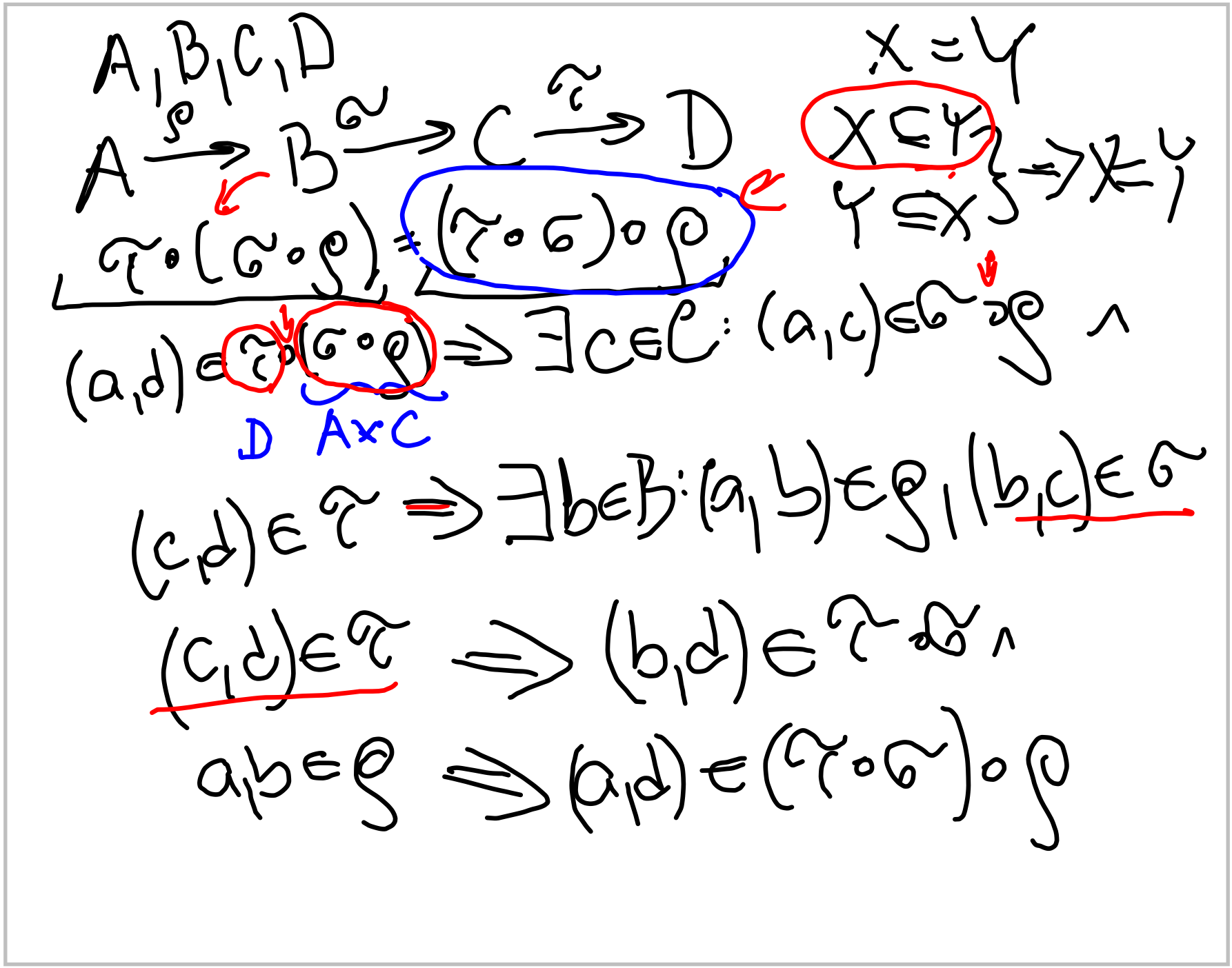
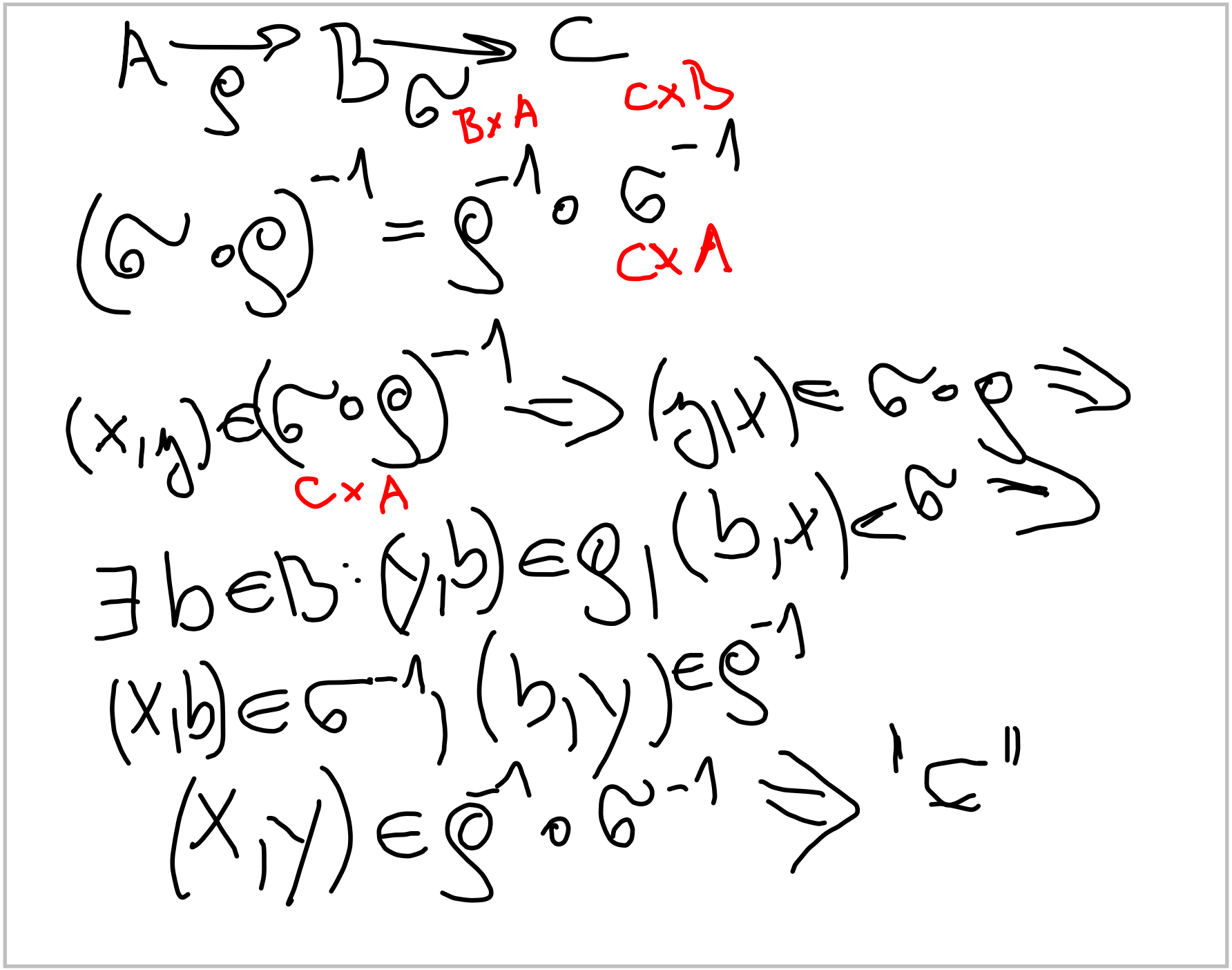

A, B, C, D
A → B → C → D
τ∘(σ∘ρ) = (τ∘σ)∘ρ
X = Y
X ⊆ Y
Y ⊆ X
⇒ X = Y
(a,d) ∈ τ∘(σ∘ρ) ⇒ ∃c∈C: (a,c) ∈ σ∘ρ ∧
D
A×C
(c,d) ∈ τ ⇒ ∃b∈B: (a,b) ∈ ρ, (b,c) ∈ σ
(c,d) ∈ τ ⇒ (b,d) ∈ τ∘σ ∧
a,b ∈ ρ ⇒ (a,d) ∈ (τ∘σ)∘ρ


A
B
C
ϱ
σ
$B \times A$
$C \times B$
$(\sigma \circ \varrho)^{-1} = \varrho^{-1} \circ \sigma^{-1}$
$C \times A$
$(x,y) \in (\sigma \circ \varrho)^{-1} \Rightarrow (y,x) \in \sigma \circ \varrho \Rightarrow$
$\exists b \in B : (y,b) \in \varrho \wedge (b,x) \in \sigma \Rightarrow$
$(x,b) \in \sigma^{-1} \wedge (b,y) \in \varrho^{-1}$
$(x,y) \in \varrho^{-1} \circ \sigma^{-1} \Rightarrow$ "$\subseteq$"

$a \mathcal{R} b \Leftrightarrow a \mid b$

R: $a \in \mathbb{N}$: $a = a \cdot 1 \Rightarrow a \mid a$ R✓

S: $a \mathcal{R} b \Rightarrow a \mid b \Rightarrow$

$2 \mid 4$ S ✗

$4 \nmid 2$

A $a \mathcal{R} b \wedge b \mathcal{R} a \Rightarrow a \mid b \wedge b \mid a \Rightarrow$

$a = b$

T: $a \mathcal{R} b \wedge b \mathcal{R} c \Rightarrow a \mid b \Rightarrow b \mid c$

$b = a \cdot k, \; c = b \cdot l \Rightarrow c = a \cdot (k \cdot l) \Rightarrow$

$a \mid c \Rightarrow a \mathcal{R} c$

U: $a \mathcal{R} b \vee b \mathcal{R} a$ $\quad a \mid b \vee b \mid a$

✗ $\quad 3 \nmid 4 \quad 3 \nmid 7, 7 \nmid 3$

USPOŘÁDÁNÍ NA $\mathbb{N}$

$\equiv$ $m \in \mathbb{Z}$

$a \equiv b \Leftrightarrow m \mid b-a$

kong. modulo m

R: $\forall a \in \mathbb{Z}: m \mid a-a \Rightarrow m \mid 0$ V

$a \equiv a \ (m)$

S $a \equiv b \ (m) \Rightarrow m \mid b-a \Rightarrow$

$m \mid (-1) \cdot (a-b) \Rightarrow m \mid a-b \Rightarrow$

$b \equiv a \ (m)$

AS: $a \equiv b \ (m) \wedge b \equiv a \ (m)$

$m \equiv 0 \ (m)$ $\quad m \mid 0-m$

$0 \equiv m \ (m)$ $\quad m \mid m-0$

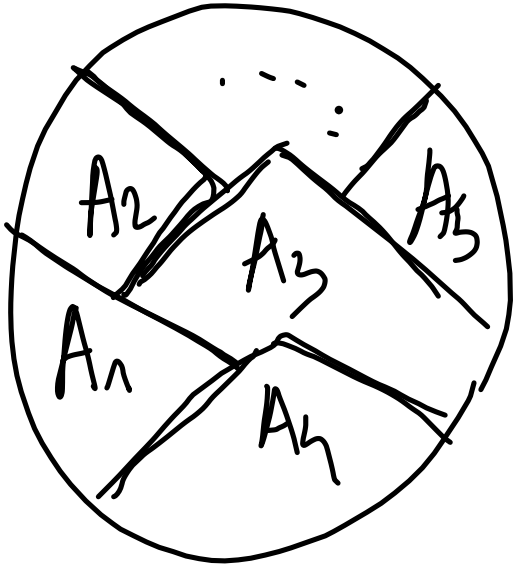
$0 \neq m$

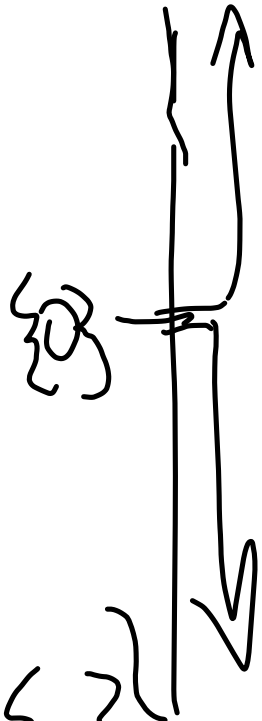
T: $a \equiv b \ (m)$ $\quad m \mid b-a \Rightarrow$

$b \equiv c \ (m)$ $\quad m \mid c-b$

$\Leftarrow$ $\boxed{\begin{array}{l} b-a = m \cdot k \\ c-b = m \cdot l \end{array}}$ $\quad c-a = m(k+l)$

$m \mid c-a \Rightarrow a \equiv c \ (m)$

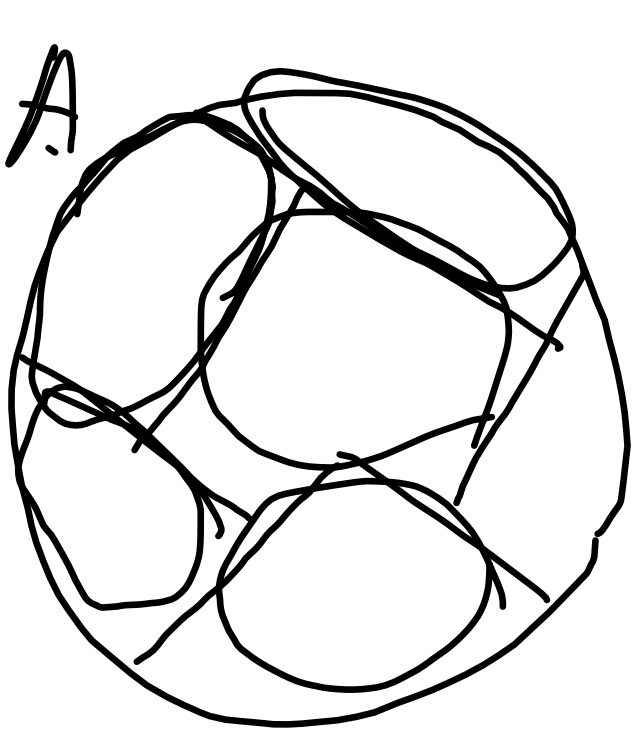




$\{\mathbb{Z}^+, \mathbb{Z}^-, \{0\}\}$

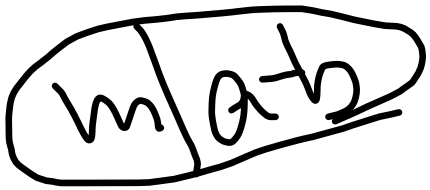
$(-\infty, 10) < \{10\} < (10, 11) < \{11\} < (11, 15) < \{15\} < (15, \infty)$

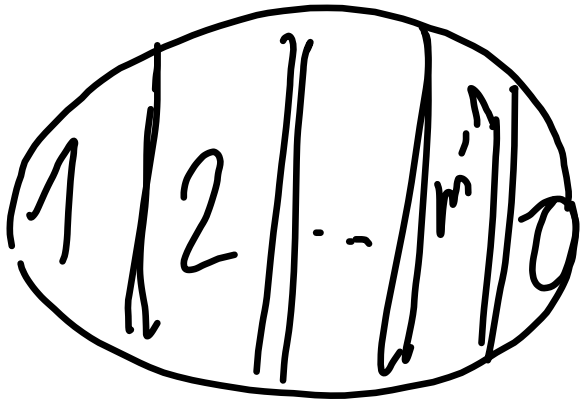
A, B disjunktní $\Leftrightarrow A \cap B = \emptyset$

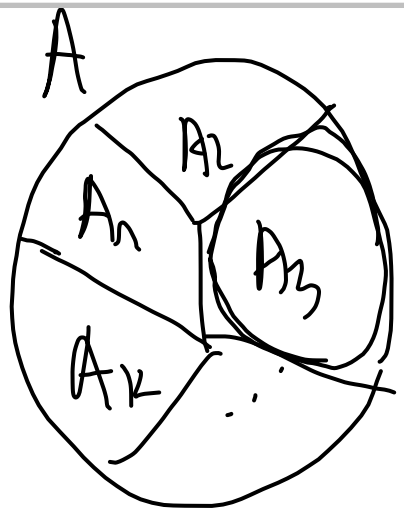


$\sim$ EKVIVAL. A

$$X_a = \{(a,b) \in A \times A \mid a \not\sim b\}$$





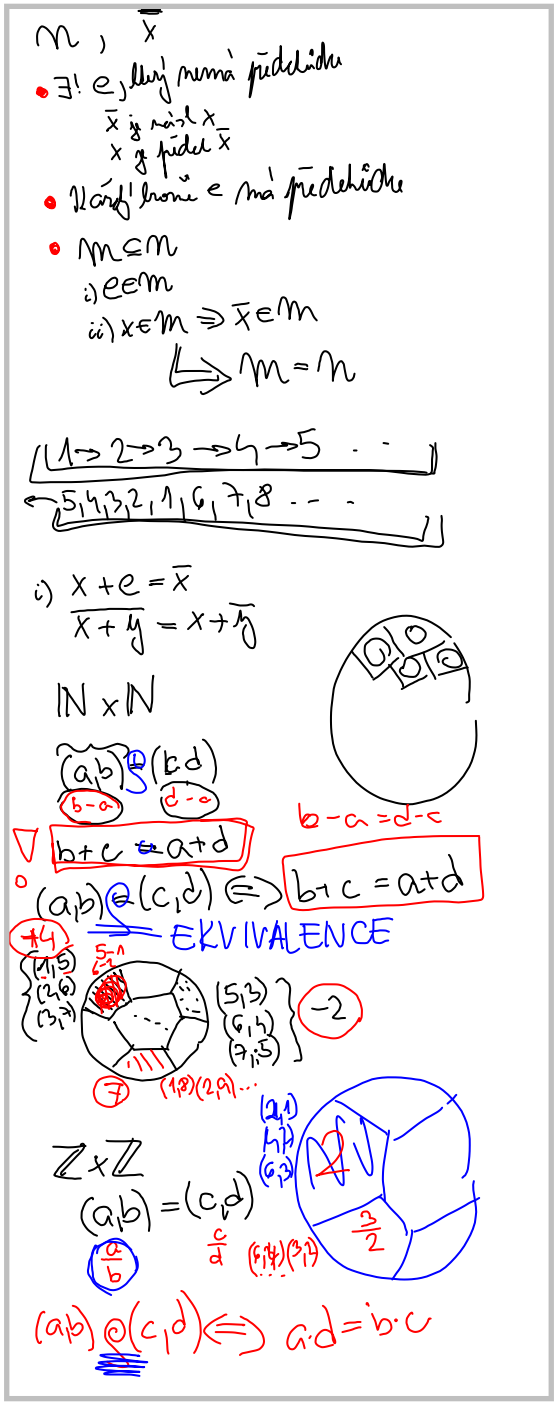


$x \varrho y \Leftrightarrow \exists A_i : x, y \in A_i$

R: $x \in A : x \in A_i$ pro $i \Rightarrow x \sim x \in A_i$
$\Rightarrow x \varrho x$ ✓

S: $x \varrho y \Rightarrow x, y \in A_i \Rightarrow y, x \in A_i$
$y \varrho x$ ✓

T: $x \varrho y \wedge y \varrho z \Rightarrow x, y \in A_i \wedge y, z \in A_i$
$\Rightarrow x, z \in A_i$ ✓


$\mathbb{N}$, $\bar{x}$
∃! e, který nemá předchůdce
$\bar{x}$ je následník x
x je předek $\bar{x}$
Každý kromě e má předchůdce
$M \subset \mathbb{N}$
i) $e \in M$
ii) $x \in M \Rightarrow \bar{x} \in M$
$\Rightarrow M = \mathbb{N}$
1→2→3→4→5 . . .
5,4,3,2,1,6,7,8 . . .
i) $x + e = \bar{x}$
$\overline{x+y} = x + \bar{y}$
$\mathbb{N} \times \mathbb{N}$
(a,b) ~ (c,d)
b-a
d-c
b-a = d-c
b+c = a+d
(a,b) ~ (c,d) ⇔ b+c = a+d
EKVIVALENCE
(1,5)
(2,6)
(3,7)
(5,3)
(6,4)
(7,5)
-2
(1,8)(2,9)...
$\mathbb{Z} \times \mathbb{Z}$
(a,b) = (c,d)
$\frac{a}{b}$
$\frac{c}{d}$
(2,1)
(4,2)
(6,3)
$\mathbb{Q}$
$\frac{3}{2}$
(a,b) ~ (c,d) ⇔ a·d = b·c

a je horní závora B

$\forall b \in B: b \leq a$

$\mathbb{Z}$, $B \subseteq \mathbb{Z}$ $B = \{1, 2, 3, 5\}$

5 je HZ B

10 HZ B

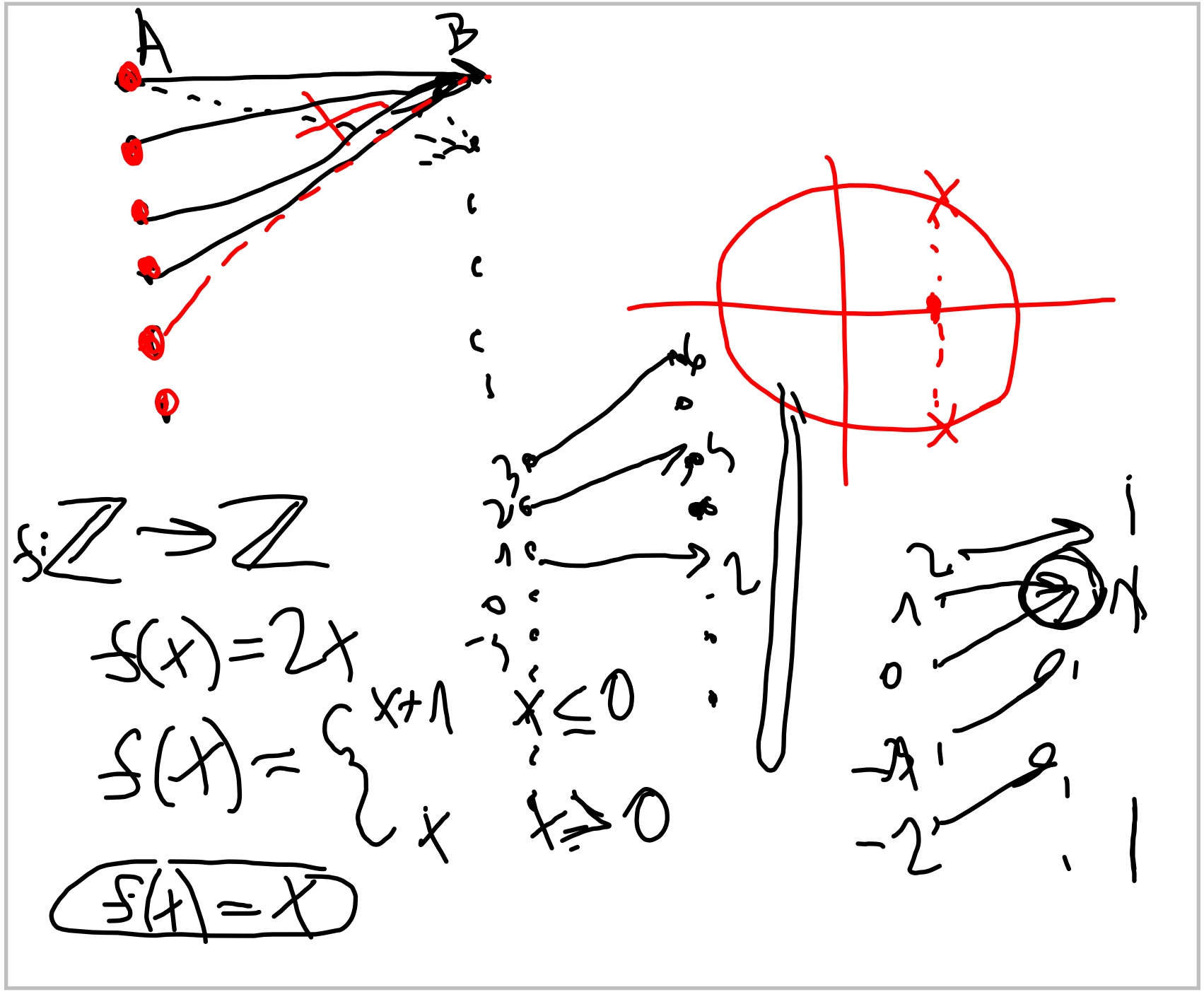
4 HZ B

a je dolní z. B

$\forall b \in B:$

$a \leq b$

$-5, -1, 0, 1$ jsou d.z.


A
B
$f: \mathbb{Z} \to \mathbb{Z}$
$f(x) = 2x$
$f(x) = \begin{cases} x+1 & x \leq 0 \\ x & x \geq 0 \end{cases}$
$f(x) = x$

$f: A \to B$ inj

$g: B \to C$ inj $\Rightarrow$ $g \circ f$ je inj

$\exists x, y \in C: (g \circ f)(x) = (g \circ f)(y)$

$g(f(x)) = g(f(y))$

$\Downarrow$

$f(x) = f(y)$

$x = y \Rightarrow$ INJ

$x = y$

$g \circ f$ inj $\Rightarrow f$ inj

$g(f(x)) = g(f(y))$

$(g \circ f)(x) = (g \circ f)(y)$

$x = y$

f, g $\quad g \circ f$

$x \in C \Rightarrow \exists b \in B : g(b) = x \Rightarrow$

$\exists a \in A : f(a) = b \Rightarrow g(f(a)) = x$

$\rightsquigarrow g \circ f(a) = x$