$$(a+1)x_1 + x_2 + x_3 = a^2+3a$$
$$x_1 + (a+1)x_2 + x_3 = a^3+3a^2$$
$$x_1 + x_2 + (a+1)x_3 = a^4+3a^3$$

$$\left(\begin{array}{ccc|c} a+1 & 1 & 1 & a(a+3) \\ 1 & a+1 & 1 & a^2(a+3) \\ 1 & 1 & a+1 & a^3(a+3) \end{array}\right) \sim$$

$$\begin{array}{c} -a-1 \\ \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 1 & a+1 & a^3(a+3) \\ 1 & a+1 & 1 & a^2(a+3) \\ a+1 & 1 & 1 & a(a+3) \end{array}\right) \sim \quad -(a+1)^2+1$$

$$+\left(\begin{array}{ccc|c} 1 & 1 & a+1 & a^3(a+3) \\ 0 & a & -a & a^2(a+3)(1-a) \\ 0 & -a & -a^2-2a & * \end{array}\right)$$

$* \quad -a^3(a+3)(a+1) + a(a+3) =$

$$\underbrace{a(a+3)\left(-a^2(a+1)+1\right)}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & a+1 & a^3(a+3) \\ 0 & a & -a & a^2(a+3)(1-a) \\ 0 & 0 & -a(a+3) & a(a+3)\left(a(1-a)+(-a^2(a+1)+1)\right) \end{array}\right)$$

$$\underbrace{\qquad}_{0}$$

i) $a \neq 0$

$$\begin{array}{c|ccc|c} 3.r. & 0 & 0 & 0 & 0 \\ 2.r & 0 & 0 & 0 & 0 \\ 1.r & 1 & 1 & 1 & 0 \end{array} \Rightarrow \begin{array}{l} x_3 = t \\ x_2 = s \\ x_1 = -t-s \end{array}$$

$$(-t-s, s, t)$$

ii) $a = -3$

$$\begin{array}{r} 3.r \\ \\ -3x_2+3x_3=0 \end{array} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$$

$x_3 = x_2 \qquad x_2 = t \qquad (t, t, t)$

$x_1 = 2t - t$

$x_1 = t$

$$\begin{pmatrix} 0 & 4 & 10 & 1 \\ 4 & 8 & 18 & 7 \\ 10 & 18 & 40 & 17 \\ 1 & 7 & 17 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 4 & 8 & 18 & 7 \\ 10 & 18 & 40 & 17 \end{pmatrix} \begin{matrix} -4 \\ + \\ \\ \end{matrix} \sim$$

-10

$$\equiv \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ 0 & -20 & -50 & -5 \\ 0 & -52 & -130 & -13 \end{pmatrix} \begin{matrix} \\ \\ (:-5) \\ (:-13) \end{matrix} \sim \begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \\ \cancel{0} & \cancel{4} & \cancel{10} & 1 \\ \cancel{0} & \cancel{4} & \cancel{10} & 1 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 7 & 17 & 3 \\ 0 & 4 & 10 & 1 \end{pmatrix}$$

$h(A) = 2$

n rovnic o m ~~neznámých~~

$$n \left\{ \underbrace{\underbrace{\begin{pmatrix} & m & \\ & & \\ & & \end{pmatrix}}_{A} \begin{vmatrix} b_1 \\ \vdots \\ b_m \end{vmatrix}}_{\bar{A}} \right.$$

$h(A) = h(\bar{A}) = m$

1 řešení

$h(A) < h(\bar{A})$

∞ řešení

$h(A) < h(\bar{A}) \leq m$

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 2 & -1 \\ 1 & 1 & 3 & 1 & 0 \end{vmatrix} =$$

1. řádku:

$$1 \cdot (-1)^2 \cdot \underbrace{\begin{vmatrix} 3 & 0 & 0 & 0 \\ 5 & 6 & 7 & 8 \\ 3 & 1 & 2 & -1 \\ 1 & 3 & 1 & 0 \end{vmatrix}} + 2 \cdot (-1)^3 \begin{vmatrix} 2 & 0 & 0 & 0 \\ 4 & 6 & 7 & 8 \\ 2 & 1 & 2 & -1 \\ 1 & 3 & 1 & 0 \end{vmatrix}$$

$$= 1 \cdot 3 \cdot (-1)^2 \begin{vmatrix} 6 & 7 & 8 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{vmatrix} + (-2) 2 \cdot \begin{vmatrix} 6 & 7 & 8 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= (3-4) \begin{vmatrix} 6 & 7 & 8 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{vmatrix} = -1 \cdot \begin{vmatrix} 6 & 7 & 8 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= -1 \cdot (-1) \cdot \ldots - 0 - 21 - 48 + 6 = -55$$

ij

$$\begin{vmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 2 \cdot \cancel{\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}} \cdot \cancel{(-1)}^{\substack{1+1 \\ i+j}} +$$

$$3 \cdot \begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} \cdot (-1)^{1+2} + 1 \cdot \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} \cdot \underbrace{(-1)^4}_{+}$$

$$\begin{vmatrix} 3x & 0 & x \\ -x & 0 & 2 \\ 0 & x & -5 \end{vmatrix} = 0$$

$$-x^3 - 6x^2 = 0$$

$$-x^2(x+6) = 0$$

$$x = 0$$

$$x = -6$$

$$A = \begin{pmatrix} 1 & 2 & 3 & \cdots & & n \\ 0 & 2 & 5 & 7 & \cdots & 2n-1 \\ 0 & 0 & 3 & & \cdots & 2n-1 \\ 0 & 0 & 0 & 4 & \cdots & 2n-1 \\ \vdots & & & & & \\ 0 & 0 & \cdots & & 0 & n \end{pmatrix}$$

$$|A| = n!$$

$$|A_n| = \begin{vmatrix} x+1 & x & 0 & \dots & 0 \\ 1 & x+1 & x & \dots & 0 \\ 0 & 1 & x+1 & \dots & 0 \\ \vdots & & & & \\ 0 & \dots & 0 & 1 & x+1 \end{vmatrix} =$$

$$(x+1)\cdot(-1)^2 \cdot \overbrace{\begin{vmatrix} x+1 & x & \dots & 0 \\ 1 & x+1 & x \dots & 0 \\ \vdots & & & \\ 0 & \dots & 1 & x+1 \end{vmatrix}}^{|A_{n-1}|} +$$

$$(-1)^3 \begin{vmatrix} x & 0 & 0 & 0 & 0 \\ 1 & x+1 & x & \dots & 0 \\ \vdots & & & & \end{vmatrix} =$$

$$(x+1)\,|A_{n-1}| - x\,|A_{n-2}|$$

$$|A_n| = (x+1)|A_{n-1}| - x|A_{n-2}|$$

$$|A_n| = x^n + x^{n-1} + \dots + x + 1$$

$n=1 \qquad x+1$

$n=2 \qquad \begin{vmatrix} x+1 & x \\ 1 & x+1 \end{vmatrix} = x^2+2x+1 - x = x^2+x+1$ ✓

IP: $1 \dots n-1$

Dk: $|A|_n = (x+1)\cdot\underbrace{|A_{n-1}|}_{IP} - x\underbrace{|A_{n-2}|}_{IP} =$

$= (x+1)(x^{n-1}+x^{n-2}+\dots+x+1) - x(x^{n-2}+x^{n-3}+\dots+x+1) =$

$= x^n + x^{n-1} + x^{n-2} + \dots + x + 1$

$$\begin{pmatrix} 6 & 9 & 7 \\ 4 & 7 & 6 \\ 9 & 6 & 9 \end{pmatrix}$$

$$\underbrace{100 \cdot 1.\text{ sloupec} \quad 10 \cdot 2\text{ sloupec}}_{\text{přičtu ke třetímu sloupci}}$$

$$\begin{vmatrix} - & - & 697 \\ - & \cdot & 476 \\ - & \cdot & 969 \end{vmatrix} = \underline{17} \begin{vmatrix} \cdot & \cdot & \frac{697}{17} \\ - & - & \frac{476}{17} \\ - & \cdot & \frac{969}{17} \end{vmatrix}$$

$$\begin{pmatrix} 3 & 2 & 0 \\ 5 & 4 & 1 \\ 1 & 4 & 5 \end{pmatrix} \Rightarrow \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix} \frac{1}{|A|}$$

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 5 & 4 & 1 \\ 1 & 2 & 5 \end{pmatrix}$$

$$|A| = 62 - 50 - 6 = 6$$

$$A_{11} = \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} = 18 \qquad A_{22} = \begin{vmatrix} 3 & 0 \\ 1 & 5 \end{vmatrix} = 15$$

$$A_{12} = \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} = -24 \qquad A_{21} = \begin{vmatrix} 2 & 0 \\ 2 & 5 \end{vmatrix} = -10$$

$$A_{13} = \begin{vmatrix} 5 & 4 \\ 1 & 2 \end{vmatrix} = 6 \qquad A_{32} = \begin{vmatrix} 3 & 0 \\ 5 & 1 \end{vmatrix} = 3$$

$$A_{33} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 2 \qquad A_{31} = \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} = 2$$

$$A_{23} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = -4$$

$$A^{-1} = \frac{1}{6} \cdot \begin{pmatrix} 18 & -10 & 2 \\ -24 & 15 & -3 \\ 6 & -4 & 2 \end{pmatrix}$$

$$\begin{aligned} 2x + 3y + z &= 4 \\ x + 2y + 2z &= 6 \\ 5x + y + 4z &= 21 \end{aligned}$$

$$x_i = \frac{|A_i|}{|A|}$$

i-tý sloupec nahradíte sloupcem abs. členů

$$x_1 = \frac{\begin{vmatrix} 4 & 3 & 2 \\ 6 & 2 & 2 \\ 21 & 1 & 4 \end{vmatrix}}{|A|}$$

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 5 & 1 & 4 \end{vmatrix} = 17 + 30 - 10 - 4 - 12 = \underline{\underline{17}}$$

$$A_1 = \begin{vmatrix} 4 & 3 & 2 \\ 6 & 2 & 2 \\ 21 & 1 & 4 \end{vmatrix} = \underline{32} + \underline{126} + \underline{12} - \underline{84} - \underline{72} - \underline{8} = \underline{\underline{16}}$$

$$A_2 = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 6 & 2 \\ 5 & 21 & 4 \end{vmatrix} = 48 + 21 + 40 - 30 - 16 - 84 = \ldots$$

$$A_3 = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 6 \\ 5 & 1 & 21 \end{vmatrix} =$$