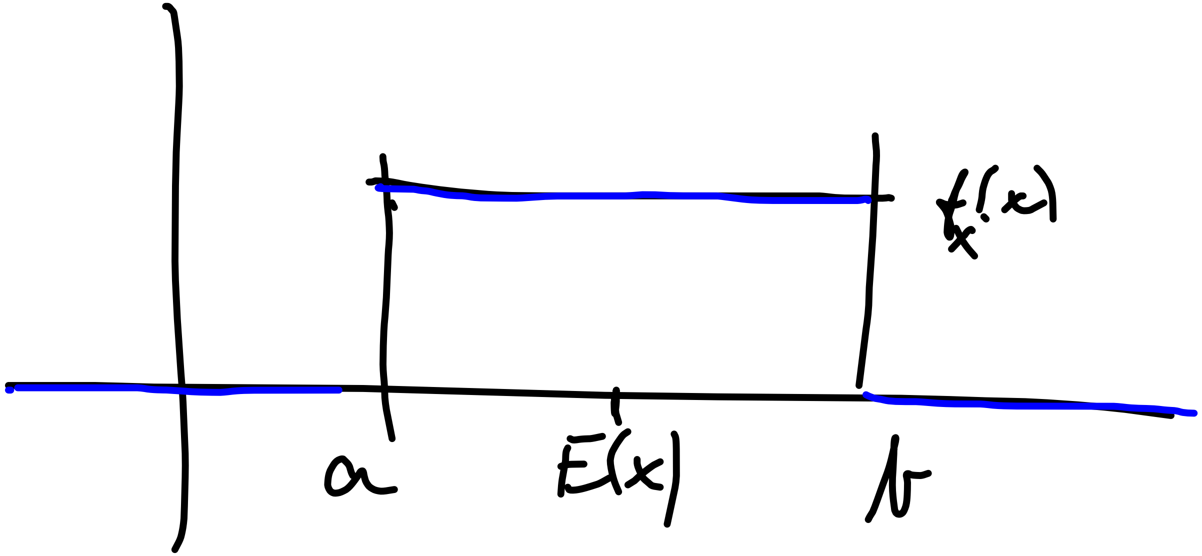

$$E(a+bX) = \int_{-\infty}^{\infty} (a+bx) f(x)\,dx = \int_{-\infty}^{\infty} a \cdot f(x)\,dx + \int_{-\infty}^{\infty} bx\,f(x)\,dx =$$

$$= a\int_{-\infty}^{\infty} f(x)\,dx + b\int_{-\infty}^{\infty} x\,f(x)\,dx =$$

$$= a + b\,EX$$

$$\operatorname{var} X = E\left[(X - EX)^2\right] = E\left[\left(\underline{X^2} - \underline{2X\cdot E(X)} + \underline{E(X)^2}\right)\right] =$$

$$= E(X^2) + E\left(-2X\cdot \underbrace{E(X)}_{\in \mathbb{R}}\right) + E\left(E(X)^2\right) =$$

$$= E(X^2) - 2E(X)E(X) + E(X)^2 =$$

$$= \boxed{E(X^2) - [E(X)]^2}$$

$$\operatorname{var}(a + bX) = E\left[\left(\cancel{a} + bX - E(\cancel{a} + bX)\right)^2\right] =$$

$$= E\left[\left(bX - bE(X)\right)^2\right] = E\left(b^2\left(X - E(X)\right)^2\right) =$$

$$= b^2 E\left[\left(X - E(X)\right)^2\right] = b^2 \cdot \operatorname{var} X$$

$$E\left(\frac{X-E(X)}{\sqrt{\operatorname{var}X}}\right) = \frac{1}{\sqrt{\operatorname{var}X}}\left[E\left(X-E(X)\right)\right] =$$

$$= \frac{1}{\sqrt{\operatorname{var}X}} \cdot \left(E(X) - E(X)\right) = 0$$

$$\operatorname{var}\left(\frac{X-E(X)}{\sqrt{\operatorname{var}X}}\right) = \operatorname{var}\left(-\frac{E(X)}{\sqrt{\operatorname{var}X}} + \frac{X}{\sqrt{\operatorname{var}X}}\right) =$$

$$= \left(\frac{1}{\sqrt{\operatorname{var}X}}\right)^2 \cdot \operatorname{var}(X) = 1$$

$$\operatorname{cov}(a+bX, c+dY) =$$

$$E\left[\left(\cancel{a}+bX - E(\cancel{a}+bX)\right)\left(\cancel{c}+dY - E(\cancel{c}+dY)\right)\right] =$$

$$= E\left[\left(bX - bE(X)\right)\left(dY - dE(Y)\right)\right] =$$

$$= bd\,E\left[\left(X-E(X)\right)\left(Y-E(Y)\right)\right] = bd\operatorname{cov}(X,Y)$$

$$\operatorname{var}(X+Y) = \operatorname{var}X + \operatorname{var}Y + 2\operatorname{cov}(X,Y)$$

$$\operatorname{cov}(X,Y) = E\left[\left(X-E(X)\right)\left(Y-E(Y)\right)\right] =$$

$$= E\left(X-E(X)\right)\cdot E\left(Y-E(Y)\right) = 0$$

$$|\rho_{X,Y}| \leq 1:$$

$$0 \leq \operatorname{var}\left(\frac{X-E(X)}{\sqrt{\operatorname{var} X}} + s\,\frac{Y-E(Y)}{\sqrt{\operatorname{var} Y}}\right) =$$

$$= 1 + 2\operatorname{cov}\left(\frac{X-E(X)}{\sqrt{\operatorname{var} X}},\ s\,\frac{Y-E(Y)}{\sqrt{\operatorname{var} Y}}\right) + s^2 =$$

$$= 1 + 2s\rho_{X,Y} + s^2 \qquad (\text{pro lib. } s \in \mathbb{R})$$

$$\Rightarrow D \leq 0 \Leftrightarrow 4\rho_{X,Y}^2 - 4 \leq 0 \Leftrightarrow$$

$$\rho_{X,Y}^2 \leq 1$$

$$M_{X+Y}(t) = E\, e^{t(X+Y)} = E\left(e^{tX} \cdot e^{tY}\right) = E(e^{tX}) \cdot E(e^{tY}) =$$

$$= M_X(t) \cdot M_Y(t)$$

$$Y_i \sim Bi(1, p) \sim A(p)$$

$$X := \sum_{i=1}^{n} Y_i \sim Bi(n, p)$$

$$M_X(t) = M_{\sum_{i=1}^{n} Y_i}(t) = \prod_{i=1}^{n} \left(p(e^t - 1) + 1\right) =$$

$$= \underbrace{\left(p(e^t - 1) + 1\right)^n}$$

$$\mu_1' = \frac{d}{dt} M_X(t)\Big|_{t=0} = n\left(p(e^t-1)+1\right)^{n-1} p \cdot e^t \,\Big|(0)$$

$$= \boxed{np}$$

$$\frac{d^2}{dt^2} M_X(t) = np\left[(n-1)\left(p(e^t-1)+1\right)^{n-2} p e^t \cdot e^t + \right.$$

$$\left. + \left[p(e^t-1)+1\right] \cdot e^t\right]$$

$$\mu_2' = \frac{d^2}{dt^2} M_X(0) = np\left((n-1)p + 1\right) = n(n-1)p^2 + np =$$

$$= n^2p^2 - np^2 + np$$

$$\mathrm{var}(X) = EX^2 - (EX)^2 = \mu_2' - n^2p^2 =$$

$$= np - np^2 = np(1-p)$$

$$h(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$M_Z(t) = e^{\frac{t^2}{2}}$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$