

$$u_1 = (1, 1, 1, 1)^T; u_2 = (-1, 4, 4, -1)^T;$$

$$u_3 = (4, -2, 2, 0)^T$$

určete ortogonální bázi

• $v_1 := u_1$

• projekce u_2 na podprostor $\langle v_1 \rangle$

$$P_1 = \frac{\langle u_2, v_1 \rangle}{\langle v_1, v_1 \rangle} \cdot v_1 = \frac{1 \cdot (-1) + 1 \cdot 4 + 1 \cdot 4 + 1 \cdot (-1)}{\sqrt{16}} \cdot v_1 = \frac{6}{4} \cdot v_1 = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right)^T$$

$$\text{Pak } v_2 := u_2 - P_1 = \left(-\frac{5}{2}, \frac{5}{2}, \frac{5}{2}, -\frac{5}{2} \right)^T$$

Podobnost projekce u_3 na $\langle v_1, v_2 \rangle$

$$p_2 = \frac{\langle u_3, v_1 \rangle}{\langle v_1, v_1 \rangle} \cdot v_1 + \frac{\langle u_3, v_2 \rangle}{\langle v_2, v_2 \rangle} \cdot v_2 =$$

$$= (2, 0, 0, 2)^T$$

$$v_3 = u_3 - p_2 = (2, -2, +2, -2)^T$$

Normalizaci dostaneme

$$v_1 = \frac{1}{\|v_1\|} \cdot v_1 = \frac{1}{\sqrt{4}} v_1 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)^T$$

$$v_2 = \frac{1}{\|v_2\|} \cdot v_2 = \frac{1}{\sqrt{4}} v_2 = \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right)^T$$

$$v_3 = \frac{1}{\|v_3\|} \cdot v_3 = \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right)^T$$

Projekce $v = (-1, 2, 3, 4)^T$:

$$p = \langle v, w_1 \rangle w_1 + \langle v, w_2 \rangle w_2 + \langle v, w_3 \rangle w_3 \\ = \dots = \left(\frac{1}{2}, \frac{7}{2}, \frac{3}{2}, \frac{3}{2} \right)^T$$

Vzdálenost v od W je:

$$\|v - p\| = \left\| \left(-\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right) \right\| = \sqrt{9} = 3$$

Odklon v od W je úhel φ :

$$\cos \varphi = \frac{\|p\|}{\|v\|} = \frac{\sqrt{\frac{84}{4}}}{\sqrt{30}} = \sqrt{\frac{7}{8}}$$
$$\varphi = \arccos \sqrt{\frac{7}{8}} \approx 0,58.$$