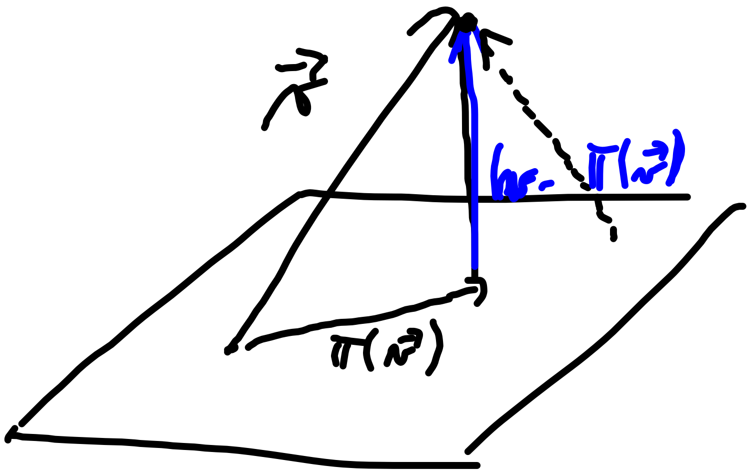

$$\left\langle \overset{g}{\overset{\shortparallel}{F}}, f_n \right\rangle = \left\langle \sum_{i=1}^{\infty} c_i f_i, f_n \right\rangle = \sum_{i=1}^{\infty} \langle c_i f_i, f_n \rangle =$$

$$= c_n \langle f_n, f_n \rangle = c_n \|f_n\|^2$$

$$c_n = \frac{1}{\|f_n\|^2} \cdot \langle g, f_n \rangle = \frac{1}{\|f_n\|^2} \cdot \int_a^b g\, f_n \, dx$$

$$f = \sum_{n=1}^{k} a_n f_n \qquad c_n = \|f_n\|^{-2} \cdot \langle g, f_n \rangle$$

$$\langle g, f_n \rangle = c_n \|f_n\|^2$$

$$\|g - f\| =$$

$$= \Big\| g - \sum_{n=1}^{k} a_n f_n \Big\|^2 = \Big\langle g - \sum_{n=1}^{k} a_n f_n , g - \sum_{n=1}^{k} a_n f_n \Big\rangle$$

$$= \langle g, g \rangle + \Big\langle \sum_{n=1}^{k} a_n f_n , \sum_{n=1}^{k} a_n f_n \Big\rangle - 2 \Big\langle g, \sum_{n=1}^{k} a_n f_n \Big\rangle$$

$$= \|g\|^2 + \sum_{n=1}^{k} \underline{\langle a_n f_n , a_n f_n \rangle} - 2 \sum_{n=1}^{k} \langle g, a_n f_n \rangle =$$

$$= \|g\|^2 + \sum_{n=1}^{k} a_n^2 \|f_n\|^2 - 2 \sum_{n=1}^{k} a_n c_n \|f_n\|^2 =$$

$$= \|g\|^2 + \sum_{n=1}^{k} \|f_n\|^2 \left[(c_n - a_n)^2 - c_n^2 \right] =$$

$$= \|g\|^2 - \sum_{n=1}^{k} \|f_n\|^2 c_n^2 + \sum_{n=1}^{k} \|f_n\|^2 (c_n - a_n)^2$$