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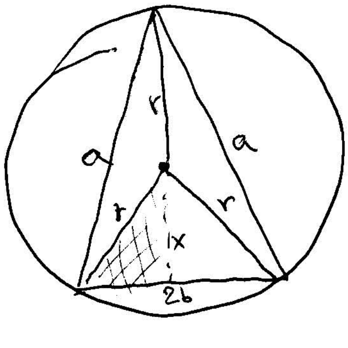
(3)

nezapomeňte ověřit, že je to maximum (a ne minimum nebo inflexní bod).



z víme $r^2=x^2+b^2$

$\Longrightarrow b=\sqrt{r^2-x^2}$

$$\boxed{S_\Delta = \frac{2b\cdot(r+x)}{2} = b(r+x) \longrightarrow \max}$$

$$S_\Delta = b(r+x) = \sqrt{r^2-x^2}\,(r+x) = r\sqrt{r^2-x^2} + x\sqrt{r^2-x^2} \qquad \Big/ \frac{\partial}{\partial x}$$

$$S'(x) = \frac{1}{2} r (r^2-x^2)^{-\frac{1}{2}}\cdot(-2x) + (r^2-x^2)^{\frac{1}{2}} + x\cdot\frac{1}{2}(r^2-x^2)^{-\frac{1}{2}}(-2x)$$

a hledám $S'(x)=0$

$$\frac{-rx}{\sqrt{r^2-x^2}} + \sqrt{r^2-x^2} + \frac{-x^2}{\sqrt{r^2-x^2}} = \frac{-rx+r^2-2x^2}{\sqrt{r^2-x^2}} = 0$$

$$\Longleftrightarrow \quad 2x^2+rx-r^2=0 \qquad x_{12} = \frac{-r\pm\sqrt{r^2+4\cdot 2\cdot r^2}}{4} = \frac{-r\pm 3r}{4} = \begin{cases} -r \\ \frac{r}{2} \end{cases}$$

$\boxed{x_1=-r \text{ nemůže být záporné}}$

tedy $x=\frac{r}{2}$, pak $b=\sqrt{r^2-x^2}=\sqrt{r^2-\left(\frac{r}{2}\right)^2}=\frac{\sqrt{3}}{2}r$

$$a=\sqrt{(r+x)^2+b^2}=\sqrt{r^2+2rx+x^2+b^2}=\sqrt{r^2+2r\frac{r}{2}+\left(\frac{r}{2}\right)^2+\frac{3}{4}r^2}=\sqrt{3}\,r$$

tedy strany Δ jsou $a=\sqrt{3}\,r$, $2b=2\frac{\sqrt{3}}{2}\cdot r=\sqrt{3}\,r$, výška je $r+x=r+\frac{r}{2}=\frac{3r}{2}$.

tedy rovnostranný Δ.

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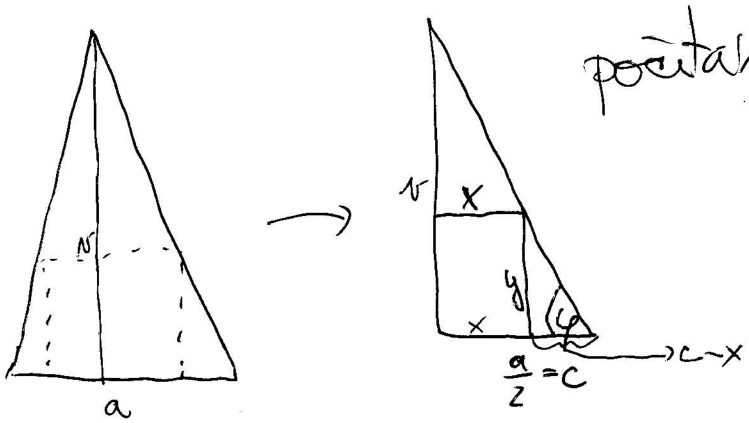

počítejme jen půlku

$$\boxed{S_{\square} = x \cdot y \longrightarrow max}$$

víme:

$$\left.\begin{aligned} tg\varphi &= \frac{v}{c} \\ tg\varphi &= \frac{y}{c-x} \end{aligned}\right\} y = \frac{v}{c}(c-x)$$

$$S_{\square} = x \cdot y = x \cdot \frac{v}{c}(c-x) = xv - x^2\frac{v}{c} \longrightarrow max$$

$$\frac{\partial}{\partial x}$$

$$S'(x) = v - 2x\frac{v}{c}$$

hledám $S'(x) = 0 = v - 2x\frac{v}{c}$, to nastává pro $\boxed{x = \frac{c}{2}}$

$$S_{max} = x \cdot y = \frac{c}{2} \cdot \frac{v}{c}\left(c - \frac{c}{2}\right) = \frac{v}{2}\left(\frac{c}{2}\right) = \frac{vc}{4}$$

to je spočtená ta půlka

celé: $\boxed{S_{max} = 2 \cdot xy = \frac{vc}{2} = \frac{v \cdot \frac{a}{2}}{2} = \frac{va}{4}}$

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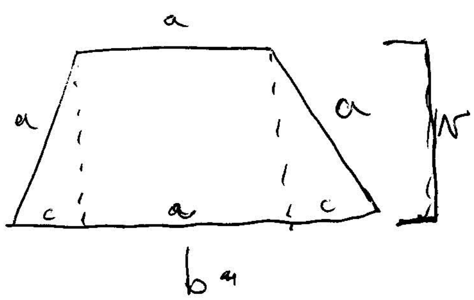
1



$b = a + 2c$

$\boxed{S = v \cdot a + c \cdot v \longrightarrow \max}$

$\boxed{v^2 = a^2 - c^2}$ $\quad$ $\boxed{a \text{ znám}}$

$$S(c) = v(a+c) = \sqrt{a^2-c^2}\,(a+c) \qquad \Big/ \frac{\partial}{\partial c}$$

$$S'(c) = \frac{1}{2}(a^2-c^2)^{-\frac{1}{2}}(-2c)(a+c) + (a^2-c^2)^{\frac{1}{2}}$$

$$= -\frac{ac+c^2}{\sqrt{a^2-c^2}} + \sqrt{a^2-c^2} = \frac{-ac-c^2+a^2-c^2}{\sqrt{a^2-c^2}}$$

hledám $S'(c)=0$, tj

$$\frac{-ac-c^2+a^2-c^2}{\sqrt{a^2-c^2}} = 0 \iff \underbrace{ac+c^2-a^2+c^2}_{2c^2+ac-a^2} = 0$$

$$c_{12} = \frac{-a \pm \sqrt{a^2-4 \cdot 2(-a^2)}}{2 \cdot 2} = \frac{-a \pm \sqrt{9a^2}}{4} = \begin{cases} \frac{a}{2} \\ -a \end{cases}$$

$-a$ není možné (c je také vzdálenost)

tedy $c = \frac{a}{2}$, pak $\boxed{b = a + 2c = a + 2\frac{a}{2} = 2a}$