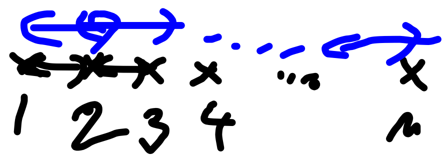
1, Pravděpodobnost, že dělník půjde při k-té cestě mezi stroji i a j je pro všechny dvojice i, j, $1 \leq i \neq j \leq n$ stejná, tj $\frac{1}{\binom{n}{2}}$.

X … délka cesty



$$EX = 1 \cdot \frac{n-1}{\binom{n}{2}} + 2 \cdot (n-2) \cdot \frac{1}{\binom{n}{2}} + \dots + (n-2) \cdot \frac{2}{\binom{n}{2}} + (n-1) \cdot \frac{1}{\binom{n}{2}} =$$

$$= \frac{1}{\binom{n}{2}} \left[1 \cdot (n-1) + 2 \cdot (n-2) + \dots + (n-1) \cdot 1 \right]$$

$$
n-1\left\{\begin{array}{lcl}
1+2+3+\cdots+(n-1) & = & \frac{n(n-1)}{2} \\
1+2+3+\cdots+(n-2) & = & \frac{(n-1)(n-2)}{2} \\
\vdots & & \vdots \\
1+2 & = & 3 \\
1 & = & 1
\end{array}\right\} =
$$

$$
= \sum_{i=1}^{n-1} \frac{i(i+1)}{2} = \frac{1}{2}\left(\sum_{i=1}^{n-1} i^2 + \sum_{i=1}^{n-1} i\right) =
$$

$$
= \frac{1}{2}\left(\frac{(n-1)(n)(2n-1)}{6} + \frac{n(n-1)}{2}\right) =
$$

$$
= \frac{1}{2}\left(\frac{n(n-1)(2n-1+3)}{6}\right) = \frac{1}{12}\left(n(n-1)(2n+2)\right) =
$$

$$
= \frac{1}{6}\left(n(n-1)(n+1)\right) =: S
$$

$$
EX = \frac{S}{\binom{n}{2}} = \frac{(n+1)}{3}
$$

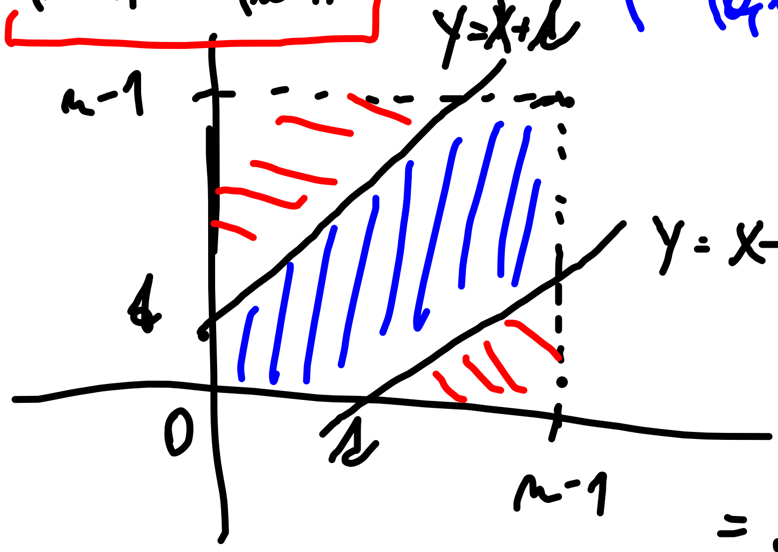
$$Z := |X-Y|$$

$$F_Z(s) = P[Z<s] = P[|X-Y|<s] = P[X-s<Y<X+s] =$$

$$= P[-s<X-Y<s]$$

$$= \frac{S}{(n-1)^2} = \frac{2s}{(n-1)} - \frac{s^2}{(n-1)^2}$$

((X,Y) má rovnoměrné rozdělení na čtverci $(0,n-1)\times(0,n-1)$)



$$S = (n-1)^2 - (n-1-s)^2 =$$

$$= 2(n-1)s - s^2$$

pro $0 < s < (n-1)$

$$f_Z(s) = \frac{2}{(n-1)} - \frac{2s}{(n-1)^2}$$

$$EZ = \int_0^{n-1} \left(\frac{2s}{n-1} - \frac{2s^2}{(n-1)^2}\right) ds = \left[\frac{s^2}{n-1} - \frac{2s^3}{3(n-1)^2}\right]_0^{(n-1)} =$$

$$= n-1 - \frac{2}{3}(n-1) = \frac{n-1}{3}$$

$$EX = 1{,}5$$

$$EX^2 = \int_1^2 x^2\,dx = \frac{7}{3}$$

$$E\left(\frac{1}{X}\right) = \int_1^2 \frac{1}{x}\,dx = \ln(2)$$

$$E\left(\frac{1}{X^2}\right) = \int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 = \frac{1}{2}$$

$$\operatorname{var} X = \frac{1}{12}, \quad \operatorname{var}\frac{1}{X} = E\left(\frac{1}{X^2}\right) - \left[E\left(\frac{1}{X}\right)\right]^2 = \frac{1}{2} - \ln^2(2) \ldots$$

X má hustotu $f(x)$ na (a, b), pak $g(X)$ má stř. hodnotu $\int_a^b g(x) \cdot f(x)\,dx$

$$E\left(X\cdot\frac{1}{X}\right) = E(1) = 1$$

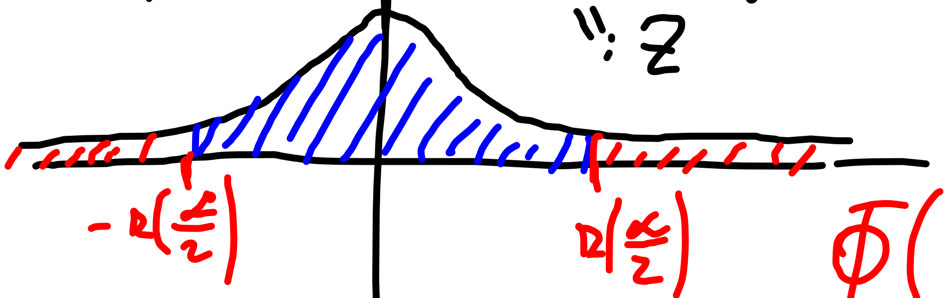
$$\frac{\bar{X}-\mu}{\sigma}\sqrt{n} \sim N(0,1)$$

$\bar{X}$ = výběrový průměr $= \frac{1}{n}\sum_{i=1}^{n} X_i$

Distr. fce $\frac{\bar{X}-\mu}{\sigma}\sqrt{n}$ je Φ (distr. fce norm. rozd.!)

$=: Z$

u ... kvantilová fce normálního rozdělení



$\Phi(u(\alpha)) = 1-\alpha$

$$P\left[|Z| < u\left(\frac{\alpha}{2}\right)\right] = P\left[\frac{\bar{X}-\mu}{\sigma}\sqrt{n} < u\left(\frac{\alpha}{2}\right)\right] =$$

$$= P\left[\,|\bar{x}-\mu| < \frac{\sigma}{\sqrt{n}}\, z\left(\frac{\alpha}{2}\right)\right] = P\left[\bar{X}-\frac{\sigma}{\sqrt{n}} z\left(\frac{\alpha}{2}\right) < \mu < \bar{X}+\frac{\sigma}{\sqrt{n}} z\left(\frac{\alpha}{2}\right)\right]$$

$$= P\left[870{,}3 - \frac{2{,}1}{\sqrt{5}}(1{,}96) < \mu < 870{,}3 + \dots\right] \quad \left| \; z(0{,}025) \doteq 1{,}96 \right]$$

$$= P\left[X \in (868{,}47\,;\,872{,}13)\right]$$

$$\left(\frac{X-\mu}{S}\sqrt{n}\right) \sim t_{n-1} \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$$

v desetinách promile

$$\bar{X} = 5‰_0\,, \quad S^2 = \frac{1}{3-1}\left(2^2+2^2\right) = 4$$

$$S = 2$$

$$\frac{5-\mu}{2}\sqrt{3} \sim t_2$$

$\alpha = 0{,}05$

$$0{,}95 = 1 - \alpha = P\left(|Z| < t_2\left(\frac{\alpha}{2}\right)\right) = P\left(\left|\frac{5-\mu}{2}\sqrt{3}\right| < t_2\left(\frac{\alpha}{2}\right)\right) =$$

$$\left[t_2(0{,}025) \doteq 4{,}3\right]$$

$$= P\left[|5-\mu| < \frac{2}{\sqrt{3}} \cdot 4{,}3\right] =$$

$$= P\left[\mu \in \left(5 - \frac{2}{\sqrt{3}} \cdot 4{,}3 \, , \, 5 + \frac{2}{\sqrt{3}} 4{,}3\right)\right] =$$

$$P\left[\mu \in (0{,}15 \, ; \, 9{,}85)\right]$$