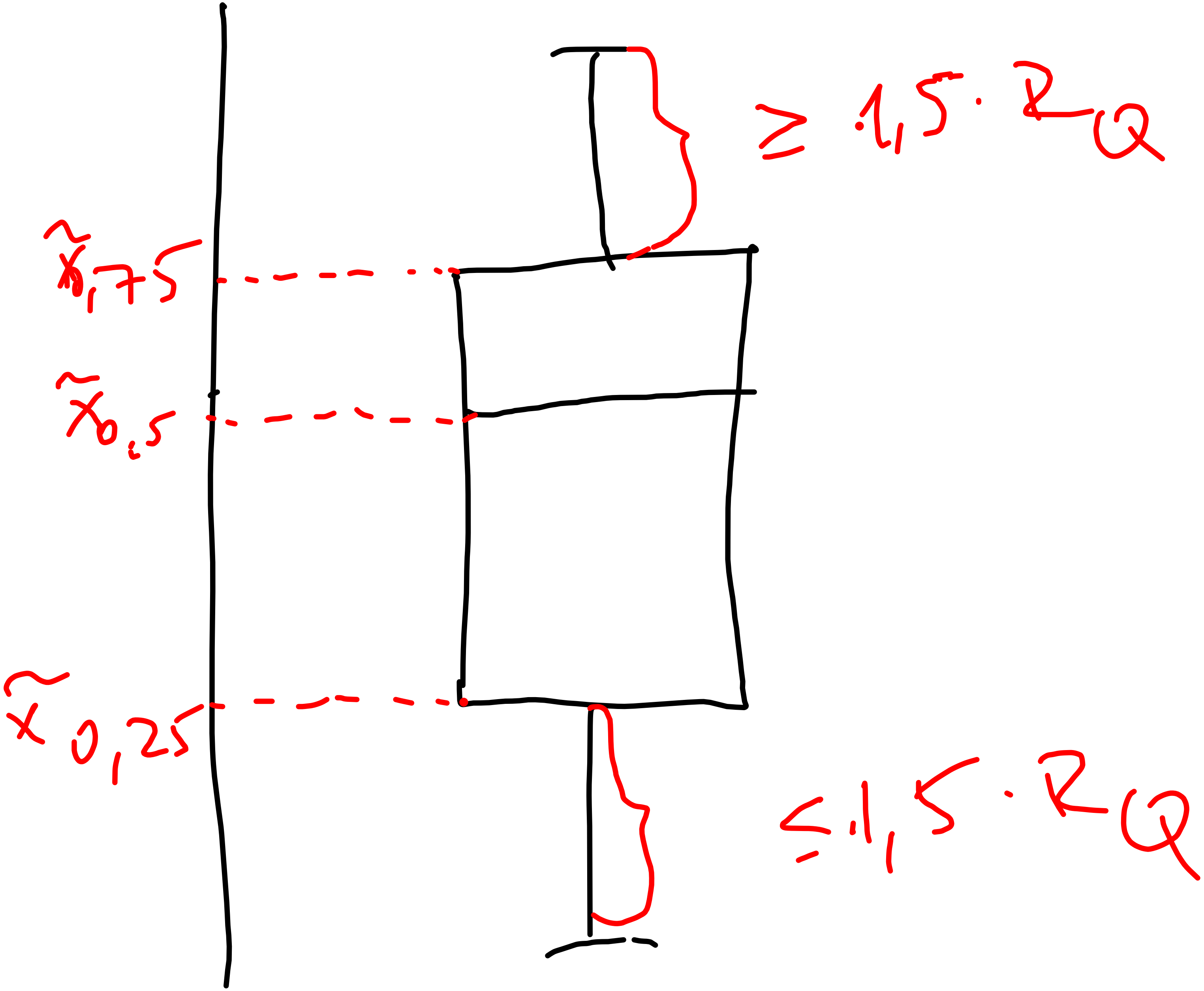


$\tilde{x}_{0,75}$
$\tilde{x}_{0,5}$
$\tilde{x}_{0,25}$
$\geq 1{,}5 \cdot R_Q$
$\leq 1{,}5 \cdot R_Q$

3 4 7 7 7 7 8 8 8 8 9 9 9 9 9 9
10 10 10 11

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{20}\sum_{i=1}^{20} x_i = 8{,}1$$

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\cdot\bar{x}^2\right) =$$

$$= \frac{1}{19}\left(1384 - 20\cdot 8{,}1^2\right) =$$

$$= 3{,}78$$

$p = 0{,}5 \quad n = 20 \quad p\cdot n = 10$

$$\tilde{x}_{0{,}5} = \frac{1}{2}\left(x_{(10)} + x_{(11)}\right) = \frac{1}{2}(8+9) =$$
$$= 8{,}5$$

$p = 0{,}25 \quad n\cdot p = 5$

$$\tilde{x}_{0{,}25} = \frac{1}{2}\left(x_{(5)} + x_{(6)}\right) = 7$$

$p = 0{,}75 \quad n\cdot p = 15$

$$\tilde{x}_{0{,}75} = \frac{1}{2}\left(x_{(15)} + x_{(16)}\right) = 9$$

$$p = 0{,}125 \quad n\cdot p = 2\cancel{0}\cdot\frac{125}{100\cancel{0}} = \frac{125}{50} = \frac{25}{10} =$$
$$= \frac{5}{2} =$$
$$2{,}5$$

$$[n\cdot p] = 2$$

$$\tilde{x}_{0{,}125} = x_{(2+1)} = x_{(3)} = 7$$

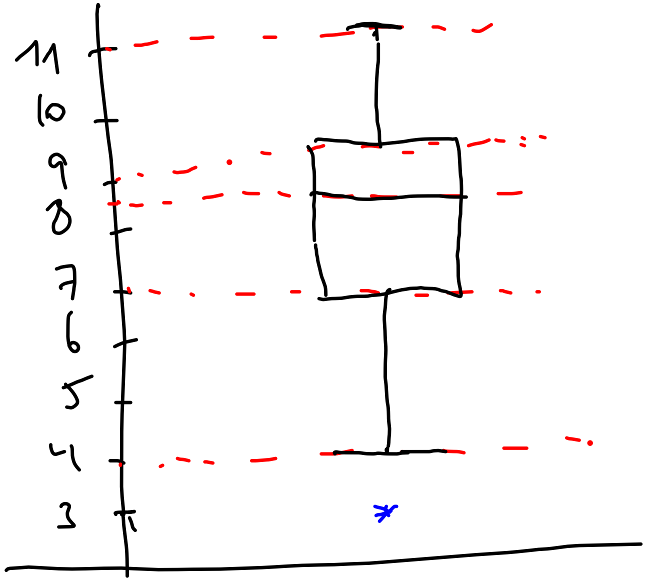
$$R_Q = \tilde{x}_{0{,}75} - \tilde{x}_{0{,}25} = 9 - 7 = 2$$

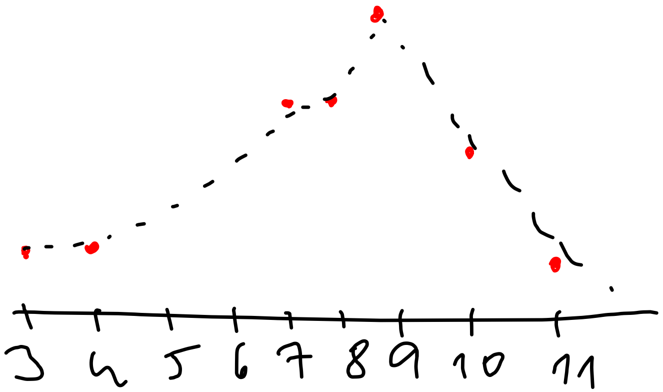
$$1{,}5\cdot R_Q = 2\cdot 1{,}5 = 3$$

Název: V 14-8:10 (2 z 10)

$\tilde{x}_{0,5} = 8,5 \quad \tilde{x}_{0,25} = 7 \quad \tilde{x}_{0,75} = 9$

$1,5 \cdot R_Q = 3$



$$M = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad X_i \sim F(x_i)$$

$$E(X_i) = \mu$$

$$D(X_i) = \sigma^2$$

$$i = 1, \ldots, n$$

$$E(M) = E\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i\right) =$$

$$= \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{1}{n} \cdot n \cdot \mu =$$

$$= \mu$$

$$D(M) = D\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} D\left(\sum_{i=1}^{n} X_i\right) =$$

$$\overset{sn}{=} \frac{1}{n^2} \sum_{i=1}^{n} D(X_i) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$X \sim N(72, 9^2)$$

$$P(X > 80) = P(X - 72 > 80 - 72) =$$

$$= P\left(\underbrace{\frac{X-72}{9}}_{U \sim N(0,1)} > \frac{8}{9}\right) =$$

$$= P\left(U > \frac{8}{9}\right) = 1 - P\left(U \le \frac{8}{9}\right) =$$

$$= 1 - 0{,}811 = \underline{\underline{0{,}189}}$$

b) $M_{10} = \frac{1}{10} \sum_{i=1}^{10} X_i$

$$P(M_{10} > 80) = P(M_{10} - 72 > 80 - 72)$$

$$= P\left(\underbrace{\frac{M_{10} - 72}{\frac{9}{\sqrt{10}}}}_{U \sim N(0,1)} > \frac{8}{\frac{9}{\sqrt{10}}}\right) =$$

$\frac{\sigma}{\sqrt{n}}$

$$= P\left(U > \frac{\sqrt{10} \cdot 8}{9}\right) = 1 - P(U \le 2{,}8\dot{1})$$

$$= 1 - 0{,}99752 = \underline{\underline{0{,}0025}}$$

Název: V 14-8:37 (5 z 10)

$$X_1, X_2, \ldots, X_n$$

$$E(X_i) = \mu$$

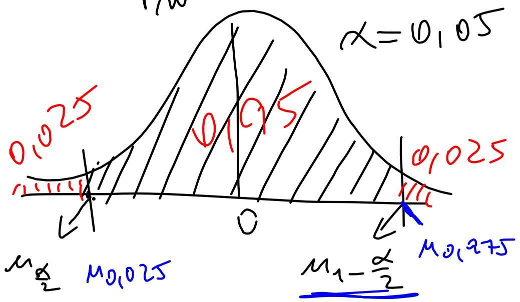
$$M = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad E(M) = \mu$$

$$T = X_1 \qquad E(T) = \mu$$

$$D(M) = \frac{\sigma^2}{n} \qquad D(T) = \sigma^2$$

$$M = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad E(M)=\mu \qquad D(M)=\frac{\sigma^2}{n}$$

$$U = \frac{M-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$




$$P\left(u_{\frac{\alpha}{2}} \le U \le u_{1-\frac{\alpha}{2}}\right) \geq 1-\alpha$$

$$P\left(u_{\frac{\alpha}{2}} \le \frac{M-\mu}{\frac{\sigma}{\sqrt{n}}} \le u_{1-\frac{\alpha}{2}}\right) =$$

$$= P\left(\frac{\sigma}{\sqrt{n}}\, u_{\frac{\alpha}{2}} \le M-\mu \le \frac{\sigma}{\sqrt{n}} \cdot u_{1-\frac{\alpha}{2}}\right)$$

$$= P\left(-M+\frac{\sigma}{\sqrt{n}}\, u_{\frac{\alpha}{2}} \le -\mu \le -M + \frac{\sigma}{\sqrt{n}}\, u_{1-\frac{\alpha}{2}}\right)$$

$$= P\left(M-\frac{\sigma}{\sqrt{n}}\, u_{\frac{\alpha}{2}} \ge \mu \ge M-\frac{\sigma}{\sqrt{n}}\, u_{1-\frac{\alpha}{2}}\right)$$

$$u_{\frac{\alpha}{2}} = -u_{1-\frac{\alpha}{2}}$$

$$P\left(\underbrace{M-\frac{\sigma}{\sqrt{n}}\, u_{1-\frac{\alpha}{2}}}_{D} \le \mu \le \underbrace{M+\frac{\sigma}{\sqrt{n}} \cdot u_{1-\frac{\alpha}{2}}}_{H}\right)$$

$$M \xrightarrow{data} m = 870{,}3\ m\cdot s^{-1}$$

$$\sigma = 2{,}1\ m\cdot s^{-1}$$

$$X \sim N(\mu; 2{,}1^2) \quad n = 5$$

$$U = \frac{M - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$u_{0{,}975}$

$$D = M - \frac{\sigma}{\sqrt{n}} \cdot u_{1-\frac{\alpha}{2}} =$$

$$= 870{,}3 - \frac{2{,}1}{\sqrt{5}} \cdot 1{,}96 =$$

$$= 868{,}4593$$

$$H = M + \frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}} =$$

$$= 870{,}3 + \frac{2{,}1}{\sqrt{5}} \cdot 1{,}96 =$$

$$= 872{,}1409$$

$$\mu \in (868{,}4593; 872{,}1409)$$

$X_i \sim N(\mu; 0{,}04)$

$\alpha = 0{,}05$

$\mu \in (D, H)$

$H - D \leq 0{,}16$

$\left(M + \frac{\sigma}{\sqrt{n}} \cdot u_{1-\frac{\alpha}{2}}\right) - \left(M - \frac{\sigma}{\sqrt{n}} u_{1-\frac{\alpha}{2}}\right) \leq 0{,}16$

$\frac{2\sigma}{\sqrt{n}} \cdot u_{1-\frac{\alpha}{2}} \leq 0{,}16$

$$\frac{2\sigma \cdot u_{1-\frac{\alpha}{2}}}{0{,}16} \leq \sqrt{n}$$

$$\frac{2 \cdot 0{,}2 \cdot 1{,}96}{0{,}16} \leq \sqrt{n}$$

$4{,}9 \leq \sqrt{n}$

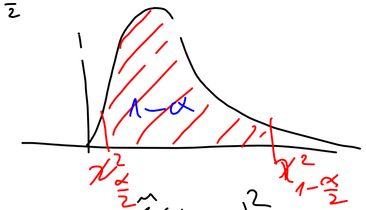
$n \geq 24{,}01$

$\boxed{n = 25}$

$n = 4 \qquad \mu = 0{,}30 \qquad \alpha = 0{,}05$

$$\boxed{W = \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{\sigma^2} \sim \chi^2(n)}$$

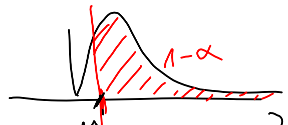
$$P\left(\chi^2_{\frac{\alpha}{2}}(n) \le W \le \chi^2_{1-\frac{\alpha}{2}}(n)\right)$$




$$P\left(\chi^2_{\frac{\alpha}{2}}(n) \le \frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2} \le \chi^2_{1-\frac{\alpha}{2}}\right) =$$

$$= P\left(\frac{1}{\chi^2_{\frac{\alpha}{2}}(n)} \ge \frac{\sigma^2}{\sum_{i=1}^{n}(X_i-\mu)^2} \ge \frac{1}{\chi^2_{1-\frac{\alpha}{2}}}\right)$$

$$= P\left(\underbrace{\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\chi^2_{1-\frac{\alpha}{2}}}}_{D} \le \sigma^2 \le \underbrace{\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\chi^2_{\frac{\alpha}{2}}(n)}}_{H}\right)$$




$$D = \frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\chi^2_{1-\alpha}(n)} =$$

$\chi^2_{0{,}95}(4)$

$$= \frac{6{,}0 \cdot 10^{-4}}{9{,}488} = 6{,}324 \cdot 10^{-5}$$

$\sigma^2 \in (D; \infty) \qquad \sigma^2 \ge D$

$\sigma \ge \sqrt{D}$

$\sigma \ge 0{,}0078$