$$\varphi(0) = 0$$

$$\underline{0} = 0 \cdot \underline{0}$$

$$\varphi(\underline{0}) = \varphi(0 \cdot \underline{0}) = 0 \cdot \varphi(\underline{0}) = \underline{\underline{0}}$$

$$\varphi(-x) = -\varphi(x)$$

$$\Updownarrow$$

$$\varphi(-x) + \varphi(x) = 0$$

$$\Updownarrow$$

$$\varphi(\underbrace{-x + x}_{0}) = 0 \quad \Uparrow \quad \varphi(0) = 0$$

$$\varphi(-x) = \varphi((-1)x) = -1\,\varphi(x) = -\varphi(x)$$

$\varphi(x) = a x \quad a \in \mathbb{K}$ pevné

$$\varphi(x+y) = \varphi(x) + \varphi(y)$$

$$\| \qquad \| \qquad \|$$

$$a(x+y) = ax + ay$$

$$\varphi(cx) = c\,\varphi(x)$$

$$\| \qquad \|$$

$$a(cx) \overset{?}{=} c(ax)$$

$$\| \qquad \|$$

$$\underline{(a \cdot c)}\, x = \underline{(ca)}\, x$$

$$(\varphi \circ \psi)(ax+by) = \varphi(\psi(ax+by)) =$$

$$\underset{L2}{=} \varphi(a\psi(x) + b\psi(y))$$

$$\underset{L2}{=} a\,\underline{\varphi(\psi(x))} + b\,\varphi(\psi(y))$$

$$= a\,(\varphi \circ \psi)(x) + b\,(\varphi \circ \psi)(y)$$

$\varphi : V \to U$, $S \subseteq V$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ nad

$\Rightarrow \varphi(S)$ nad

$0 \in S \Rightarrow \underset{\varphi(0)}{\overset{}{0 \overset{}{=}}} \in \varphi(S)$

$x = \varphi(v_1)$

$x, y \in \varphi(S)$, $a, b \in K$ $\quad y = \varphi(v_2)$

$ax + by \overset{?}{\in} \varphi(S)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad v \in S$

$a\varphi(v_1) + b\varphi(v_2) = \varphi(av_1 + bv_2) \in \varphi(S)$

$\varphi : V \to U \quad , \quad T \leq U$ p.p.

? $\varphi^{-1}(T)$ je VP ?

$\varphi^{-1}(T) = \{ v \in V : \varphi(v) \in T \}$

$\underline{\varphi(o) = o \in T}$ $\quad \overset{\in}{o}$

$v_1, v_2 \in \varphi^{-1}(T), \; a, b \in K$

$\overset{?}{\Rightarrow} a v_1 + b v_2 \in \varphi^{-1}(T)$

$\varphi(a v_1 + b v_2) \overset{?}{\in} T$

$\|$

$a \underbrace{\varphi(v_1)}_{\in T} + b \underbrace{\varphi(v_2)}_{\in T} \in T$

V, X $\quad V^X$, $Y \subseteq X$, V^Y

$\varphi : V^X \to V^Y$ $\qquad \varphi(f) : Y \to V$

$f \mapsto f|_Y$

$\underline{\varphi(af_1 + bf_2)(y)} \overset{?}{=} \underline{[a\,\varphi(f_1) + b\,\varphi(f_2)](y)}$

$(af_1 + bf_2)|(y) = (af_1 + bf_2)|(y)$

φ je prosté $\Leftrightarrow$ $\mathrm{Ker}\,\varphi = \{0\}$

$\Rightarrow$: $x \in \mathrm{Ker}\,\varphi \Leftrightarrow \varphi(x) = 0 = \varphi(0)$

$$\Downarrow$$

$$x = 0$$

$\Leftarrow$ $\varphi(x) = \varphi(y) \Rightarrow x = y$

$\varphi(x) - \varphi(y) = 0$

$\varphi(x - y) = 0$

$x - y = 0 \Rightarrow x = y$

$\dim V = n$

$\operatorname{Ker}\varphi \subseteq V$

$$\overbrace{\underbrace{u_1, \dots, u_k}_{\text{LN}}, u_{k+1}, \dots, u_n}^{V}$$

$\operatorname{Ker}\varphi$

$\varphi(u_1), \dots, \varphi(u_k) = 0$, $\varphi(u_{k+1}), \dots, \varphi(u_n)$

$\operatorname{Im}\varphi = \varphi(V)$

$v = \sum c_i \cdot u_i$

$\underline{\varphi(v)} = \sum c_i \cdot \varphi(u_i)$

$$\left\{\begin{array}{l} c_{k+1}\varphi(u_{k+1}) + \dots + c_m\varphi(u_m) = 0 \\ \overset{?}{\Longrightarrow}\ c_{k+1} = \dots = c_m = 0 \\ \varphi(\underbrace{c_{k+1}u_{k+1} + \dots + c_m u_m}_{\in \operatorname{Ker}\varphi}) = 0 \end{array}\right.$$

$$\sum_{i=k+1}^{m} c_i u_i = \sum_{i=1}^{k} c_i u_i$$

$$u_1, \dots, u_m \Longrightarrow$$

$$c_1 = c_2 = c_k = c_{k+1} = c_m = 0$$

φ je prosté $\iff$ $\operatorname{Ker}\varphi = \{0\}$ $\iff$

$\iff$ $\dim \operatorname{Ker}\varphi = 0$ $\iff$

$\dim V = \dim \operatorname{Im}\varphi$ $\iff$ $V = \operatorname{Im}\varphi$

$$\Updownarrow$$

φ je surj.

$$\varphi : V \to V$$

$$\underline{\underline{\operatorname{Im}\varphi}} \subseteq \underline{\underline{V}}$$

$$\varphi^{-1}(ax+by) = a\varphi^{-1}(x) + b\varphi^{-1}(y)$$

$$\varphi(\varphi^{-1}(ax+by)) = \varphi(a\varphi^{-1}(x)+b\varphi^{-1}(y))$$

$$\parallel \qquad\qquad\qquad \parallel$$

$$ax+by = a\varphi(\varphi^{-1}(x)) + b\varphi(\varphi^{-1}(y))$$

a) $\mathbb{V} \cong \mathbb{U}$ $\quad \varphi : \mathbb{V} \to \mathbb{U}$ lin
lin. z.

$\dim \mathbb{V} = n$

$\mathbb{V} \cong \mathbb{K}^n$ $\qquad \dim \mathbb{U} = m$

$\mathbb{U} \cong \mathbb{K}^m$

$\Rightarrow \mathbb{K}^n \cong \mathbb{K}^m \Rightarrow \underline{\underline{n = m}}$

b) $\dim \mathbb{U} = \dim \mathbb{V}$

$\mathbb{U} \cong \mathbb{K}^n \cong \mathbb{V} \Rightarrow \mathbb{U} \cong \mathbb{V}$

$V \quad id_V : V \to V \quad B = (v_1, \ldots, v_n)$

$$(id_V)_{B,B} = I_n$$

$$\left((v_1)_B, \ldots, (v_n)_B \right) = \underline{\underline{I_n}}$$

$$v_1 = 1 \cdot v_1 + 0\, v_2 + \ldots + 0\, v_n$$

$$= (v_1, \ldots, v_n)\, e_1$$

$$v_i = (v_1, \ldots, v_n)\, e_i$$

$$\varphi : \underset{\beta}{V} \to \underset{\alpha}{U}$$

$$(\varphi(x))_\alpha = \underline{(\varphi)_{\alpha\beta}}\,(x)_\beta$$

$$(\varphi(v_j))_\alpha = s_j\left((\varphi)_{\alpha,\beta}\right)$$

$$(v_j)_\beta = e_j$$

$$(\varphi)_{\alpha,\beta}\, e_j = s_j\left((\varphi)_{\alpha,\beta}\right).$$

$$\mathbf{x} = \sum x_j \mathbf{v}_j$$

$$(\varphi(\mathbf{x}))_2 = \left(\sum x_j \varphi(\mathbf{v}_j)\right)_2 =$$

$$= \sum x_j \underbrace{(\varphi(\mathbf{v}_j))_2}_{s_j((\varphi)_{2,3})} =$$

$$= (\varphi)_{2,3} \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}}_{\shortparallel \atop (\mathbf{x})_3}$$