$X_n \sim Bi(n, p)$

$$Y_n = \frac{X_n - n \cdot p}{\sqrt{np(1-p)}} \quad \text{cca} \quad N(0,1)$$

normované normální

$$P(a < Y_n < b) = \Phi(b) - \Phi(a)$$

$1 \quad 2 \quad \ldots \quad 12\,000$

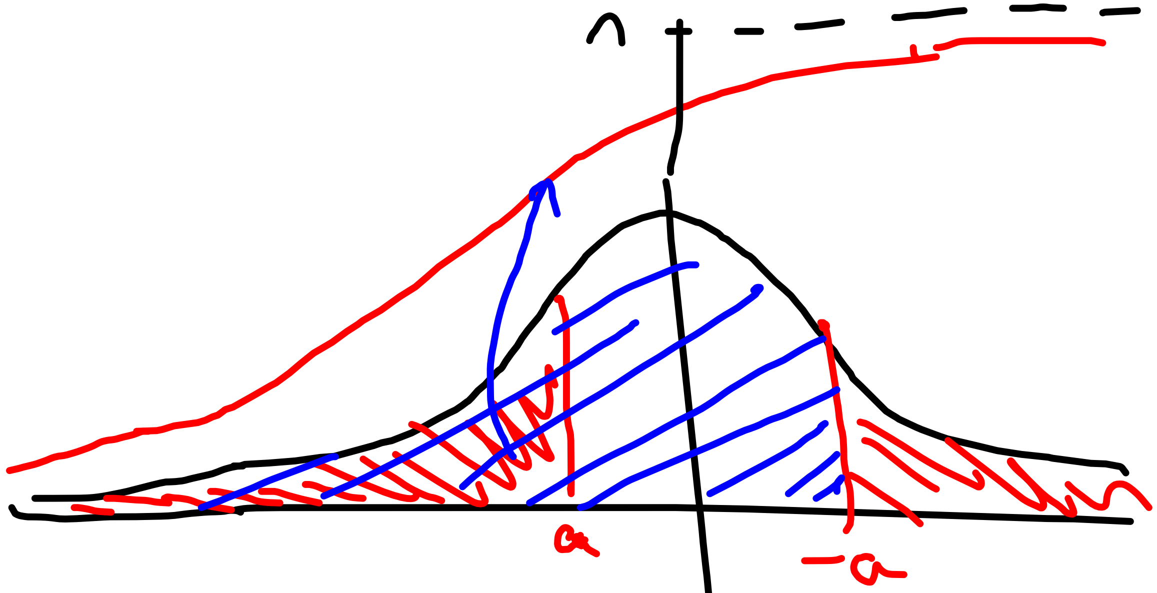
$$\sum_{k=2100}^{2100} \binom{12\,000}{k} \left(\frac{1}{6}\right)^{k} \left(\frac{5}{6}\right)^{12000-k}$$

$$Y_n = \frac{X_n - np}{\sqrt{np(1-p)}} \qquad P[a < Y_n < b] = \Phi(b) - \Phi(a)$$

$$P\left[a < \frac{X_n - np}{\sqrt{\ldots}} < b\right] = \ldots$$

$$a \leftarrow \frac{A - np}{\sqrt{np(1-p)}} \qquad P[A < X_n < B] = \Phi\left(\frac{B - np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{A - np}{\sqrt{np(1-p)}}\right)$$

$$b \leftarrow \frac{B - np}{\sqrt{\ldots}}$$



hustota

$N(0,1)$

$$F(a) = \int_{-\infty}^{a} f(x)\,dx$$

$$a = -2\sqrt{6}$$

$$F(a) = 1 - \underline{F(-a)}$$

$$\Phi(-2\sqrt{6}) = 1 - \Phi(2\sqrt{6}) \approx 0{,}0000\ldots$$

$$P\left[-\delta < \frac{X_n}{n} - p < \delta\right] =$$

$$= P\left[-\delta < \frac{X_n - pn}{n} < \delta\right] =$$

$$= P\left[-\frac{\delta\sqrt{n}}{\sqrt{p(1-p)}} < \frac{X_n - pn}{\sqrt{np(1-p)}} < \frac{\delta\sqrt{n}}{\sqrt{p(1-p)}}\right] =$$

$$= P\left[-\frac{\delta n}{\sqrt{np(1-p)}} < \frac{X_n - pn}{\sqrt{np(1-p)}} < \frac{\delta n}{\sqrt{np(1-p)}}\right]$$

$$= \Phi\left(\frac{\delta n}{\sqrt{np(1-p)}}\right) - \Phi\left(-\frac{\delta n}{\sqrt{np(1-p)}}\right)$$

$\sigma =$

$$\frac{n \cdot 0{,}05}{\sqrt{n\underbrace{p(1-p)}_{\leq \frac{1}{4}}}} \geq 1{,}645$$

$$p(1-p) \leq \frac{1}{4}$$

$$p - p^2 - \frac{1}{4} \leq 0 \quad |(-4)$$

$$4p^2 - 4p + 1 \geq 0$$

$$(2p-1)^2 \geq 0$$

$$\frac{n \cdot 0{,}05}{\frac{1}{2}\sqrt{n}} \geq 1{,}645$$

$$\sqrt{n} \cdot 0{,}05 \geq \cancel{3{,}29}\ 0{,}8225$$

$$\sqrt{n} \geq \cancel{65{,}}\ 16{,}45$$

$$\underline{n \geq 270{,}6}$$

$$E(X) = 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + \dots + 6\cdot\frac{1}{6} = \frac{21}{6} = 3{,}5$$

X ... počet ok na kostce

$$E(X^2) = 1^2\cdot\frac{1}{6} + 2^2\cdot\frac{1}{6} + \dots + 6^2\,\frac{1}{6} =$$

$$= \frac{1}{6}\left(\frac{6(6+1)(2\cdot 6+1)}{6}\right) = \underline{\underline{\frac{1}{6}\cdot 91}}$$

$$\sum_{j=0}^{n-1}\binom{n-1}{j}p^j\cdot(1-p)^{n-1-j} =$$

$$= (p+1-p)^{n-1} = 1$$

$$E(a+bX) = a + b \cdot E(X)$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$E(a+bX) = \int_{-\infty}^{\infty} (a+bx) f(x)\,dx =$$

$$= \int_{-\infty}^{\infty} a \cdot f(x)\,dx + \int_{-\infty}^{\infty} b x f(x)\,dx =$$

$$= a \cdot 1 + b \underbrace{\int_{-\infty}^{\infty} x f(x)\,dx}_{E(X)} = a + b \cdot E(X)$$

X, Y nezávislé: $\forall a, b \in \Omega$

$$P[X=a, Y=b] = P[X=a] \cdot P[Y=b]$$

$$E(XY) = \sum_{\substack{i=1\dots m \\ j=1\dots n}} x_i y_j \underbrace{P[X=x_i, Y=y_j]}_{\text{nezávislost!}} =$$

$$= \sum x_i y_j \overbrace{P[X=x_i]\, P[Y=y_j]}^{} =$$

$$= \sum x_i P[X=x_i] \cdot \sum y_j P[Y=y_j] =$$

$$= E(X) \cdot E(Y)$$

X .. počet ok na kostce

$3X+1$

$E(X) = 3{,}5$

$E(3X+1) = 3E(X)+1 = 11{,}5$

$E(X\cdot(3X+1)) = E(3X^2+X) =$

$= 3E(X^2) + E(X) = 3\cdot\frac{91}{6} + 3{,}5 =$

$= \frac{91}{2} + \frac{7}{2} = 49$

$\neq 3{,}5 \cdot 11{,}5$

$$D(a+bX) = b^2 D(X)$$

$$E\left(\left[(a+bX) - E(a+bX)\right]^2\right) =$$

$$= E\left(\left(a+bX - (a+bE(X))\right)^2\right) =$$

$$= E\left(\left[b(X - E(X))\right]^2\right) =$$

$$= b^2 E\left(\left[X - E(X)\right]^2\right) = b^2 \cdot D(X)$$

$$D(X) = E\left((X - E(X))^2\right) =$$

$$= E\left(X^2 - 2XE(X) + [E(X)]^2\right) =$$

$$= E(X^2) - 2E(X) \cdot E(X) + E(X)^2 =$$

$$= E(X^2) - E(X)^2$$