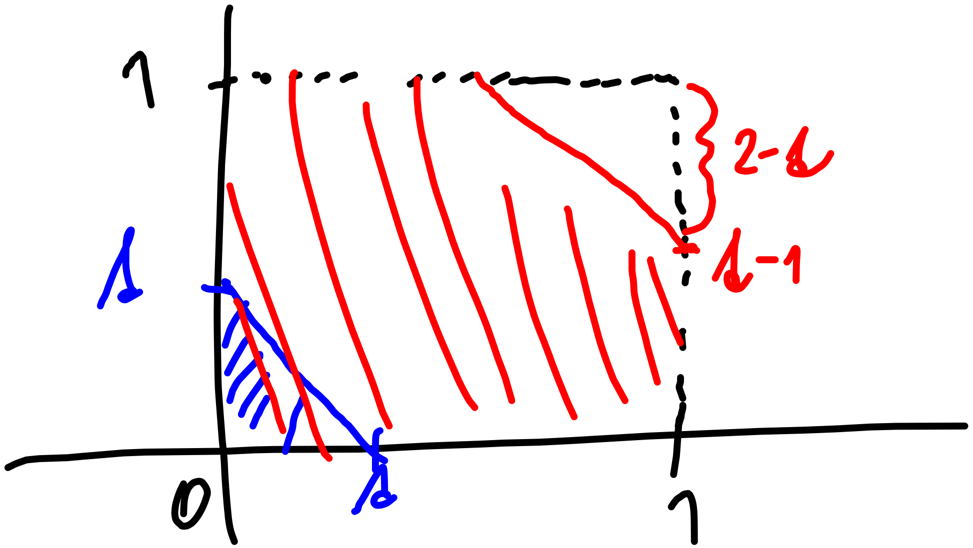
$$V = X + Y$$

$$F_V(s) = P(V < s) = P[X+Y < s] = P[Y < s - X]$$



$0 < s < 1$

$1 \le s < 2$

$= \frac{1}{2}s^2$

$= 1 - \frac{1}{2}(2-s)^2 = -\frac{1}{2}s^2 + 2s - 1$

Hustota pst.

$f(s) = s$

$f(s) = 2 - s$

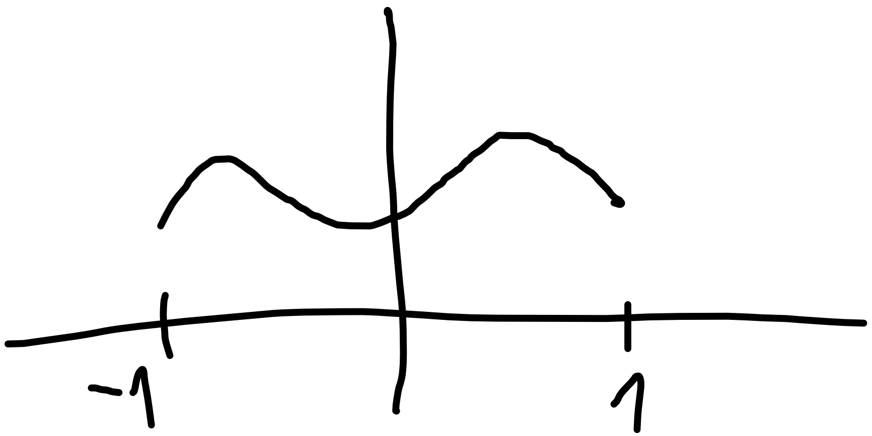
$$EV = \int_0^2 s \cdot f(s)\,ds = \int_0^1 s^2 + \int_1^2 2s - s^2 = 1$$

$$F_U(s) = P[X - Y \leq s] = P[Y - X < s] = P[X - Y > -s]$$

$$= 1 - P[X - Y \leq -s] = 1 - F_U(-s)$$

$$f_U(s) = f_U(-s)$$

Hustota této veličiny U je sudá funkce



$$EU = \int_{-1}^{1} s \cdot f(s)\, ds =$$

$$= \int_{-1}^{0} s \cdot f(s)\, ds + \int_{0}^{1} s \cdot f(s)\, dx =$$

$$= -\int_{0}^{1} s\, f(-s)\, ds + \int_0^1 \ldots = 0$$

$$\operatorname{cov}(U,V) = E(UV) - E(U)\cdot E(V) = 0 - 1\cdot 0 = 0$$

$$\Downarrow$$

$$\rho_{U,V} = 0$$

$$E(UV) = E\big((X+Y)(X-Y)\big) = E\left(X^2 - Y^2\right)$$

$$F_{UV}(1) = P\left[X^2 - Y^2 < 1\right] = P\left[Y^2 - X^2 < 1\right] =$$

$$= P\left[X^2 - Y^2 > -1\right] = 1 - F_{UV}(-1)$$

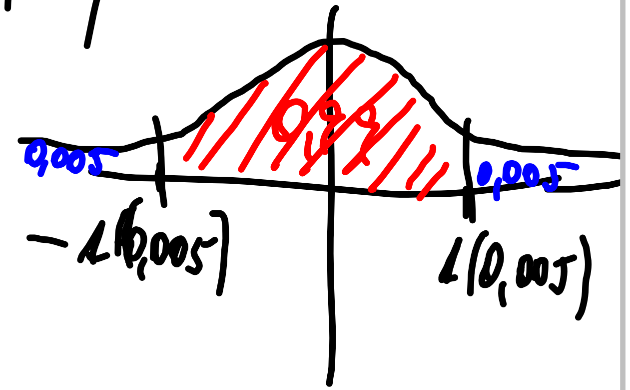
$$f_{UV}(1) = f_{UV}(-1) \Rightarrow E(UV) = E(X^2 - Y^2) = 0$$

$X_1, \dots, X_3$: 23%, 24%, 22%, 27%

$\bar{X} = 24\%$

$S^2 = \frac{1}{3}\left(1^2 + 2^2 + 3^2\right) = \frac{14}{3}$ $\qquad S = \sqrt{\frac{14}{3}}$

$T = \frac{\bar{X} - \mu}{\sigma}\sqrt{n} \sim N(0,1)$



$0{,}99 = P\left[|T| < t(0{,}005)\right]$

$= P\left[\left|\frac{\bar{X} - \mu}{\sigma}\sqrt{n}\right| < t(0{,}005)\right] =$

$= P\left[|\bar{X} - \mu| < \frac{t(0{,}005) \cdot \sigma}{\sqrt{n}}\right] =$

$$\underbrace{P\left[\mu \in \left(\bar{X} - \frac{\sigma \cdot u(0{,}005)}{\sqrt{n}}, \bar{X} + \frac{\sigma \cdot u(0{,}005)}{\sqrt{n}}\right)\right]} =$$

$$= P\left[\mu \in (22{,}18 ; 25{,}82)\right]$$

$$\frac{\bar{X} - \mu}{S}\sqrt{n} \sim t_{n-1}$$

$$\left(\bar{X} - \frac{S \cdot t(0{,}025)}{\sqrt{n}}, \bar{X} + \frac{S \cdot t(0{,}025)}{\sqrt{n}}\right)$$

$$= (17{,}69 , 30{,}3)$$

$$P[\mu \geq 25] = P[-\mu \leq -25] =$$

$$= P\left[\frac{\bar{X}-\mu}{\sigma}\cdot\sqrt{n} \leq \frac{\bar{X}-25}{\sigma}\cdot\sqrt{n}\right] =$$

$$= P\left[T \leq -\frac{1}{\sqrt{2}}\cdot 2\right] = P\left[T \leq -\sqrt{2}\right] =$$

$$= \Phi(-\sqrt{2}) = 1-\Phi(\sqrt{2}) = 8\,\%$$

$$P\left[\frac{\bar{X}-\mu}{s}\cdot\sqrt{n} \leq \frac{\bar{X}-25}{s}\cdot\sqrt{n}\right]$$

$$= \underbrace{P\left[\frac{\bar{X}-\mu}{s}\cdot\sqrt{n} \leq -0{,}825\right]} = 0{,}21$$

$n_1 = 5$: $Y_{11}, Y_{12}, \ldots, Y_{15}$ ... měření v JZD Tempo

$n_2 = 7$: $Y_{21}, Y_{22}, \ldots, Y_{27}$ ... — " — Boj

$\bar{Y}_1$ ... výběrový průměr 1. vzorku

$\bar{Y}_2$ ... — " —

$$S^2 = \frac{1}{n_1+n_2-2}\left(\sum_{i=1}^{n_1}(Y_{1i}-\bar{Y}_1)^2 + \sum_{i=1}^{n_2}(Y_{2i}-\bar{Y}_2)\right)$$

$$T = \frac{\bar{Y}_2-\bar{Y}_1}{S}\sqrt{\frac{n_1 n_2}{n_1+n_2}} \sim t_{n_1+n_2-2}$$

$$0,95 = P\left[\,|T| < t_{10}(0,025)\,\right]$$

$$\bar{Y}_1 = 15 \qquad S^2 = \frac{1}{10}\left(4+1+4+9+4+1+1+4+1+16\right) = 4,5$$

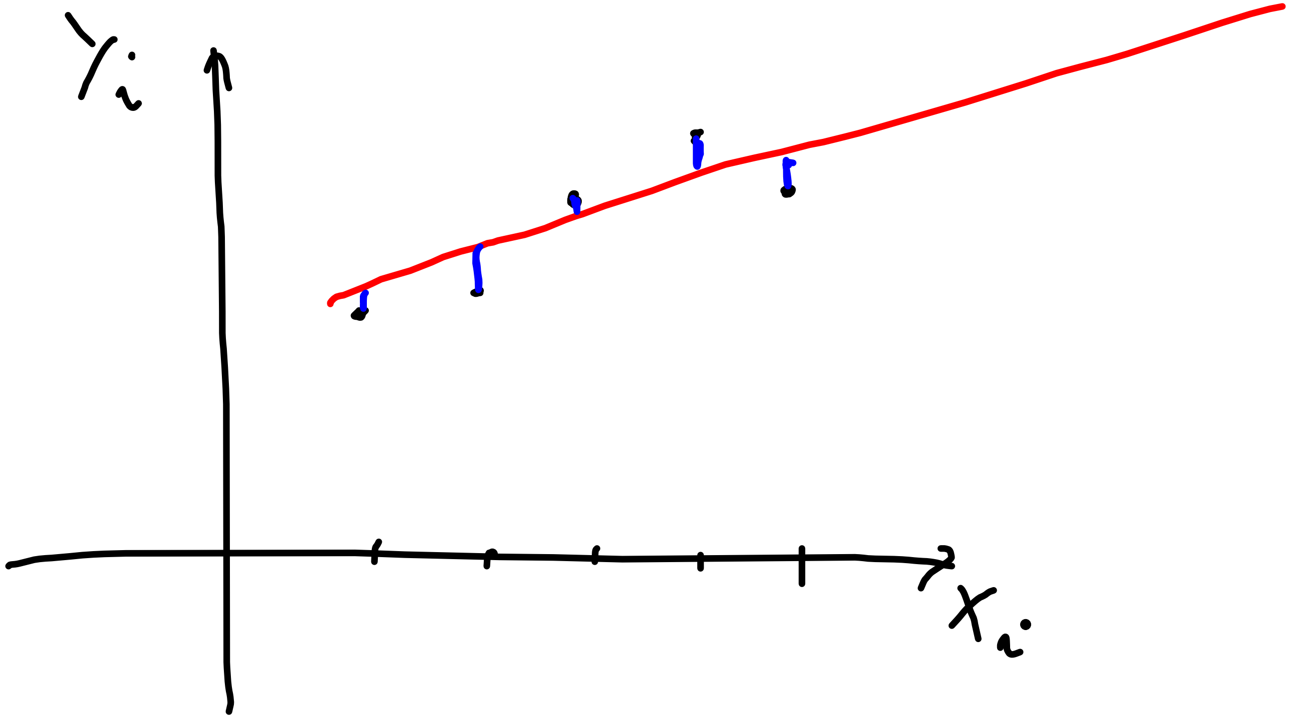
$$\bar{Y}_2 = 14$$

$$S \doteq 2,12$$

$$T = \frac{15-14}{2,12}\sqrt{\frac{5\cdot 7}{5+7}} = \frac{1}{2,12}\sqrt{\frac{35}{12}} \doteq +0,805$$

$$t_{10}(0,025) > 0,805 \Rightarrow$$

Hypotézu na pětiprocentní hladině nezamítáme.



Předpokládáme, že mezi naměřenými daty X_i a Y_i je afinní závislost, tedy

$$Y_i = b \cdot X_i + c$$

$$b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}_i)}{\sum_{i=1}^{n} (x_i - \bar{x})^2} =$$

$$c = \bar{y} - b \cdot \bar{x}$$

$$\bar{x} = 22$$

$$\bar{y} = 3$$

$$b = \frac{2 \cdot 5 + 1 \cdot 2 + 6 \cdot 2 + 1 \cdot 1}{66} = \frac{25}{66}$$

$$c = 3 - \frac{25}{66} \cdot 22 = 3 - \frac{25}{3} = -\frac{16}{3}$$

$$y = \frac{25}{66} x - \frac{16}{3}$$ pro $x = 30$

je očekávaná spotřeba 6 kilolitrů.