$$P(X > 3\delta), \quad X \sim Ex\left(\frac{1}{\delta}\right)$$

$$P(X > 3\delta) = 1 - P(X \leq 3\delta) =$$

$$= 1 - F_X(3\delta)$$

$$f(x) = \begin{cases} \frac{1}{\delta} e^{-\frac{x}{\delta}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\underline{F(x)} = \int_{-\infty}^{x} \frac{1}{\delta} e^{-\frac{t}{\delta}} dt = \frac{1}{\delta} \int_{0}^{x} e^{-\frac{t}{\delta}} dt =$$

$$= \frac{1}{\delta} \cdot \left[-\delta e^{-\frac{t}{\delta}}\right]_0^x =$$

$$= \frac{1}{\delta} \left(-\delta e^{-\frac{x}{\delta}} + \delta e^{0}\right) = \underline{1 - e^{-\frac{x}{\delta}}}$$

$$\underline{P(X > 3\delta)} = 1 - F(3\delta) =$$

$$= 1 - \left(1 - e^{-\frac{3\delta}{\delta}}\right) = e^{-3}$$

$$= \underline{0{,}04979}$$

$$\underbrace{P(X > 3\sigma)}_{\varepsilon = 3} \leq \frac{1}{3}$$

X – počet vadných výrobků

$X \sim Bi(n, \theta) \quad E(X) = n \cdot \theta$

$\sim Bi(3000; 0.04)$

$Y = \frac{X}{n} \quad E(Y) = E\left(\frac{X}{n}\right) = \frac{E(X)}{n} = \theta = 0{,}04$

$D(Y) = D\left(\frac{X}{n}\right) = \frac{D(X)}{n^2} = \frac{n \cdot \theta(1-\theta)}{n^2} =$

$= \frac{\theta(1-\theta)}{n} = 0{,}0000128$

$P(|Y - E(Y)| < 0{,}01)$

$P(|X - E(X)| \geq t) \leq \frac{D(X)}{t^2}$

$1 - P(|X - E(X)| < t) \leq \frac{D(X)}{t^2}$

$P(|X - E(X)| < t) \geq 1 - \frac{D(X)}{t^2}$

$P(|Y - E(Y)| < 0{,}01) \geq 1 - \frac{D(Y)}{0{,}01^2} =$

$= 1 - \frac{0{,}0000128}{0{,}01^2} = 0{,}872$

b) $n = 30\,000 \quad \theta = 0{,}004$

$P(|Y - E(Y)| < 0{,}01) \geq 1 - \frac{D(Y)}{0{,}01^2}$

$D(Y) = \frac{\theta(1-\theta)}{n} = \frac{0{,}004 \cdot (1 - 0{,}004)}{30000} =$

$= 0{,}0000001328$

$1 - \frac{D(Y)}{0{,}01^2} = 1 - \frac{0{,}0000001328}{0{,}01^2} =$

$\doteq 0{,}998672$

$X_i \to X = \sum_{i=1}^{100} X_i$

$E(X_i) = 40 \to \underline{E(X)} = E\left(\sum_{i=1}^{100} X_i\right) =$

$= \sum_{i=1}^{100} E(X_i) = \underline{4000}$

$D(X_i) = 900$

$\Downarrow$

$\underline{D(X)} = D\left(\sum_{i=1}^{100} X_i\right) \overset{SN}{=} \sum_{i=1}^{100} D(X_i) =$

$= \underline{90000}$

$\underline{\sqrt{D(X)} = 300}$

$P(X < x) = 0{,}95$

$P(X < x) = P(X - 4000 < x - 4000) =$

$= P\left(\frac{X - 4000}{300} < \frac{x - 4000}{300}\right) =$

$U \sim N(0,1)$

$= P\left(U < \frac{x - 4000}{300}\right) = \Phi\left(\frac{x - 4000}{300}\right)$

$= 0{,}95$

$\Phi(1{,}645) = 0{,}95$

$\Phi^{-1}(0{,}95) = 1{,}645$

$\frac{x - 4000}{300} = 1{,}645$

$x = 300 \cdot 1{,}645 + 4000$

$\underline{x = 4493{,}5}$

X – počet narozených chlapců

$X \sim Bi(n, \Theta)$ $\quad n = 10\,000$, $\Theta = 0{,}515$

$Y_1, Y_2, Y_3, \dots, Y_{10000} = X$

$n\Theta(1-\Theta) > 9 \qquad 10\,000 \cdot 0{,}515(1-0{,}515) = 2497{,}75 > 9$ ✓

$$\frac{1}{n+1} < \Theta < \frac{n}{n+1}$$

$0{,}00009999 < 0{,}515 < 0{,}9999$ ✓

$E(X) = n \cdot \Theta = 10\,000 \cdot 0{,}515 = 5150$

$D(X) = n \cdot \Theta(1-\Theta) = 2497{,}75$

$\sqrt{D(X)} = 50$

a) $P(X < 5000) = P(\underset{-150}{X - 5150 <})$

$= P\left(\frac{X-5150}{50} < -3\right) = P(U < -3) =$

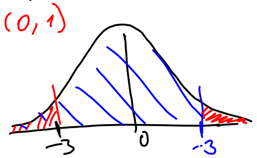
$U \sim N(0,1)$

$= \phi(-3) =$

$= 1 - \phi(3) =$

$= 1 - 0{,}99865 =$

$= 0{,}00135$



2. $P(5000 < X < 5300) = P(-150 < X - 5150 < 150)$

$= P\left(-3 < \frac{X-5150}{50} < 3\right) = P(-3 < U < 3)$

$U \sim N(0,1)$

$= \phi(3) - \phi(-3) = 0{,}99865 - 0{,}00135$

$= 0{,}9973$

3. $Y = \frac{X}{n} \qquad E(Y) = \Theta = 0{,}515$

$D(Y) = \frac{\Theta(1-\Theta)}{n} = 0{,}000025$

$P(0{,}515 < Y < 0{,}517) =$

$= P\left(\frac{0{,}515 - 0{,}515}{0{,}005} < \frac{Y - 0{,}515}{0{,}005} < \frac{0{,}517 - 0{,}515}{0{,}005}\right)$

$$P\left(0 < \frac{Y - 0{,}515}{0{,}05} < 0{,}4\right) =$$

$\sim U \sim N(0{,}1)$

$$= P(0 < U < 0{,}4) =$$

$$= \Phi(0{,}4) - \Phi(0) = 0{,}65542 - 0{,}5 =$$

$$= \underline{0{,}15542}$$

$$P(5150 < X < 5170) = P(0 < U < 0{,}4)$$

Y – počet domácností s videem

$$Y \sim Bi(900; 0,8)$$

$n\Theta(1-\Theta) > 9$ $\qquad 900 \cdot 0,8 \cdot 0,2 = 144 > 9$ ✓

$\frac{1}{n+1} < \Theta < \frac{n}{n+1}$ $\qquad 0,00111 < 0,8 < 0,998$ ✓

$E(X) = n \cdot \Theta) = 900 \cdot 0,8 = 720$

$P(Y \leq y) = 0,95$

$$P(Y \leq y) = P(Y - 720 \leq y - 720) = P\left(\frac{Y-720}{12} \leq \frac{y-720}{12}\right) = P\left(U \leq \frac{y-720}{12}\right)$$

$U \sim N(0;1)$

$$= \Phi\left(\frac{y-720}{12}\right) = 0,95$$

$$\Phi(1,645) = 0,95$$

$$\frac{y-720}{12} = 1,645 \Rightarrow y = 739,74$$

$$y = 739$$

Y_n – počet poruch na n výrobků

$$n = 100 \qquad \theta = 0{,}05$$

$$n \cdot \theta (1-\theta) > 9$$

$$100 \cdot 0{,}05 \cdot 0{,}95 = 4{,}75 < 9 \quad \times$$

$$n \geq 30 \ \checkmark \qquad \theta \leq 0{,}1 \ \checkmark$$

$$P(Y_{100} \geq 5) = 1 - \underbrace{P(Y_{100} \leq 4)}_{} =$$

$$= 1 - \sum_{y=0}^{4} \binom{n}{y} \theta^{y} (1-\theta)^{n-y} \approx 1 - \sum_{y=0}^{4} \frac{(n\theta)^{y}}{y!} e^{-n\theta} =$$

$$= 1 - \sum_{y=0}^{4} \frac{5^{y}}{y!} e^{-5} = 1 - (0{,}00674 + 0{,}03369 + 0{,}08422 + 0{,}14037 + 0{,}17547)$$

$$= \underline{0{,}55951}$$