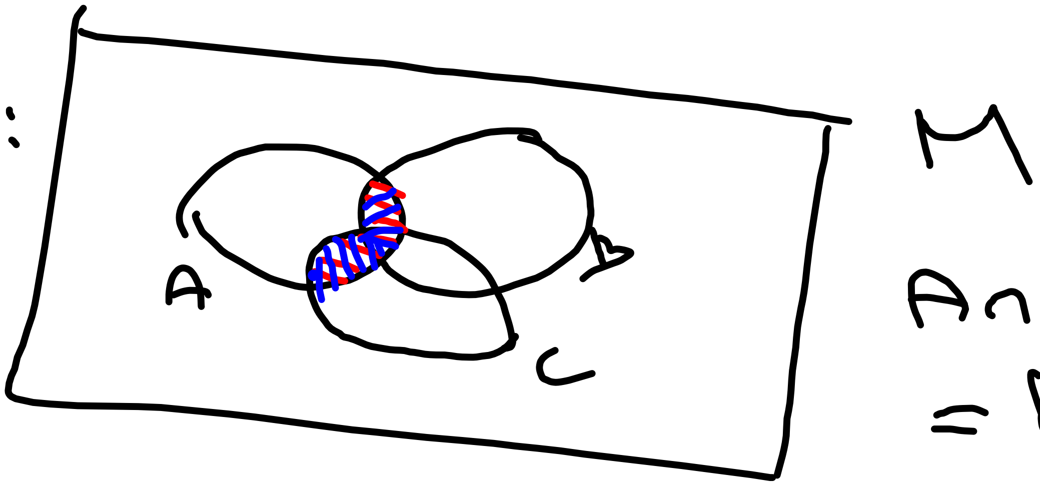
ozn. $B = A'$, hledáme $B'$

$B \cap B' = 0 \qquad B \cup B' = 1$

$B \cap A = A' \cap A \overset{(6)}{=} 0 \qquad B \cup A = A' \cup A \overset{(6)}{=} 1$

$\Rightarrow A$ je $B'$ a tedy $A = (A')'$

Venn:




$A \cap (B \cup C)$

$= (A \cap B) \cup (A \cap C)$

A) spec. v množinové algebře:

$$x \in (A \cap B)' \Longleftrightarrow x \in M \setminus (A \cap B) \Longleftrightarrow$$
$$\Longleftrightarrow x \notin A \cap B \Longleftrightarrow x \notin \{y ; y \in A \wedge y \in B\}$$
$$\Longleftrightarrow x \notin A \vee x \notin B \Longleftrightarrow x \in M \setminus A$$
$$\vee\ x \in M \setminus B$$

$$\Longleftrightarrow x \in A' \cup B'$$

---

obecně v Bool. alg.:

$$(A \wedge B)' \overset{?}{=} A' \vee B'$$

potřebujeme
$$(A \wedge B) \wedge (A' \vee B') = 0$$
$$(A \wedge B) \vee (A' \vee B') = 1$$

$$(A \wedge B) \wedge (A' \vee B') \overset{(3)}{=} ((A \wedge B) \wedge A') \vee$$

$$\vee ((A \wedge B) \wedge B') =$$

$$\overset{(2)}{=} ((B \wedge A) \wedge A') \vee ((A \wedge B) \wedge B') =$$

$$\overset{(1)}{=} (B \wedge (A \wedge A')) \vee (A \wedge (B \wedge B')) \overset{(6)}{=}$$

$$\overset{(6)}{=} (B \wedge 0) \vee (A \wedge 0) \overset{(a)}{=} 0 \vee 0 \overset{(4)}{=} 0$$

analogicky i $(A \wedge B) \vee (A' \vee B') = 1$

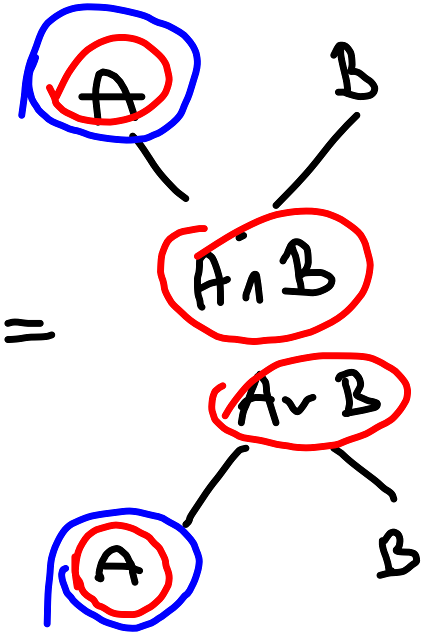
(b) absorpce (pohlcení)

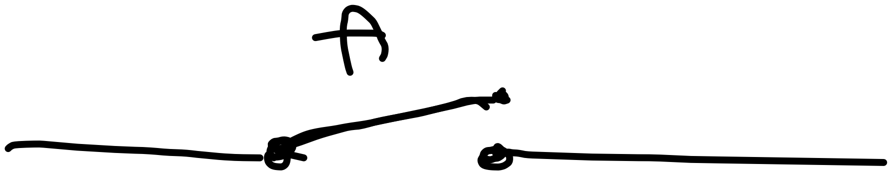
$$A \vee (A \wedge B) = A$$

$$A \vee (A \wedge B) \overset{(5)}{=} (A \wedge 1) \vee (A \wedge B) =$$

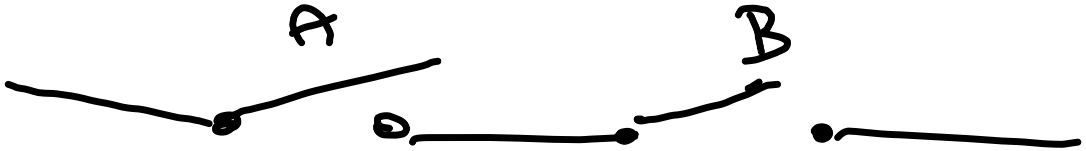
$$\overset{(3)}{=} A \wedge (1 \vee B) =$$

$$\overset{(a)}{=} A \wedge 1 \overset{(5)}{=} A$$

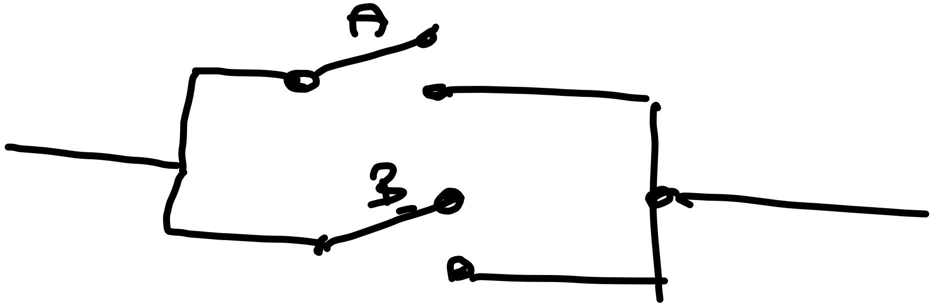





přepínač



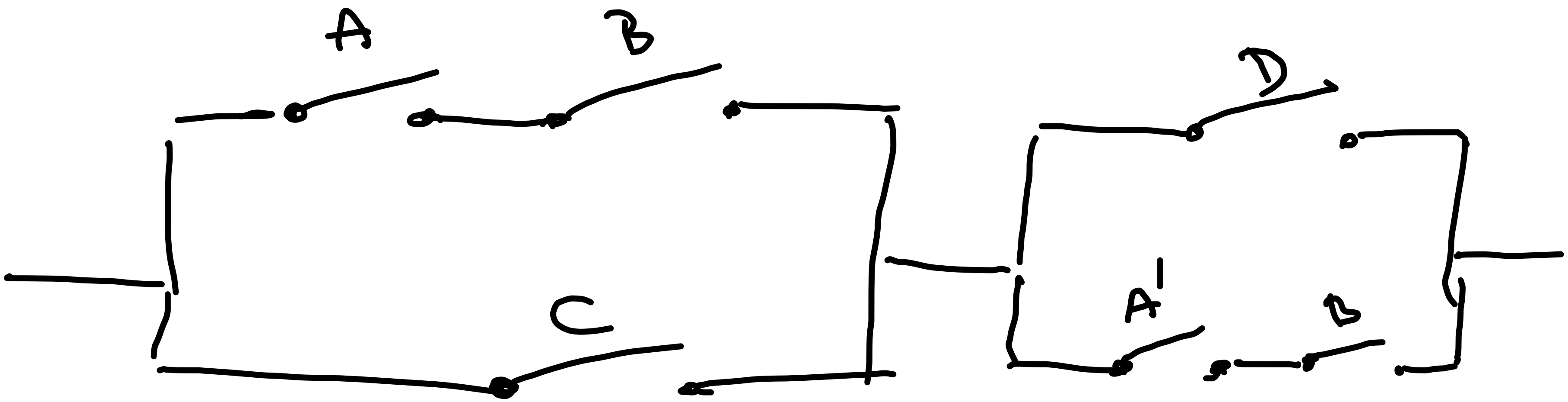

$A \wedge B$




$A \vee B$

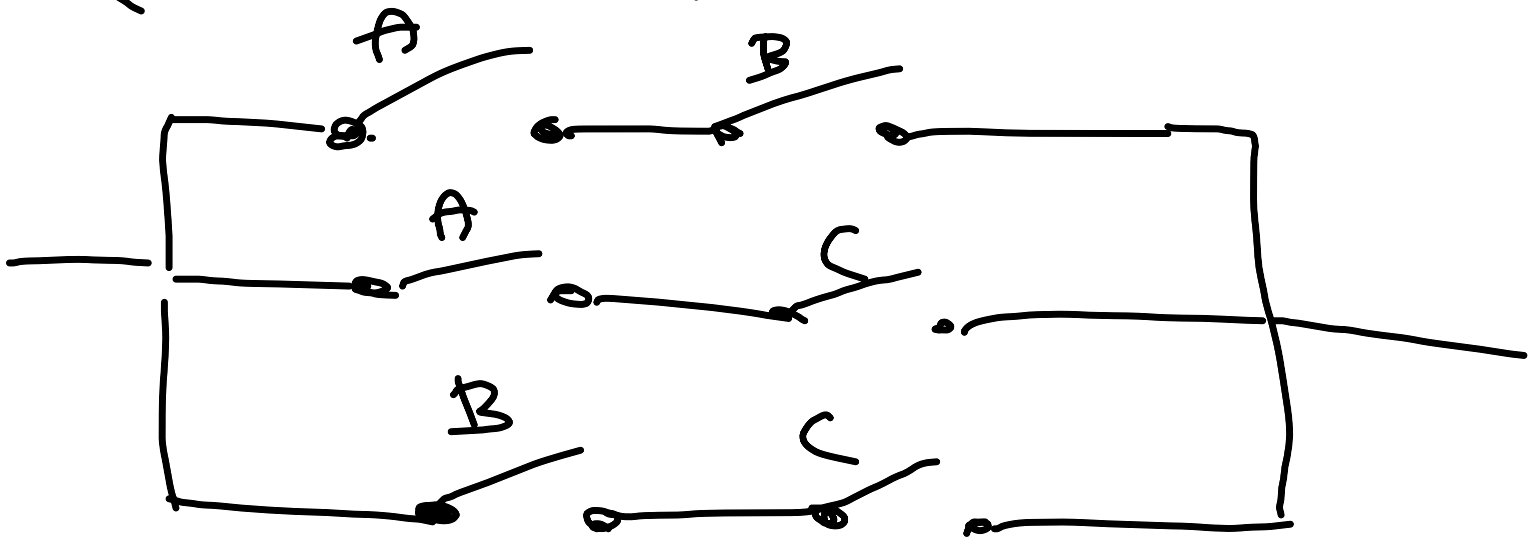
$A'$ prochází $\Longleftrightarrow$ $A$ neprochází

$$\left[(A \wedge B) \vee C\right] \wedge \left[D \vee (A' \wedge B)\right]$$



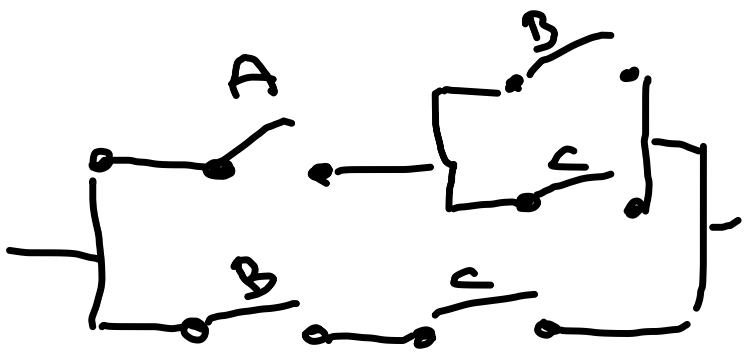
aa4. $A, B, C$

$(A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$



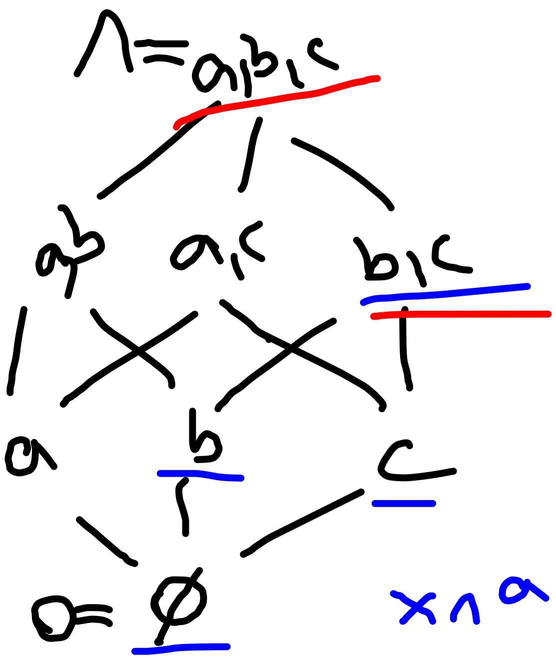

zjednodušil:

$[A \wedge (B \vee C)] \vee (B \wedge C)$



# Normalizace booleovských výrazů pomocí atomů

$M=\{a,b,c\}$




$\{a\}' = \{b,c\}$

$x \wedge a = 0$

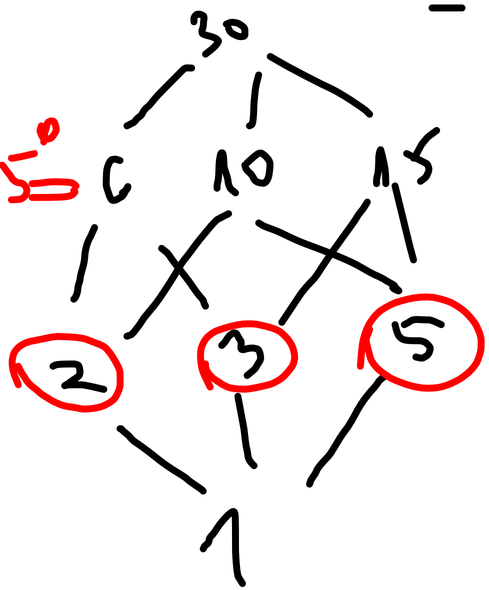
$y \vee a = 1$

$D_{30}$

$30 = 2^1\cdot 3^1\cdot 5^1$

$\tau(30) = (1+1)(1+1)(1+1) = 8$

$2^1\cdot 3^1\cdot 5^0 = 6$




př.. $\tau(45) = \tau(5\cdot 3^2) = (1+1)(2+1) = 2\cdot 3 = 6$

zapište:

a) všechny neizomorfní 7 prvkové B.A.

b) —— " —— 8 prvkové B.A.

ad a) neexistuje, každá B má $2^{|A|}$ prvků, kde A jsou atomy B

ad b) $D_{30} \cong 2^{M}$

$$((A \wedge B) \vee C)' \wedge (A' \vee (B \wedge C \wedge D))$$

$$((A \wedge B)' \wedge C') \wedge (A' \vee (B \wedge C \wedge D))$$

$$((A' \vee B') \wedge C') \wedge (A' \underline{\vee} (B \wedge C \wedge D))$$

$$((A' \wedge C') \vee (B' \wedge C')) \wedge (A' \underline{\vee} (B \wedge C \wedge D))$$

$$(A' \wedge C' \wedge A') \vee (B' \wedge C' \wedge A') \vee (A' \wedge \cancel{C' \wedge B} \wedge C \wedge D)$$

$$\vee (B' \wedge \cancel{C' \wedge B} \wedge C \wedge D)$$

$$= \boxed{A' \wedge C'} \vee (\boxed{A' \wedge C'} \wedge B') = X \vee (X \wedge B') = X = A' \wedge C'$$

suprenum atomů

$$(A' \wedge C' \wedge B \wedge D) \vee (A' \wedge C' \wedge B' \wedge D)$$

$$\vee (\ldots \quad B' \wedge D') \vee (\ldots \quad B \wedge D')$$