$$F_X(x) = P(X \leq x)$$

$$F_{\boldsymbol{X}}(x_1, \dots, x_n) = P(X_1 \leq x_1, \dots, X_n \leq x_n)$$

$$F_{X_1}(x_1) = P(X_1 \leq x_1)$$

marginální distr. funkce

$i \neq j$ $\quad F_{X_i, X_j}(x_i, x_j) = P(X_i \leq x_i, X_j \leq x_j)$

$$p(x_i) = P(X = x_i) = \sum_{j=1}^{\infty} P(X = x_i, Y = y_j)$$

nezávislost

$$P(X_1 = x_1, \dots, X_n = x_n) = \prod_{i=1}^{n} P(X_i = x_i)$$

$$\forall (x_1, \dots, x_n) \in \mathbb{R}^n$$

$$\Downarrow$$

$$F_X(x_1, \dots, x_n) = F_{X_1}(x_1) \cdots F_{X_n}(x_n)$$

Y \ X	2	3	4	5	6	7	8	9	10	11	12	marg. Y
0	1	0	1	0	1	0	1	0	1	0	1	6
1	0	2	0	2	0	2	0	2	0	2	0	10
2	0	0	2	0	2	0	2	0	2	0	0	8
3	0	0	0	2	0	2	0	2	0	0	0	6
4	0	0	0	0	2	0	2	0	0	0	0	4
5	0	0	0	0	0	2	0	0	0	0	0	2
marg X	1	2	3	4	5	6	5	4	3	2	1	36

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$E(X,Y) = \left(\ldots, \frac{70}{36}\right)$

vše lomeno 36

nezávislé? Nejsou $P(X=7, Y=2) \overset{?}{=} P(X=7) \cdot P(Y=2)$

$0 \neq 1/6 \cdot 2/9$

$$D(X) = E\left(\left(X - E(X)\right)^2\right)$$

$$D(X) = E(X^2) - E(X)^2$$

$$C(X,Y) = E\left(\left(X - E(X)\right)\left(Y - E(Y)\right)\right)$$

$$D(X) = C(X,X)$$

$$C(X,Y) = E(XY) - E(X)\cdot E(Y)$$

X, Y nezávislé $\Rightarrow C(X,Y) = 0$

$\not\Leftarrow$

$$\boxed{R(X,Y) = \frac{C(X,Y)}{\sqrt{D(X)}\cdot\sqrt{D(Y)}} \qquad |R(X,Y)| \le 1}$$

$Y \setminus X$	0	1	
0		$P(X=1, Y=0)$ *	$1-P(Y=1)$
1		$P(X=1, Y=1)$	$P(Y=1)$
	$1-P(X=1)$	$P(X=1)$	

$$* = P(X=1) - P(X=1, Y=1)$$

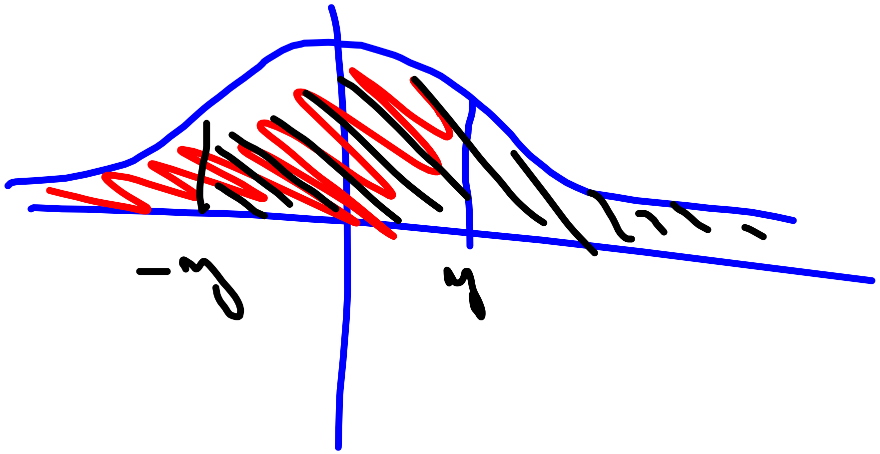
$$\begin{aligned} P(X=1, Y=0) &= \\ &= P(X=1) - P(X=1, Y=1) \\ &= P(X=1) - P(X=1) \cdot P(Y=1) \\ &= P(X=1)\left[1 - P(Y=1)\right] \\ &= P(X=1) \cdot P(Y=0) \end{aligned}$$

$$E(XY) = P(X=1, Y=1)$$

$$E(XY) = \sum_{(x,y) \in (0,1)^2} xy \cdot P(X=x, Y=y) = P(X=1, Y=1)$$

$$P(X<y)=\Phi(y)$$

$$P(-X<y)=P(X>-y)=1-\Phi(-y)=$$
$$=\Phi(y)$$



$X \sim N(0,1)$

$A = A(1/2) \ldots 1, -1$

$Y = A \cdot X$

$\Downarrow \; Y \sim N(0,1)$

$C(X,Y)$

$E(A \cdot X^2) = E(A) \cdot E(X^2)$

$P(X \leq 1, Y=0) \neq P(X=1) \cdot P(Y=0)$

$X \sim N(0,1)$

$X^2 \sim \chi^2(1)$

$E(X^2) = 1$

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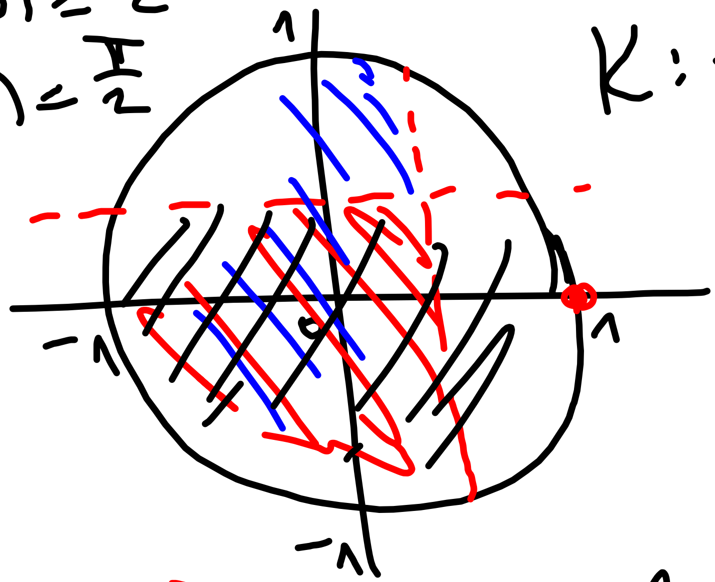
$P(X \geq 0, Y \geq 0) = \frac{\pi}{4}$ $\qquad \frac{\pi}{4} \neq \frac{\pi}{2} \cdot \frac{\pi}{2}$

$P(X \geq 0) = \frac{\pi}{2}$

$P(Y \geq 0) = \frac{\pi}{2}$



$K: x^2 + y^2 \leq 1$

$$\int_y \int_x \underbrace{f(x,y)}_{= c} \, dx \, dy = 1$$

$$1 = \int_y \int_x c \, dx \, dy$$

$P(X \leq \frac{1}{2}, Y \leq \frac{1}{2}) \stackrel{?}{=} P(X \leq \frac{1}{2}) \cdot P(Y \leq \frac{1}{2})$

$c \cdot S_K = 1$, přičom $S_K = \pi$

$$c = \frac{1}{\pi}$$

$F_{(x,y)} \neq F_X \cdot F_Y$

$X = 1 \dashrightarrow Y = 0$

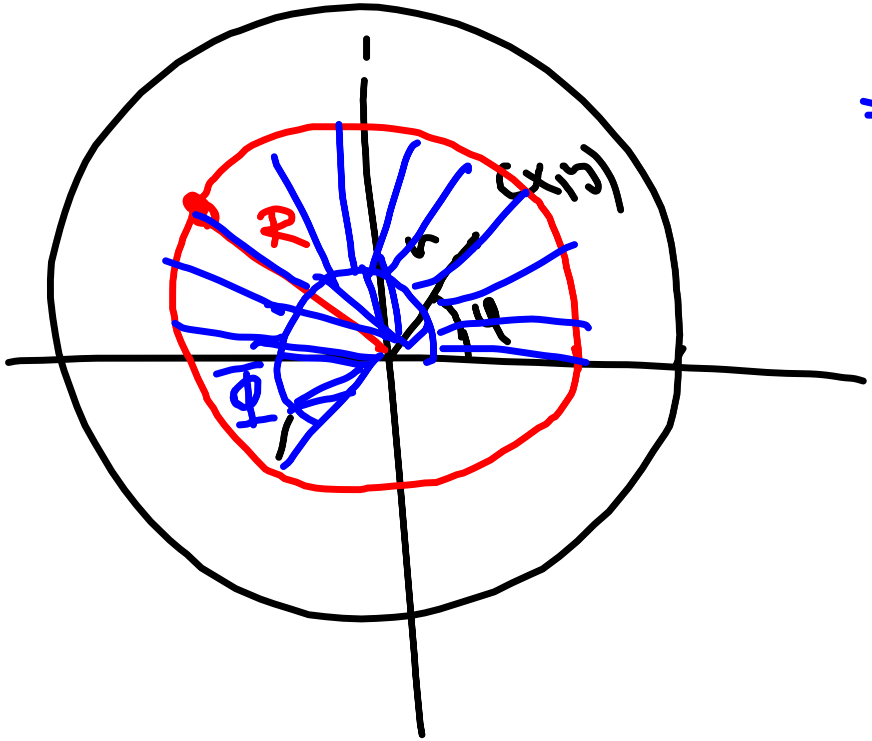
$$(x,y) \longmapsto (r,\varphi)$$

$$x = r\cos\varphi \qquad r = \sqrt{x^2+y^2}$$

$$y = r\sin\varphi$$

$$P(r \le R, \varphi \le \Phi)$$

$$= \frac{\pi R^2}{2\pi} \cdot \frac{\Phi}{\Phi} \cdot \frac{1}{\pi}$$



$$E(\Phi) = \int_0^{2\pi} \varphi \cdot h(\varphi)\, d\varphi = \left[\frac{\varphi^2}{2}\right]_0^{2\pi} \cdot \frac{1}{2\pi} = \pi$$

$$\boxed{h(\varphi) = \frac{1}{2\pi} \text{ na } \langle 0, 2\pi)}$$

$$D(\Phi) = E(\Phi^2) - E(\Phi)^2$$

$$E(\Phi^2) = \int_0^{2\pi} \varphi^2 \cdot h(\varphi)\, d\varphi = \frac{1}{2\pi} \cdot \left[\frac{\varphi^3}{3}\right]_0^{3\pi} =$$

$$= \frac{1}{2\pi} \cdot \frac{8\pi^3}{3} = \underline{\underline{\frac{4}{3}\pi^2}}$$

$$D(\Phi) = \frac{4}{3}\pi^2 - \pi^2 = \frac{\pi^2}{3}$$

n. veličina X

$$D(a+b\cdot X) = b^2\cdot D(X)$$

náh. vektor X

$$var(a+B\cdot X) = B\cdot var(X)\cdot B^T$$

$U = a + B\cdot Z$

$E(U) = a$

$var(U) = V = B\cdot B^T$

$$f(u) = \frac{1}{(\sqrt{2\pi})^m\sqrt{|V|}}$$

$U = \mu + \sigma\cdot Z$

$E(U) = \mu$

$D(U) = \sigma^2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$D(X) = E\left(\left(X - E(X)\right)^2\right)$$

rovn. rozdělení disk.

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \left(x_i - \underbrace{E(X)}_{\frac{1}{n}\sum x_i}\right)^2$$

$$S^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} \left(x_i - \mu\right)^2$$