$$X \sim N(\mu, \sigma^2)$$

$$Y = a + bX \qquad b \neq 0$$

$$f_*(y) \qquad y = g(x) = a + bx$$

$$x = \frac{y-a}{b} = \tau(y)$$

$$\left|\frac{d\tau}{dy}\right| = \left|\frac{1}{b}\right| \qquad f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(\tau(y)) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left\{-\frac{1}{2\sigma^2} \cdot [\tau(y) - \mu]^2\right\} =$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}\left(\frac{y-a}{b} - \mu\right)^2\right\} =$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left\{-\frac{(y-(a+b\mu))^2}{2b^2\sigma^2}\right\}$$

$$f_*(y) = \frac{1}{|b|} \cdot \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{[y-(a+b\mu)]^2}{2b^2\sigma^2}\right\}$$

$$Y \sim N(a+b\mu, b^2\sigma^2) \qquad \underline{X \sim N(0,1)}$$

$$X \sim N(\mu, \sigma^2) \longrightarrow U \sim N(0,1)$$

$$a + b\mu = 0$$
$$b^2\sigma^2 = 1 \Rightarrow a = -\frac{\mu}{\sigma}, \quad b = \frac{1}{\sigma}$$

$$U = a + bX$$

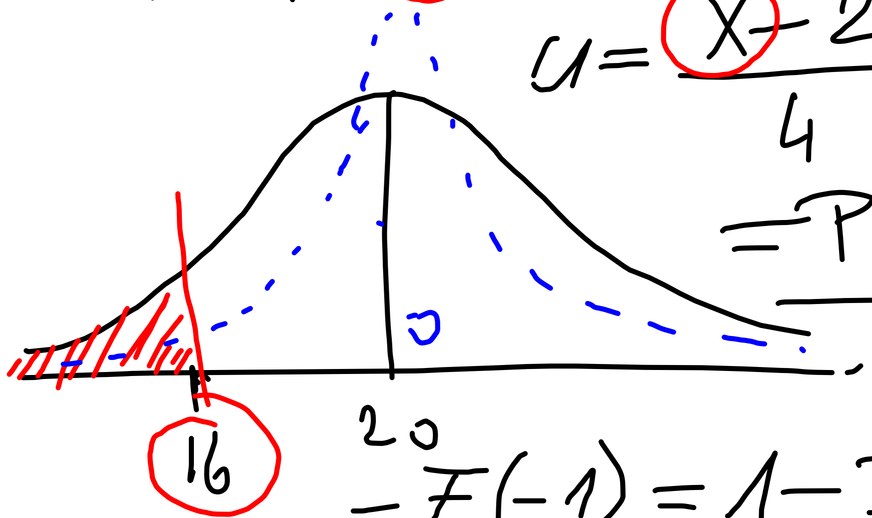
$$U = -\frac{\mu}{\sigma} + \frac{X}{\sigma} = \frac{X - \mu}{\sigma}$$

$X \sim N(20, 16)$

a) $X < 16$

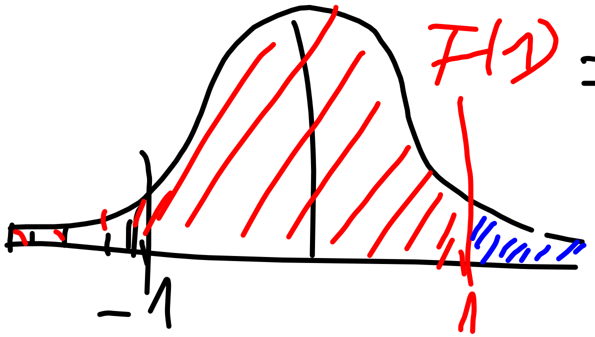
$P(X < 16) = P\left(U < \frac{16-20}{4}\right) =$

$U = \frac{X - 20}{4}$



$= P(U \leq -1) =$

$= F(-1) = 1 - F(1) =$



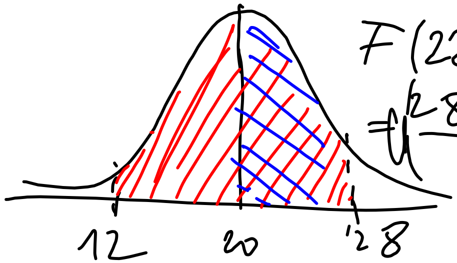
$F(1) = 1 - 0{,}84135 =$

$= 0{,}15865$

b) $P(X > 20) = 0{,}5$

$$P\left(U > \frac{20-20}{4}\right) = P(U > 0)$$

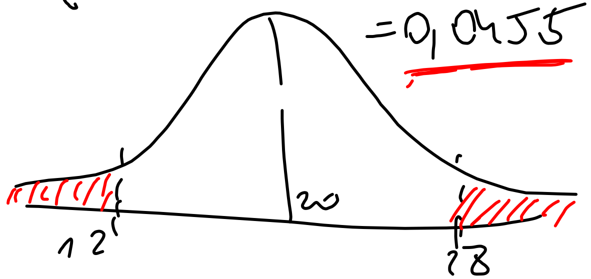
c) $P(12 < X < 28)$



$$F(28) - F(20) = U\left(\frac{28-20}{4}\right) - 0{,}5 = U(2) - 0{,}5 = \underbrace{0{,}97725 - 0{,}5}$$

$$\Rightarrow P(12 < X < 28) = 2 \cdot \qquad = 0{,}9545$$

d) $P(X < 12 \vee X > 28) = 1 - 0{,}9545 = 0{,}0455$



$$f(x) = \begin{cases} 2x e^{-x} & x > 0 \\ 0 & \text{jinak} \end{cases}$$

$$Y = X^2$$

$$y = g(x) = x^2 \longrightarrow \tau(y) = y^{\frac{1}{2}} \quad y > 0$$

$$x > 0$$

$$\left| \frac{d\tau(y)}{dy} \right| = \frac{1}{2} y^{-\frac{1}{2}}$$

$$f(\tau(y)) = 2 y^{\frac{1}{2}} e^{-y}$$

$$f^*(y) = \cancel{2} y^{\frac{1}{2}} e^{-y} \cdot \frac{1}{\cancel{2}} \cancel{y^{-\frac{1}{2}}} = \begin{cases} e^{-y} & y > 0 \\ 0 & \text{jinak} \end{cases}$$

$Y = X_1 + X_2 \qquad f(x_1, x_2)$

$\qquad\qquad\qquad\qquad f_1(x_1)\, f_2(x_2)$

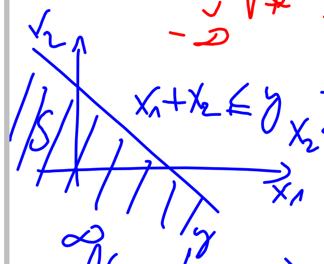
$f_*(y) \overset{?}{=}$

$F_*(y) = P(Y \le y) = P(X_1 + X_2 \le y) =$

$= \iint_S f(x_1, x_2)\, dx_1\, dx_2 =$

$\int_{-\infty}^{y} f_*(y)\, dy$

$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y-x_1} f_1(x_1)\, f_2(x_2)\, dx_2 \right) dx_1 =$



$x_1 + x_2 \le y$

$x_2 \le y - x_1$

$t = x_1 + x_2$

$dt = dx_2$

$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y} f_1(x_1)\, f_2(t - x_1)\, dt \right) dx_1 =$

$= \int_{-\infty}^{y} \left(\int_{-\infty}^{\infty} f_1(x_1) \cdot f_2(t - x_1)\, dx_1 \right) dt = F_*(y)$

$f_*(y) = \dfrac{dF_*(y)}{dy} = \int_{-\infty}^{\infty} f_1(x_1) \cdot f_2(y - x_1)\, dx_1$

konvoluce f_1 a f_2

$$X \to \pi(x) = \begin{cases} \frac{\lambda^x}{x!} e^{-\lambda} & x = 0, 1, \ldots \\ 0 & \text{jinak} \end{cases}$$

$$\underline{Y = 4X}$$

$$\pi_Y(y) = P(Y = y) = P(4X = y) =$$

$$= P\left(X = \frac{y}{4}\right) = \pi\left(\frac{y}{4}\right) = \begin{cases} \frac{\lambda^{\frac{y}{4}}}{\left(\frac{y}{4}\right)!} e^{-\lambda} & y = 0, 4, 8, \ldots \\ 0 & \text{jin} \end{cases}$$

$$D = E\left[(X - E(X))^2\right] =$$

$$= E\left[X^2 - 2XE(X) + E^2(X)\right] =$$

$$= E(X^2) - E(2XE(X)) + E(E^2(X))$$

$$E(a) = a$$

$$E(a \cdot X) = aE(X)$$

$$= E(X^2) - 2E(X)E(X) + E^2(X) =$$

$$= E(X^2) - 2E^2(X) + E^2(X) =$$

$$= E(X^2) - E^2(X)$$

$X \quad f(x) = \begin{cases} \frac{1}{a} & 0 < x < a \\ 0 & \text{jinak} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^a x \cdot \frac{1}{a}\,dx =$$

$$= \left[\frac{x^2}{2a}\right]_0^a = \frac{a^2}{2a} = \frac{a}{2}$$

$$D(X) = E\big(\underbrace{[X - E(X)]^2}_{g(x)}\big) =$$

$$= \int_{-\infty}^{\infty} (x - E(x))^2 \cdot f(x)\,dx =$$

$$= \int_0^a \left(x - \frac{a}{2}\right)^2 \cdot \frac{1}{a}\,dx = \frac{1}{a}\int_0^a \left(x^2 - xa + \frac{a^2}{4}\right) dx =$$

$$= \frac{1}{a}\left[\frac{x^3}{3} - \frac{ax^2}{2} + \frac{a^2}{4}x\right]_0^a =$$

$$= \frac{1}{a} \cdot \frac{a^3}{3} - \frac{1}{a} \cdot \frac{a^3}{2} + \frac{1}{a} \cdot \frac{a^3}{4} =$$

$$= \frac{a^2}{3} - \frac{a^2}{2} + \frac{a^2}{4} = \frac{a^2}{12}$$

$$D(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^a x^2 \cdot \frac{1}{a}\,dx =$$

$$= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$$

$$D(X) = \frac{a^2}{3} - \left(\frac{a}{2}\right)^2 = \frac{a^2}{3} - \frac{a^2}{4} = \frac{a^2}{12}$$

$$E(X^4) = \int_0^a x^4 \cdot \frac{1}{a}\,dx = \left[\frac{x^5}{5a}\right]_0^a = \frac{a^4}{5}$$

---

1. $E(2X+3) = E(2X) + E(3) =$
$= 2E(X) + 3 = a + 3$

2. $E(3X^2 - 2X + 1) = 3E(X^2) - 2E(X) + 1$
$= a^2 - a + 1$

3. $D(2X+3) = 4 \cdot D(X) = 4 \cdot \frac{a^2}{12} = \frac{a^2}{3}$

4. $D(X^2+1) = D(X^2) = D(Y) = E(Y^2) - E^2(Y) = E(X^4) - E^2(X^2) = \frac{a^4}{5} - \left(\frac{a^2}{3}\right)^2 = \frac{a^4}{5} - \frac{a^4}{9} = \frac{4a^4}{45}$

$$\pi(x) = \begin{cases} \frac{\lambda^x}{x!} \cdot e^{-\lambda} & x = 0,1,2,\dots \\ 0 & \text{jinak} \end{cases}$$

$$E(X) = \sum_{x=-\infty}^{\infty} x \cdot \pi(x) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x}{x!} e^{-\lambda} =$$

$$= \sum_{x=1}^{\infty} \frac{\lambda^x \cdot e^{-\lambda}}{(x-1)!} = e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!} =$$

$x-1=y$

$$= \lambda \cdot e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \lambda$$

$$D(X) = E(X^2) - [E(X)]^2$$

$$E[X(X-1)] = \sum_{x=0}^{\infty} x(x-1) \cdot \pi(x) =$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{\lambda^x}{x!} \cdot e^{-\lambda} = e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} =$$

$y = x-2$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+2}}{y!} = \lambda^2 e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda^2 e^{-\lambda} e^{\lambda} =$$

$$= \lambda^2$$

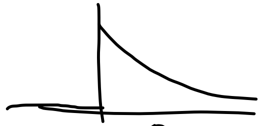
$$E(X(X-1)) = E(X^2 - X) = E(X^2) - E(X)$$

$$E(X^2) = E(X(X-1)) + E(X)$$

$$D(X) = E(X^2) - E^2(X) = \lambda^2 + \lambda - \lambda^2 =$$

$$= \lambda$$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{jinak} \end{cases}$$



$$E(X) = \int_0^\infty x f(x)\,dx = \int_0^\infty x\cdot\lambda\, e^{-\lambda x}\,dx =$$

$u = \lambda x \quad u' = \lambda$

$v' = e^{-\lambda x} \quad v = -\frac{1}{\lambda} e^{-\lambda x}$

$$= \overbrace{\left[-x e^{-\lambda x}\right]_0^\infty}^{\to 0} + \int_0^\infty e^{-\lambda x}\,dx =$$

$$= \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^\infty = \frac{1}{\lambda}\cdot e^0 = \frac{1}{\lambda}$$

$$D(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \int_0^\infty x^2 f(x)\,dx = \int_0^\infty x^2 \lambda\, e^{-\lambda x}\,dx =$$

$u = \lambda x^2 \quad u' = 2\lambda x$

$v' = e^{-\lambda x} \quad v = -\frac{1}{\lambda} e^{-\lambda x}$

$$= \overbrace{\left[-x^2 e^{-\lambda x}\right]_0^\infty}^{\to 0} + \int_0^\infty \underbrace{2x\, e^{-\lambda x}\,dx}_{\frac{2\cdot E(X)}{\lambda}} = \frac{2\cdot\frac{1}{\lambda}}{\lambda} = \frac{2}{\lambda^2}$$

$$D(X) = E(X^2) - E^2(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 =$$

$$= \frac{1}{\lambda^2}$$