$B_n = \#$ ekvivalencí na $\{1,2,\dots n\}$

$B_n = ?$ pro $n = 1,2,3,4$

$n=1$ $\{1\}$ jen jedna ekvivalence $\{(1,1)\}$

$B_1 = 1$

$n=2$ $\{1,2\}$ 2 ekvivalence

rozklady

1 | 2 $\quad$ 12

$\{(1,1),(2,2)\}$ $\quad$ $\{(1,1),(1,2),(2,1),(2,2)\}$

$B_2 = 2$

$B_3 = ?$ $\{1,2,3\}$

1 | 2 | 3 $\quad$ 123

12 | 3 $\quad$ 13 | 2 $\quad$ 1 | 23

$B_3 = 5$

$n=4$ $\{1,2,3,4\}$

1 | 2 | 3 | 4 $\quad$ 1234

12 | 34 $\quad$ 13 | 24 $\quad$ 14 | 23

4 | 123 $\quad$ 1 | 234 $\quad$ 2 | 134 $\quad$ 3 | 124

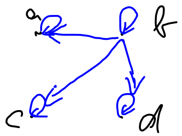
1 | 2 | 34 $\quad$ 1 | 3 | 24 $\quad$ 1 | 4 | 23

2 | 3 | 14 $\quad$ 2 | 4 | 13 $\quad$ 3 | 4 | 12

$B_4 = 15$

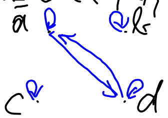
$R_a = \{(a,a), (b,b), (c,c), (d,d), (b,a), (b,c), (b,d)\}$

$R_= = \{(a,a), (b,b), (c,c), (d,d)\}$



R_a - reflexivní, antisymetrická, tranzitivní $\Rightarrow$ (neúplné) uspoř.

$R_b = R_= \cup \{(d,a), (a,d)\}$



R_b - refl., sym., tranz. $\Rightarrow$ ekvivalence

$\{\{a,d\}, \{b\}, \{c\}\}$

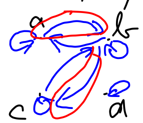
$R_c = R_=$

R_c - refl., sym., antisym., tranzitivní

ekvivalence i (neúplné) uspoř.

(jediná taková)

$R_d = R_= \cup \{(b,a), (a,b), (b,c), (c,b)\}$



$a R_d b \ \& \ b R_d c$

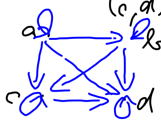
$\not\Downarrow$

$a R_d c$

R_d není tranzitivní $\Rightarrow$ ani ani

$R_e \not\supseteq R_= \Rightarrow$ není reflex. $\Rightarrow$ ani ani

$R_f = R_= \cup \{(a,b), (a,c), (a,d), (b,c), (b,d), (c,d)\}$



úplné uspořádání

$\left.\begin{array}{l}\text{reflex.}\\ \text{antisym.}\\ \text{tranzitivní}\end{array}\right\}$ uspoř.

Hasseův diagram

1) $M: \{f: \mathbb{R} \to \mathbb{R}\}$

$(f \sim g) \iff f(0) = g(0)$

reflexivní, sym., tranzitivní

ANO, je to eko.

2) M stejná

$(f \sim g) \iff f(0) = g(1)$

není ani reflex, ani sym, ani tranz.

NE.

3), 4) symetrické, tranzitivní

3) nereflex. 4) reflex.

NE ANO

5) $M = \mathbb{N}$, $(a \sim b) \iff S(a) + S(b) = 20$

není reflexivní $\Rightarrow$ NE

6) $M = \mathbb{N}$ $(m \sim n) \iff 7 | a - b$

reflex. : $7 | a - a = 0 \Rightarrow a \sim a$

symetrie : $a \sim b \Rightarrow 7 | a - b \Rightarrow 7 | b - a \Rightarrow b \sim a$

tranz. : $(a \sim b, b \sim c) \Rightarrow (7 | a - b \ \& \ 7 | b - c) \Rightarrow$

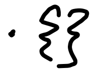
$\Rightarrow 7 | (a - b) + (b - c) = a - c \Rightarrow a \sim c$

ANO

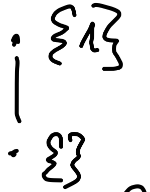
$\mathbb{N} \to \{0, 1, \dots 6\}$

$a \mapsto$ zbytek po dělení 7 čísla a

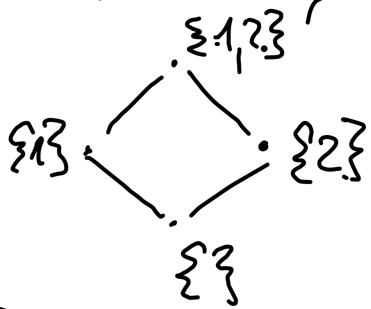
$A = \{\}$ $\quad 2^A = P(A) = \{\{\}\}$



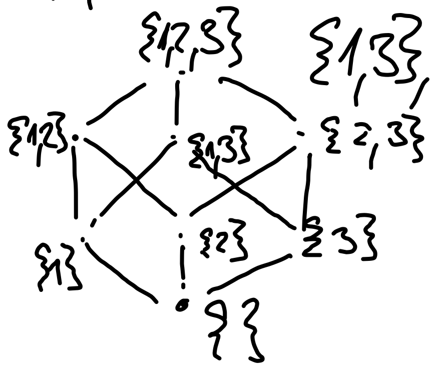
$A = \{1\}$ $\quad 2^A = \{\{\}, \{1\}\}$



$A = \{1,2\}$ $\quad 2^A = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$



$A = \{1,2,3\}$ $\quad 2^A = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$



Hypotéza:

Jediné ekvivalence na $\mathbb{Z}$, respektující + jsou: zbytkové třídy modulo $n \in \mathbb{Z}$

$\{\mathbb{Z}\}$, $\{\ldots\{0\},\{1\},\{2\},\ldots\}$

$a \sim b,\ c \sim c \Rightarrow a+c \sim b+c$

vezměme a, b taková, že $a \sim b$, $|a-b|$ je minimální takový rozdíl ($a \neq b$). Pak je rozklad podle $\sim$ dán zbytkovými třídami mod $|a-b|$

3 2 1 0 −1 −2

4 2 0 −2 3 1 −1

2 −1 4 1 −2 3 0 −3

… pro $\forall n \in \mathbb{N}$

(mocniny 7)

(mocniny 3)

(mocniny 2)

zbytek

16 8 4 2

2 4 8

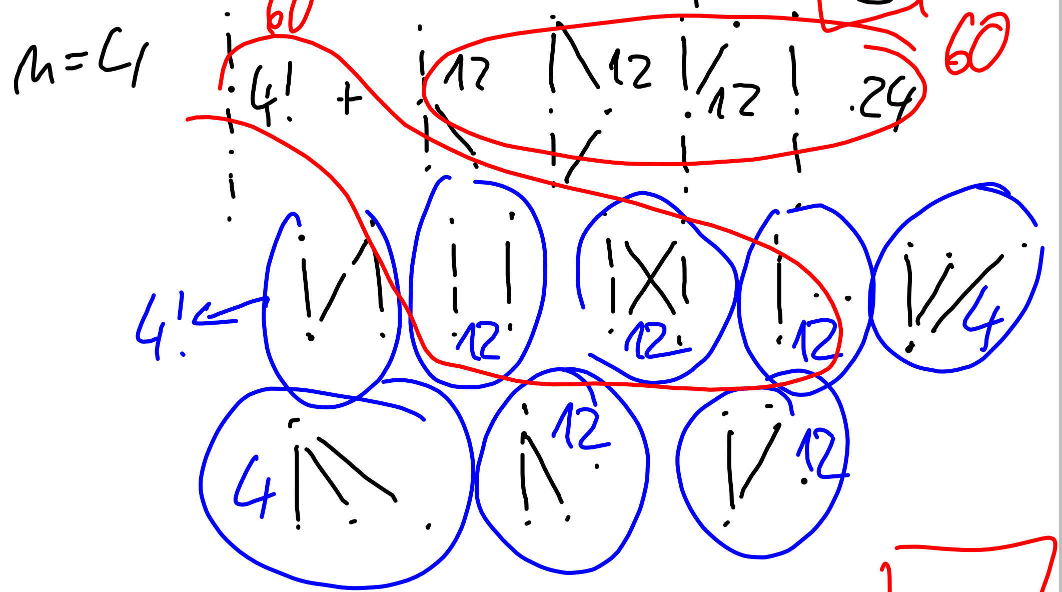
úplných usp. na $\{1,2,\ldots,n\}$ je $n!$

$n=1$ $\boxed{1}$

$n=2$ $2 + 1 = \boxed{3}$

$n=3$ $3! = 6 + 3 + 3 + 1 = \boxed{13}$

$n=4$



minimálně $120 + 56 = \boxed{176}$