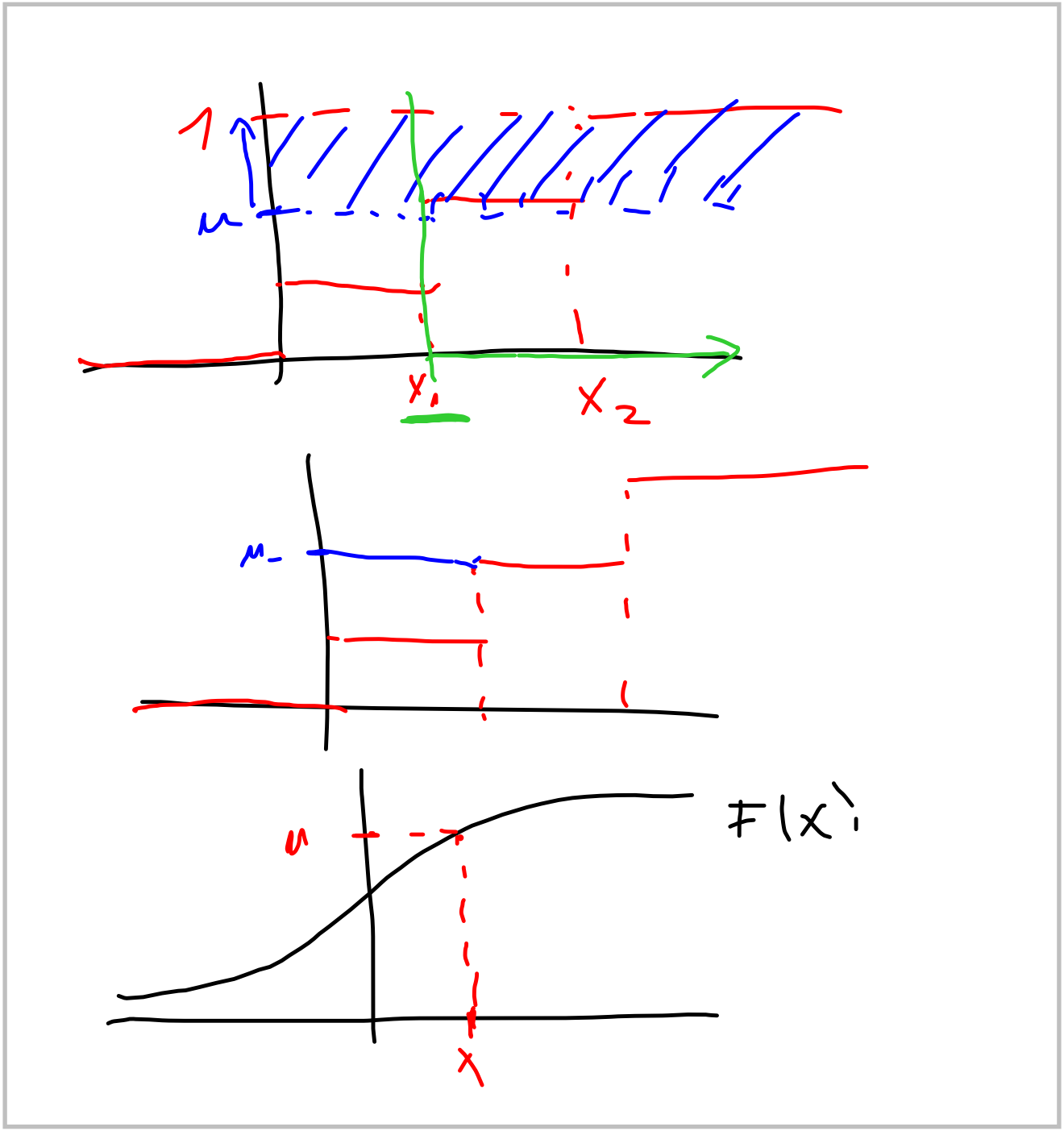
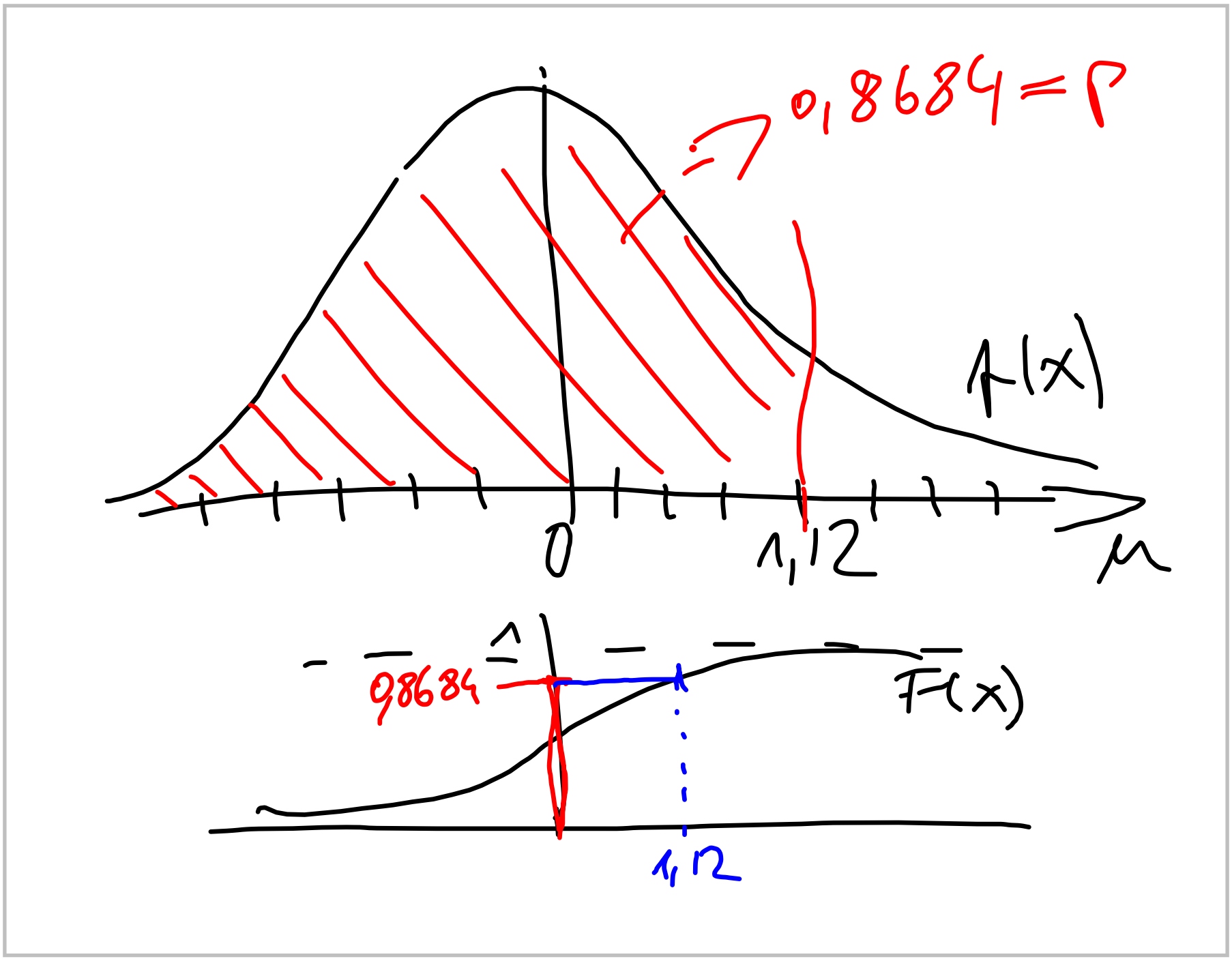
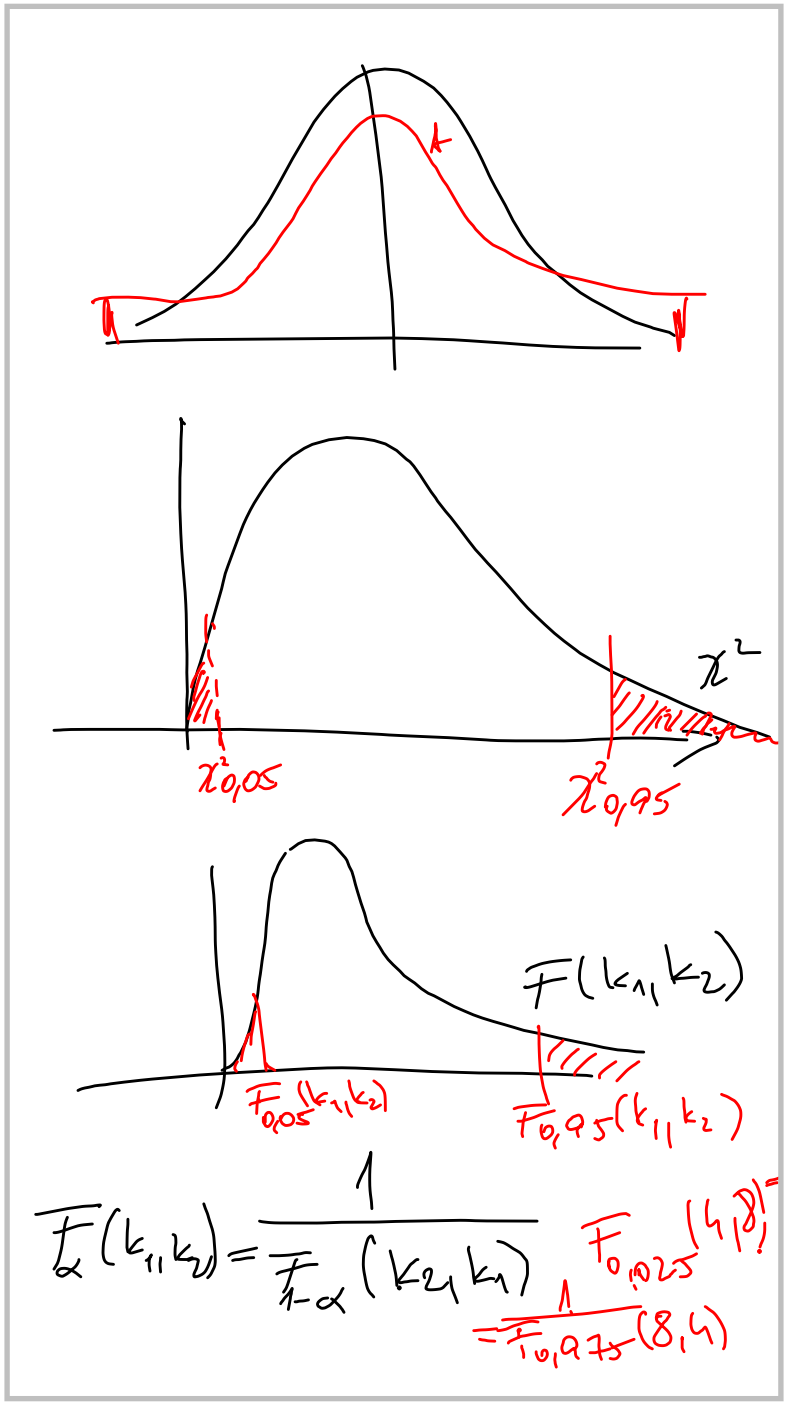
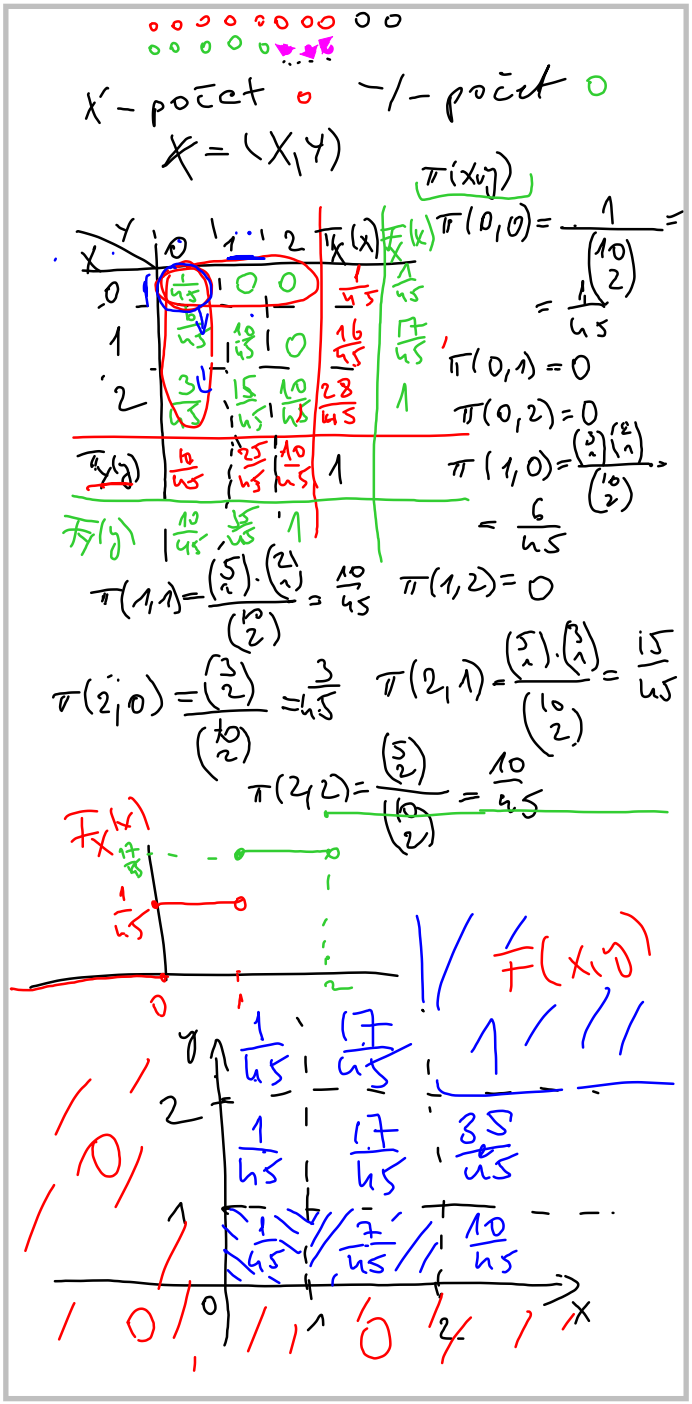

1
u
x1
x2
u
u
F(x)
x


0,8684 = P
f(x)
0
1,12
μ
0,8684
F(x)
1,12


A
χ^2
$\chi^2_{0,05}$
$\chi^2_{0,95}$
$F(k_1,k_2)$
$F_{0,05}(k_1,k_2)$
$F_{0,95}(k_1,k_2)$
$F_\alpha(k_1,k_2) = \frac{1}{F_{1-\alpha}(k_2,k_1)}$
$F_{0,025}(4,8) = \frac{1}{F_{0,975}(8,4)}$


X - počet ○ Y - počet ○
X = (X, Y)
π(x,y)
π(0,0) = 1 / (10 2) = 1/45
π(0,1) = 0
π(0,2) = 0
π(1,0) = (5 1)(5 1) / (10 2) = 6/45
π(1,1) = (5 1)(2 1) / (10 2) = 10/45
π(1,2) = 0
π(2,0) = (3 2) / (10 2) = 3/45
π(2,1) = (5 1)(3 1) / (10 2) = 15/45
π(2,2) = (5 2) / (10 2) = 10/45
F(x,y)

(X_1, X_2, X_3)

$$f(x_1,x_2,x_3) = \begin{cases} k \cdot x_1 x_2 x_3^2 & x_1 \in (0,1) \\ & x_2 \in (0,1) \\ & x_3 \in (0,3) \\ 0 & \text{jinak} \end{cases}$$

a) $$\int_0^1 \int_0^1 \int_0^3 k\, x_1 x_2 x_3^2 \, dx_3\, dx_2\, dx_1 =$$

$$= k \int_0^1 x_1 \left(\int_0^1 x_2 \left(\int_0^3 x_3^2 \, dx_3 \right) dx_2 \right) dx_1 =$$

$$= k \int_0^1 x_1 \int_0^1 x_2 \left[\frac{x_3^3}{3} \right]_0^3 = k \int_0^1 x_1 \int_0^1 x_2 \cdot 9 \, dx_2\, dx_1 =$$

$$= 9k \int_0^1 x_1 \left[\frac{x_2^2}{2} \right]_0^1 dx_1 = 9k \int_0^1 x_1 \cdot \frac{1}{2} dx_1 =$$

$$= \frac{9}{2} k \left[\frac{x_1^2}{2} \right]_0^1 = \frac{9}{4} k = 1 \Rightarrow k = \frac{4}{9}$$

b) $P\left(0 < X_1 < \frac{1}{2} \wedge \frac{1}{3} < X_2 < \frac{2}{3} \wedge 1 < X_3 < 2\right)$

$P\left(0 < X_1 < \frac{1}{2}\right) = P\left(X_1 \le \frac{1}{2}\right) - P(X_1 \le 0) =$

$F\left(\frac{1}{2}\right) - F(0)$

$$= \int_0^{\frac{1}{2}} \int_{\frac{1}{3}}^{\frac{2}{3}} \int_1^2 \frac{4}{9} x_1 \cdot x_2 \cdot x_3^2 \, dx_3\, dx_2\, dx_1 =$$

$$= \frac{4}{9} \int_0^{\frac{1}{2}} x_1 \int_{\frac{1}{3}}^{\frac{2}{3}} x_2 \int_1^2 x_3^2 \, dx_3\, dx_2\, dx_1 =$$

$$= \frac{4}{9} \int_0^{\frac{1}{2}} x_1 \int_{\frac{1}{3}}^{\frac{2}{3}} x_2 \left[\frac{x_3^3}{3} \right]_1^2 dx_2\, dx_1 =$$

$$= \frac{4}{9} \int_0^{\frac{1}{2}} x_1 \cdot \left(\frac{8}{3} - \frac{1}{3} \right) \left[\frac{x_2^2}{2} \right]_{\frac{1}{3}}^{\frac{2}{3}} dx_1 =$$

$$= \frac{28}{27} \cdot \left(\frac{4}{18} - \frac{1}{18} \right) \left[\frac{x_1^2}{2} \right]_0^{\frac{1}{2}} = \frac{28}{27} \cdot \frac{3}{18} \cdot \frac{1}{6} = \frac{7}{324} \doteq 0{,}0216$$

$$\pi(x,y) = \pi_X(x) \cdot \pi_Y(y)$$

$$\pi(0,0) = \pi_X(0) \cdot \pi_Y(0)$$

$$\frac{1}{45} = \frac{1}{45} \cdot \frac{10}{45}$$

$$F(x,y) = F_X(x) \cdot F_Y(y)$$

$$F(1,2) = F_X(1) \cdot F_Y(2)$$

$$\frac{17}{45} = \frac{17}{45} \cdot 1 \quad \checkmark$$

$$F(0,0) = F_X(0) \cdot F_Y(0)$$

$$\frac{1}{45} = \frac{1}{45} \cdot \frac{10}{45} \quad \times$$

(X_1, X_2)

$$f(x_1,x_2) = \begin{cases} 24 x_1^2 x_2 (1-x_1) & x_1 \in \langle 0,1 \rangle \\ 0 & \text{jinak} \end{cases} \quad x_2 \in \langle 0,1 \rangle$$

$$f_{X_1}(x_1) = \int_{-\infty}^{\infty} 24 x_1^2 x_2 (1-x_1)\, dx_2 =$$

$$= \int_{-\infty}^{0} 24 x_1^2 (1-x_1) x_2\, dx_2 =$$

$$= 24 x_1^2 (1-x_1) \left[\frac{x_2^2}{2}\right]_0^1 = 12 x_1^2 (1-x_1)$$

$$f_{X_2}(x_2) = \int_{-\infty}^{\infty} 24 x_1^2 x_2 (1-x_1)\, dx_1 =$$

$$= 24 x_2 \int_{-\infty}^{0} (x_1^2 - x_1^3)\, dx_1 = 24 x_2 \left[\frac{x_1^3}{3} - \frac{x_1^4}{4}\right]_0^1$$

$$= 24 x_2 \cdot \frac{1}{12} = 2 x_2$$

$$f(x_1,x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2)$$

$$24 x_1^2 x_2 (1-x_1) = 12 x_1^2 (1-x_1) \cdot 2 x_2 =$$

$$= 24 x_1^2 \cdot x_2 (1-x_1) \quad \checkmark$$