$\cdot : G \times G \to G$

Př: $- : \mathbb{N} \times \mathbb{N} \cancel{\to} \mathbb{N}$

$1 - 3 \notin \mathbb{N}$

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

neutr. prvek: jednoznačný

$e \cdot a = a \cdot e = a.$

e_1, e_2 neutrální: $e_1 = e_1 \cdot e_2 = e_2$.

inverze k a:

b tak, že $a \cdot b = b \cdot a = e$

Př: $(\mathbb{Z}, \cdot)$ není grupa

$(\mathbb{Q}, \cdot)$ není grupa (0 nemá inv)

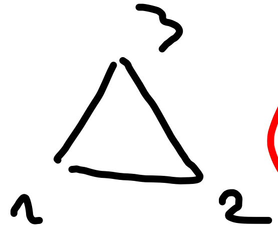
$(\mathbb{Q}^{\times}, \cdot)$ je grupa

Př: podgrupa

$(\mathbb{Q}^*, \cdot)$ je grupa

$(\mathbb{C}^*, \cdot)$ je grupa

$\mathbb{Q}^* \leq \mathbb{C}^*$ je podgrupa

Př: D_6 grupa symetrií  $(\mathbb{Z}_6, +)$

$S_6 \cong D_6 = \{id, (1,2), (1,3), (2,3), (1,2,3), (1,3,2)\}$

podgrupa: $\{id\}, D_6; \{id, (1,2)\}, \ldots$

$\{\underset{0}{id}, \underset{1}{(1,2,3)}, \underset{2}{(1,3,2)}\} \cong (\mathbb{Z}_3, +)$

$f : (G,\cdot) \to (H,\circ)$

$f(e_G) \overset{?}{=} e_H$

Buď $g \in G$ lib.

$f(e_G \cdot g) = f(e_G) \circ f(g)$

$\|$

$f(g)$

$f(g \cdot e_G) = f(g) \circ f(e_G)$

$\|$

$f(g)$

$= e_H$

$$f(a^{-1}) \circ f(a) = f(e_G) \quad [:= e_H]$$

$$\shortparallel$$

$$f(\underbrace{a^{-1} \cdot a}_{e_G}) = f(e_G)$$

podobně pravá inverze

podgrupa $f(K) \leq H$

$e_G \in K \Rightarrow f(e_G) \in f(K)$

nechť $f(k) \in f(K)$: $f(k)^{-1} = f(\underbrace{k^{-1}}_{\in K}) \in f(K)$

podobně $f(a), f(b) \in f(K) \overset{?}{\Rightarrow} f(a) \circ f(b) \in f(K)$

Dk 6:

"$\Rightarrow$" injektivní $\Rightarrow$ $f^{-1}(e_H) = \{e_G\}$

(snadno sporem: kdyby ~~$a \neq e_G$~~ $a \in f^{-1}(e_H)$, pak

$\underline{f(a)} = e_H = \underline{f(e_G)}$ $\Rightarrow$ není prosté!)

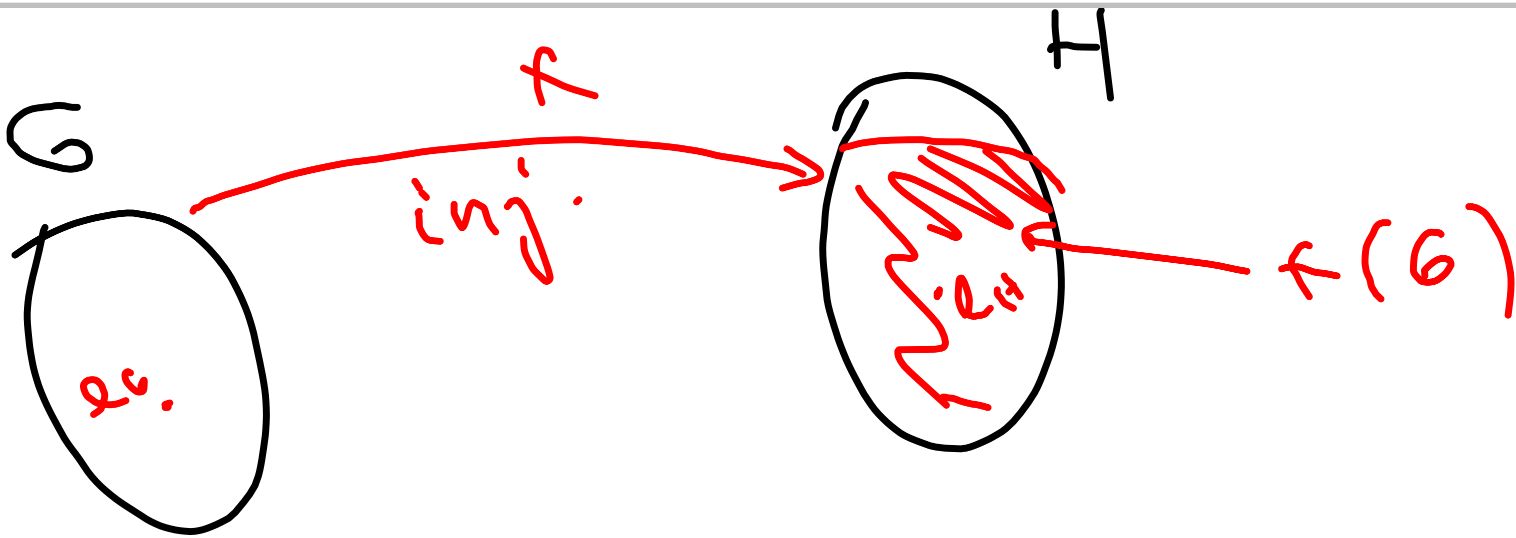
"$\Leftarrow$" $\underline{f^{-1}(e_H) = \{e_G\}}$ $\Rightarrow$ injektivní

obměna: není inj., tj. ex. $a, b \in G$ $a \neq b$

$f(a) = f(b) \Rightarrow f(a \cdot b^{-1}) = f(a) \circ f(b)^{-1} =$

$= e_H$

$\Rightarrow a \cdot b^{-1} \in f^{-1}(e_H) \wedge a \cdot b^{-1} \neq e_G$



$$G \cong f(G) \leq H$$

Př: $(\mathbb{Z}_3, +) \cong \{id, (1,2,3), (1,3,2)\} \leq D_6$

$$f: \begin{aligned} [0]_3 &\mapsto id \\ [1]_3 &\mapsto (1,2,3) \\ [2]_3 &\mapsto (1,3,2) \end{aligned}$$

$f([2]_3) = f([1]_3 + [1]_3) = f([1]) \circ f([1]) = (1,2,3) \circ (1,2,3) = (1,3,2)$

parita: 1

-1

$$\mathrm{sgn}: (S_n, \circ) \longrightarrow (\{1, -1\}, \cdot)$$

sudá $\longmapsto 1$

lichá $\longmapsto -1$

$$(\{1, -1\}, \cdot) \cong (\mathbb{Z}_2, +)$$

$\{[0]_2, [1]_2\}$

$$\exp : (\mathbb{R}, +) \to (\mathbb{R}^*, \cdot)$$

$$a \mapsto e^a$$

$$e^{a+b} = e^a \cdot e^b$$

$$e^0 = 1 \qquad (0 \mapsto 1)$$

$$\exp : (\mathbb{C}, +) \to (\mathbb{C}^*, \cdot)$$

$$e^{a+bi} = e^a \cdot (\cos b + i \sin b)$$

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

maticové grupy

$\left(Mat_{m,n}(\mathbb{R}), + \right)$

$$m\left(\underset{n}{\odot} \right) + \left(\circ \right) = $$

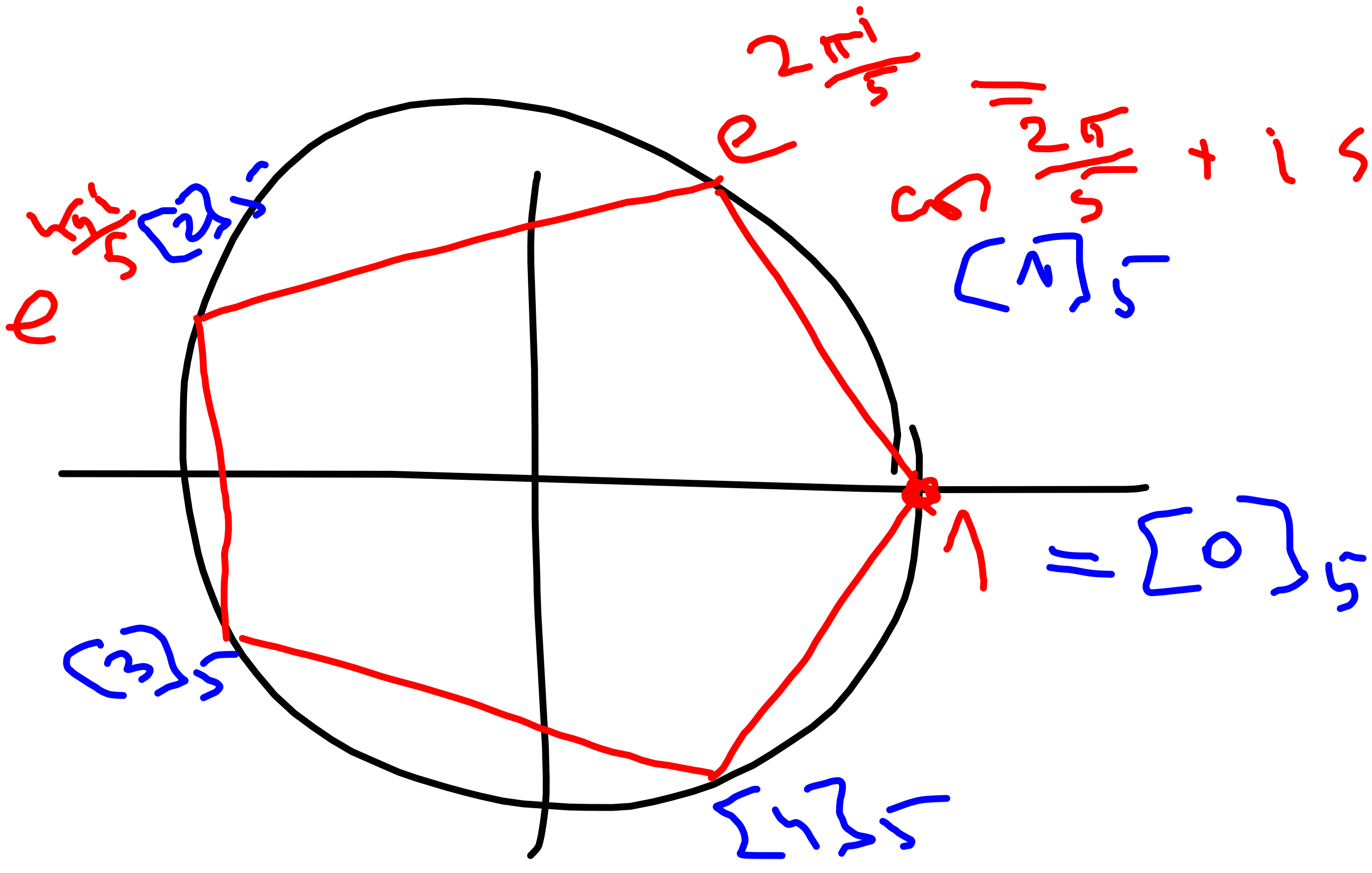
$\cong \underbrace{\mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}}_{m \cdot n}$

$GL_n(\mathbb{R})$ zobecněná lin. grupa

matice řádu n nad $\mathbb{R}$
invertibilní

$$\det : (GL_n(\mathbb{R}), \cdot) \longrightarrow (\mathbb{R}^*, \cdot)$$

$$\det(A \cdot B) = \det(A) \cdot \det(B)$$



5. té odmocniny z jedné

$\mathbb{Z}_6$ $\qquad\qquad$ $\mathbb{Z}_2 \times \mathbb{Z}_3$

$$[0]_6 \mapsto ([0]_2, [0]_3)$$
$$[1]_6 \mapsto ([1]_2, [1]_3)$$
$$[2]_6 \mapsto ([0]_2, [2]_3)$$
$$[3]_6 \mapsto ([1]_2, [0]_3)$$
$$[4]_6 \mapsto ([0]_2, [1]_3)$$
$$[5]_6 \mapsto ([1]_2, [2]_3)$$

obraz generátoru určuje homom. !

$$\mathbb{Z}_{km} \xrightarrow{f\ \cong} \mathbb{Z}_k \times \mathbb{Z}_m \qquad (k,m)=1$$

$$[a]_{km} \longmapsto \left([a]_k, [a]_m\right)$$

homom:

$$f([a]+[b]) =$$

$$[a]+[b] \longmapsto ([a]+[b], [a]+[b]) =$$

$$= \left([a],[a]\right) + \left([b],[b]\right)$$

$$= f([a]) + f([b])$$

injekt.

$$f([a]) = f([b])$$

$$([a]_k, [a]_m) = ([b]_k, [b]_m)$$

$$[a]_k = [b]_k \Leftrightarrow a \equiv b\ (k)$$

$$[a]_m = [b]_m \Leftrightarrow a \equiv b\ (m)$$

$$k \mid a-b,\ m \mid a-b \Rightarrow k \cdot m \mid a-b$$

$$(k,m)=1 \qquad \Rightarrow [a]_{km} = [b]_{km}$$

$$a^k = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{k}$$