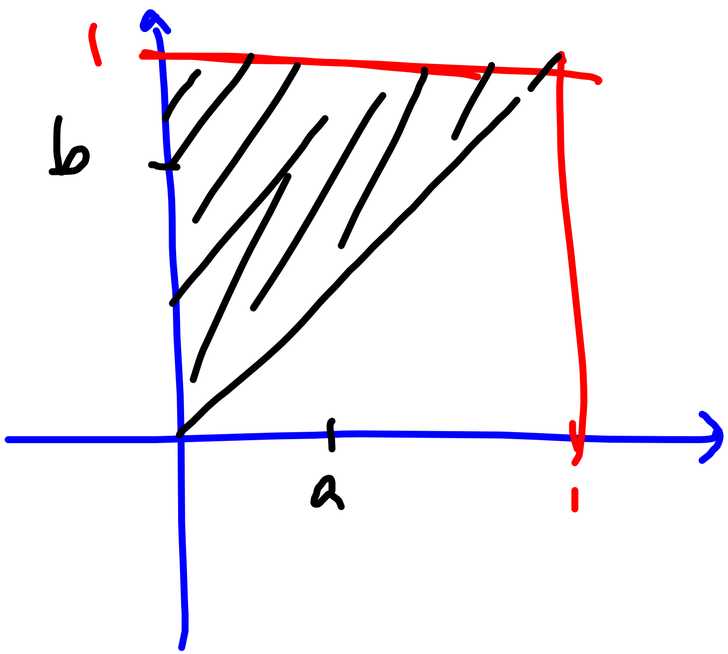
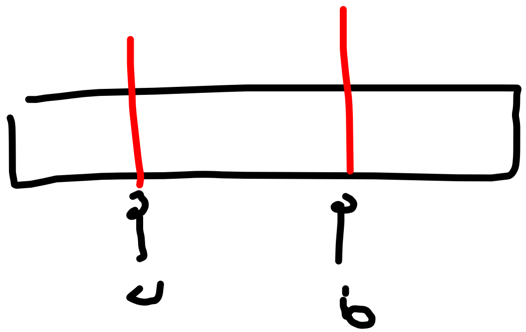
"geometrická pravděpodobnost"





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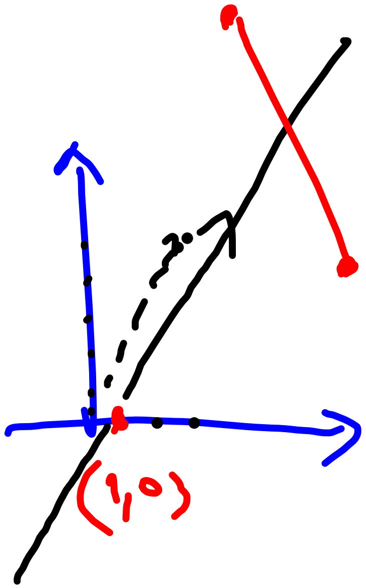


? trefíme ?

2 $ax + by = c$ rovnice přímky

předpis z parametrizace $p: \begin{cases} x = 2 + 3t \\ y = 0 + t \end{cases}$

2\. rovnice $\Rightarrow t = y \overset{\text{dosazení!}}{\Rightarrow} x = 2 + 3y \Rightarrow \boxed{x - 3y = 2}$

dosazuju $\quad r: \begin{cases} x = -1 + s \\ y = 2 + 3s \end{cases}$

$$\underbrace{-1 + s}_{x} \underbrace{- 3(2 + 3s)}_{-3y} = 2$$

$$-8s = 2 + 7 \quad \Rightarrow \quad s = -\frac{9}{8}$$

$$\Rightarrow \begin{cases} x_0 = -1 - \frac{9}{8} = -\frac{17}{8} \\ y_0 = 2 - \frac{3 \cdot 9}{8} = -\frac{11}{8} \end{cases}$$

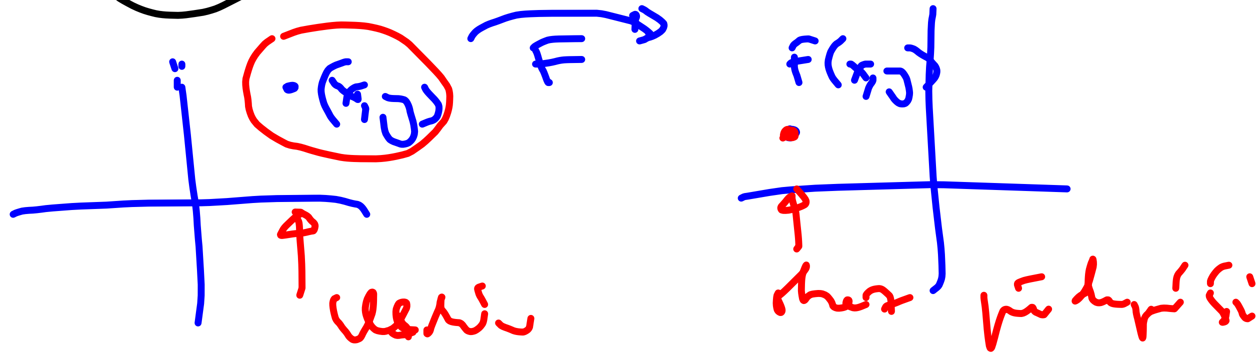
převedení r na implicitní tvar:

$$r: \begin{cases} x = -1 + s \\ y = 2 + 3s \end{cases} \Rightarrow 3x - y = -5$$

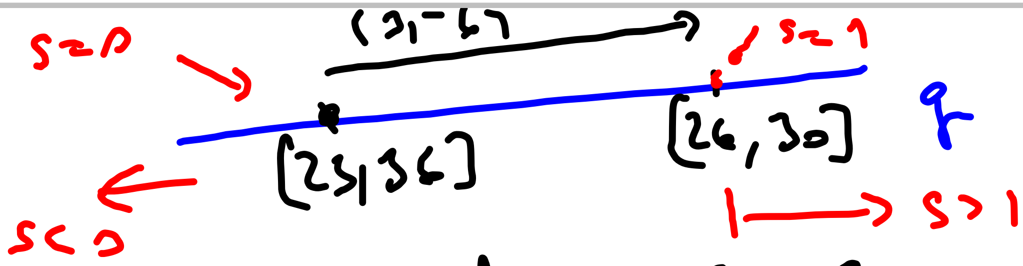
soustava rovnic:

$$3x - y = -5$$
$$x - 3y = 2$$

$$\begin{pmatrix} 3 & -1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$



Zpět k ① :



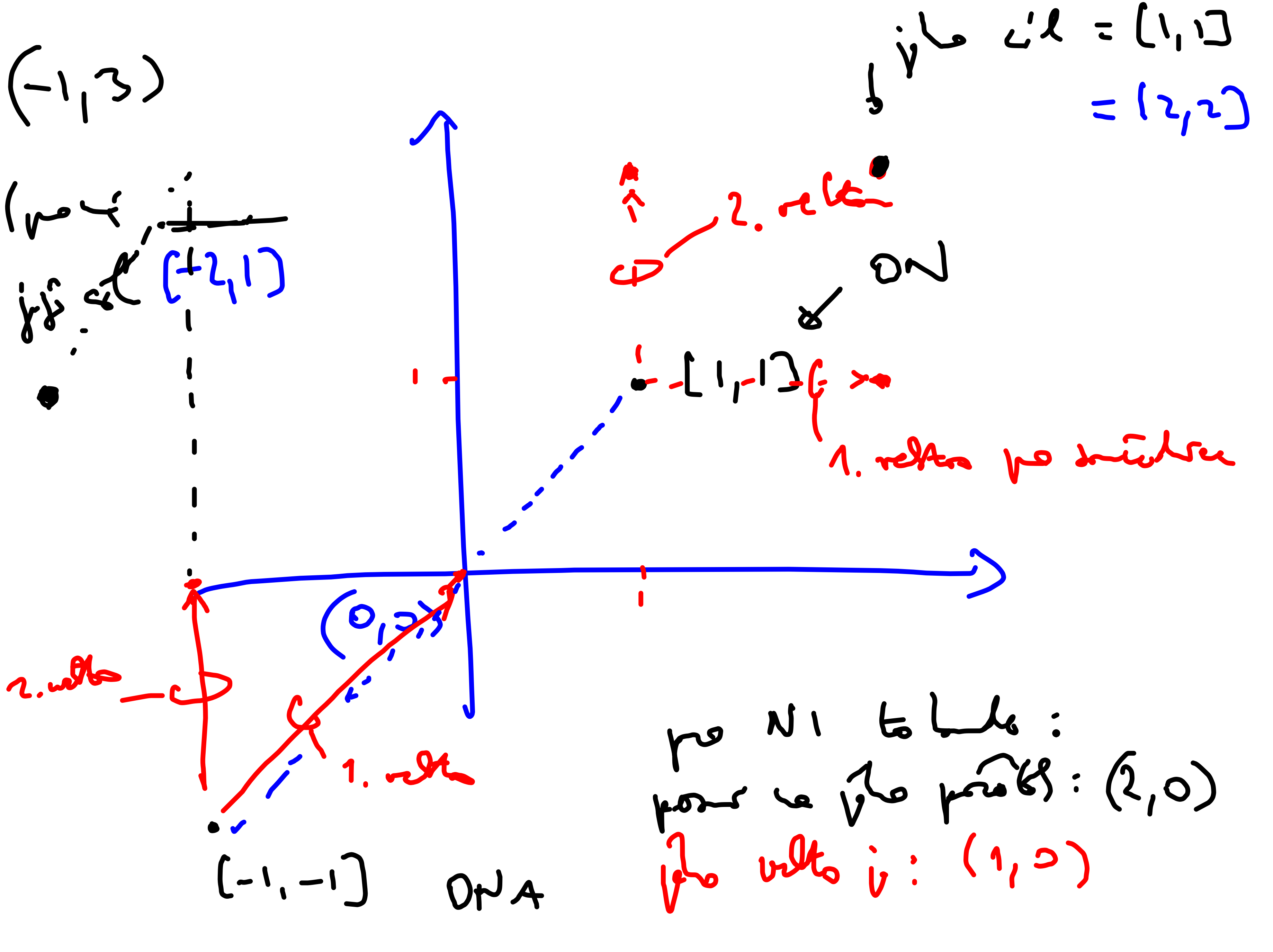
$$p: \begin{cases} x = 1 + 3t \\ y = 0 + 5t \end{cases} \qquad q: \begin{cases} x = 23 + 3s \\ y = 36 - 6s \end{cases}$$

$\Rightarrow$ buď jediný z předchozích společných nebo:

$$\left.\begin{aligned} 1 + 3t &= 23 + 3s \\ 0 + 5t &= 36 - 6s \end{aligned}\right| \qquad 2 + 11t = 82 \Rightarrow 11t = 80$$

$$\Rightarrow t = \frac{80}{11} \Rightarrow 1 + 3 \cdot \frac{80}{11} = 23 + 3s \Rightarrow$$

$$\Rightarrow s = \frac{11 + 3 \cdot 80 - 23 \cdot 11}{3 \cdot 11} = \frac{11 + 240 - 253}{33} = \underline{\frac{-2}{33}} \Rightarrow \text{NETREFÍME}$$


(-1,3)
= [1,1]
= [2,2]
[-2,1]
ON
[1,-1]
(0,2)
[-1,-1]
ONA
(2,0)
(1,0)

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1-2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

1. vektor protečený
vyjádří ve vektorech
verze

řešme: body v rovině jsou $\begin{pmatrix} x \\ y \end{pmatrix}$

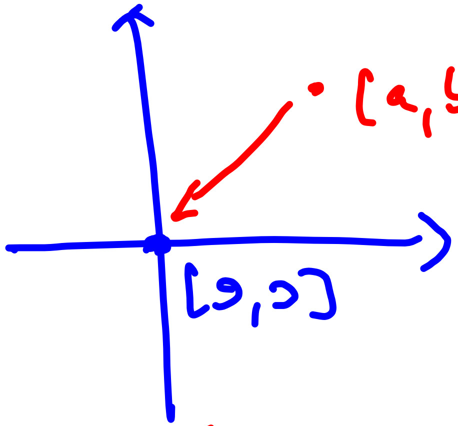
lineární zobrazení daná maticí! $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

(6) pokud $ad - bc \neq 0$, pak vždy právě jedno řešení

$$M \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

(3)



$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} -x \\ -y \end{pmatrix}$$

středová symetrie v $[1,1]$

$$\begin{pmatrix} x \\ y \end{pmatrix} \xmapsto[\text{posunutí}]{} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} \xmapsto[\text{symetrie}]{} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = \begin{pmatrix} -x+1 \\ -y+1 \end{pmatrix}$$

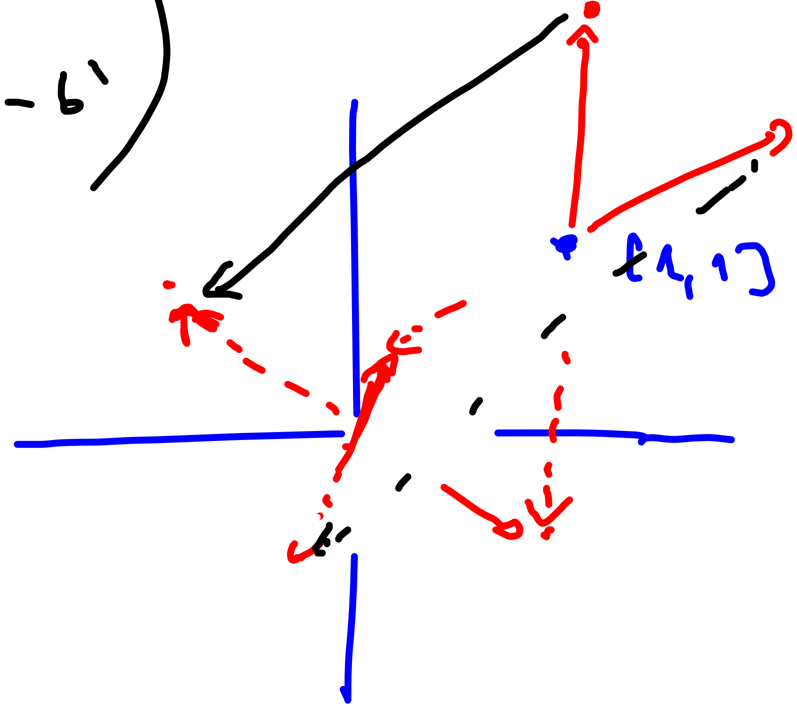
$$\xmapsto[\text{posun zpět}]{} \begin{pmatrix} -x+1 \\ -y+1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

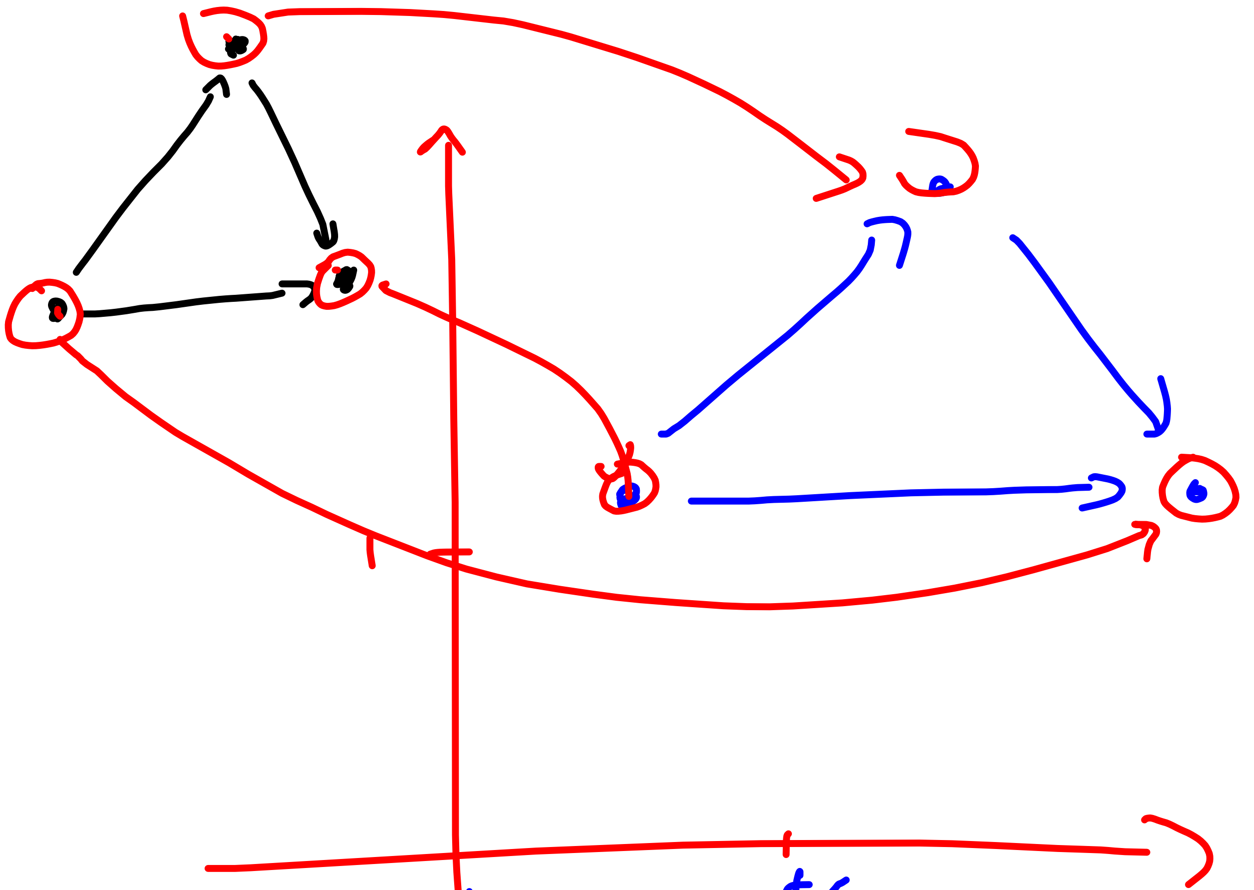
$$S_{[a,b]} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} + 2 \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$S_{[a,b]} \circ S_{[a',b']}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \xmapsto{S_{[a',b']}} \begin{pmatrix} \overbrace{-x+2a'}^{x'} \\ \underbrace{-y+2b'}_{y'} \end{pmatrix} \xmapsto{S_{[a,b]}} \begin{pmatrix} x-2a'+2a \\ y-2b'+2b \end{pmatrix}$$

$$= \begin{pmatrix} x \\ y \end{pmatrix} + 2\cdot \begin{pmatrix} a-a' \\ b-b' \end{pmatrix}$$





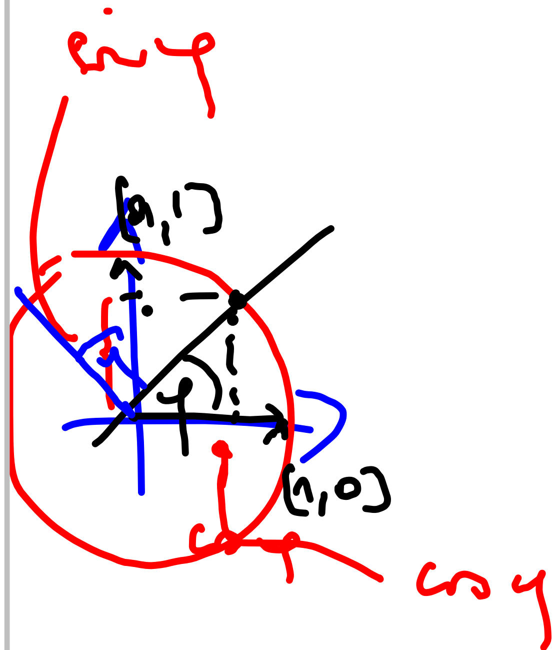
$$\begin{pmatrix} \overbrace{\begin{matrix} ax+by \\ dx+ey \end{matrix}}^{\text{lineární}} + \overbrace{\begin{matrix} c \\ f \end{matrix}}^{\text{posunutí}} \end{pmatrix} = F\cdot\begin{pmatrix} x \\ y \end{pmatrix}$$

dilatace:

$$\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix}$$

otočení:

$$\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \ldots$$

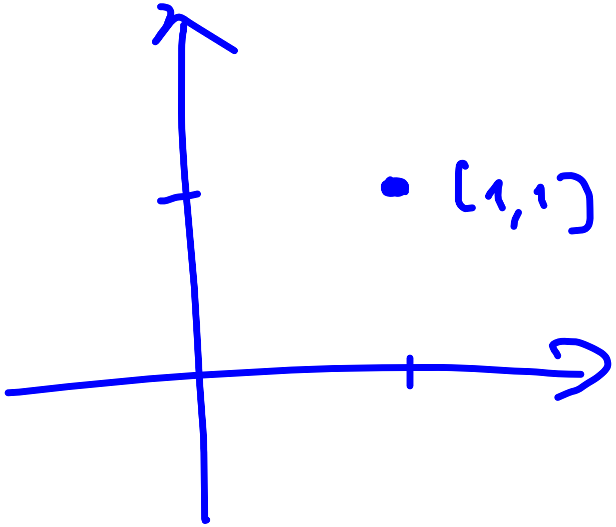


otočení o 45° prot. hodinovým kolem bodu [1, 1]

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} \longmapsto \begin{pmatrix} \sqrt{2}/2 & -\frac{\sqrt{2}}{2} \\ \sqrt{2}/2 & \frac{\sqrt{2}}{2} \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$

$$= \frac{\sqrt{2}}{2} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} x-y \\ x+y+2 \end{pmatrix}$$

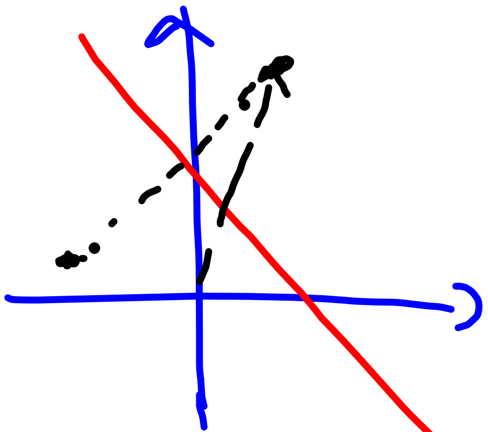
$$\longmapsto \frac{\sqrt{2}}{2} \begin{pmatrix} x-y \\ x+y+2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

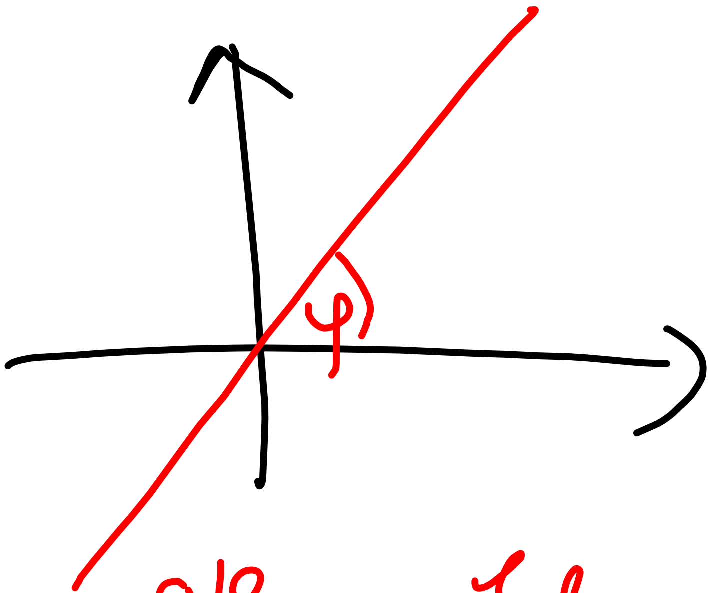


zrcadlení :

1) vždy prochází přímka a je osa x :

$$\begin{pmatrix}x\\y\end{pmatrix} \xmapsto{Z_0} \begin{pmatrix}x\\-y\end{pmatrix}$$



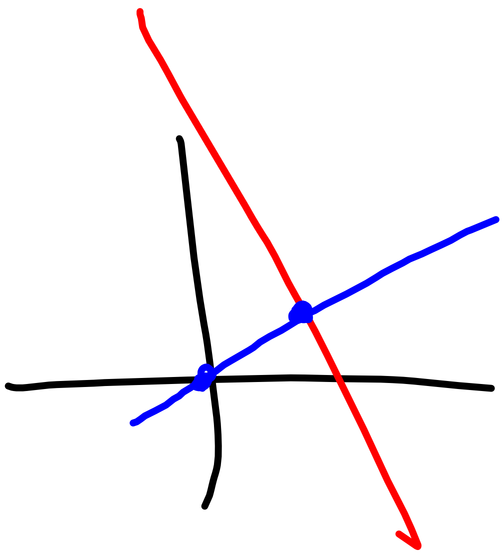


2) vůbec přímka , úhel s x φ :

první $R_\varphi : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto R_\varphi \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto$

$Z_0 \cdot R_\varphi \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto R_{-\varphi} \cdot Z_0 \circ R_\varphi \begin{pmatrix} x \\ y \end{pmatrix}$

3) lib. přímka:



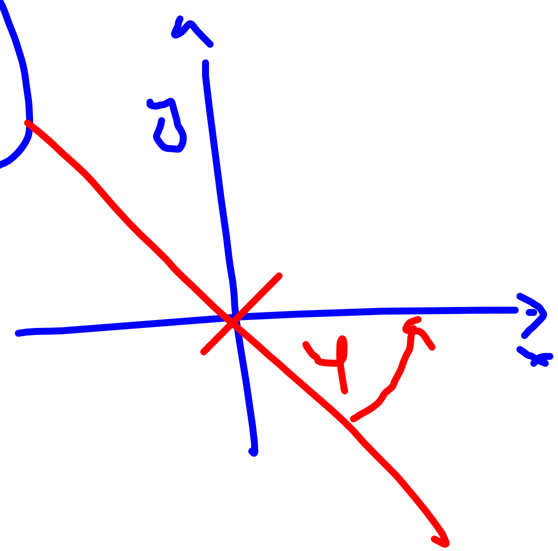
zrcadlení podle osy 2. kvadrantu:

$$\begin{pmatrix} x \\ y \end{pmatrix} \overset{\text{rotace přímky do osy } x}{\longmapsto} \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{\sqrt{2}}{2}\begin{pmatrix} x-y \\ x+y \end{pmatrix}$$

$$\overset{\text{zrcadlení podle } x}{\longmapsto} \frac{\sqrt{2}}{2}\begin{pmatrix} x-y \\ -x-y \end{pmatrix}$$

$$\overset{\text{rotace o } -45^\circ}{\longmapsto} \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x-y \\ -x-y \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -2y \\ -2x \end{pmatrix} = -\begin{pmatrix} y \\ x \end{pmatrix}$$



$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$ zachovává vzdálenosti, tj.

$v = (x_2 - x_1, y_2 - y_2)$

$[x_2, y_2]$

$[x_1, y_1]$

$$\|v\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\left\| M \cdot \begin{pmatrix} x \\ y \end{pmatrix} \right\| = \left\| \begin{pmatrix} x \\ y \end{pmatrix} \right\|$$

?

$$\Longleftrightarrow \quad a^2 + c^2 = 1, \quad b^2 + d^2 = 1, \quad ad - bc = 1, \quad ab + cd = 0$$

dokážeme zítra

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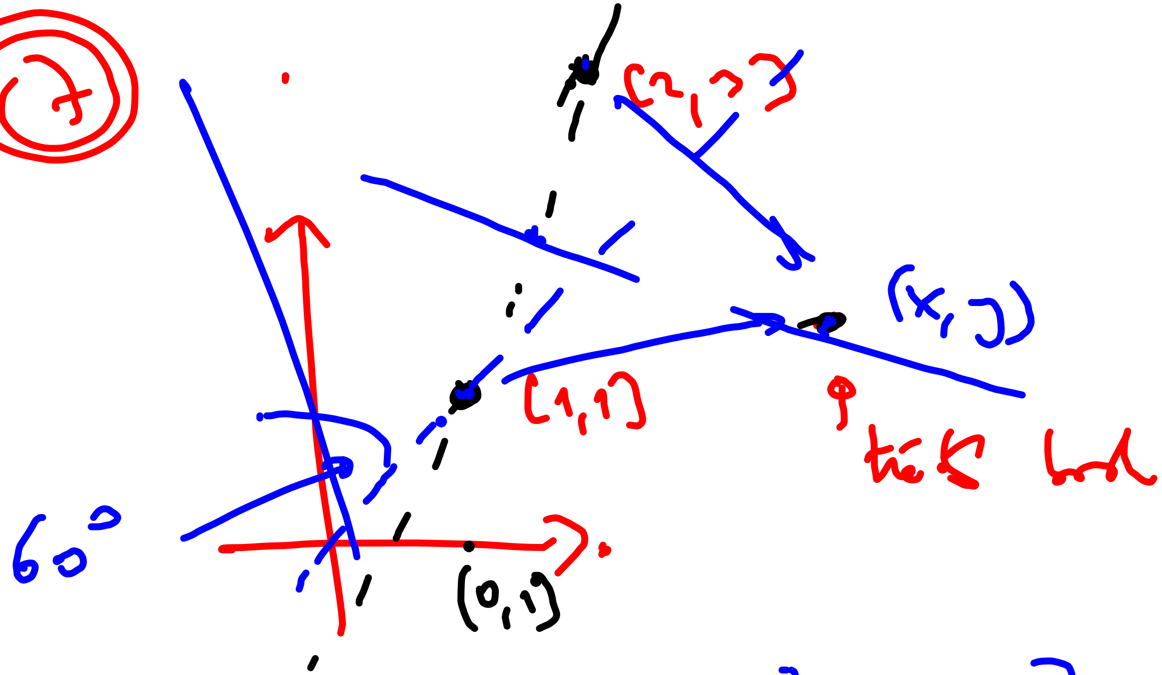
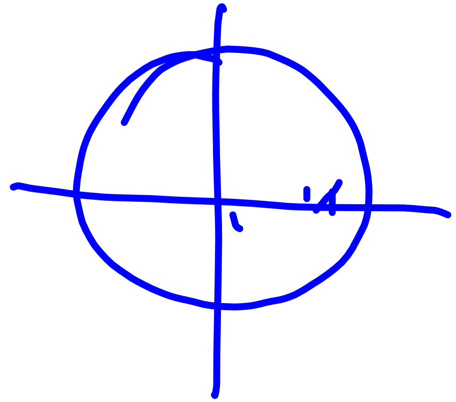
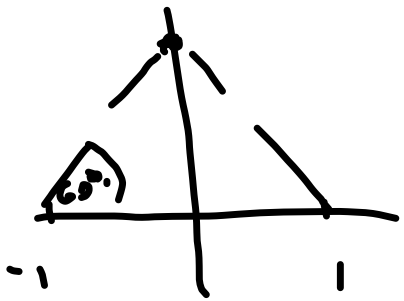




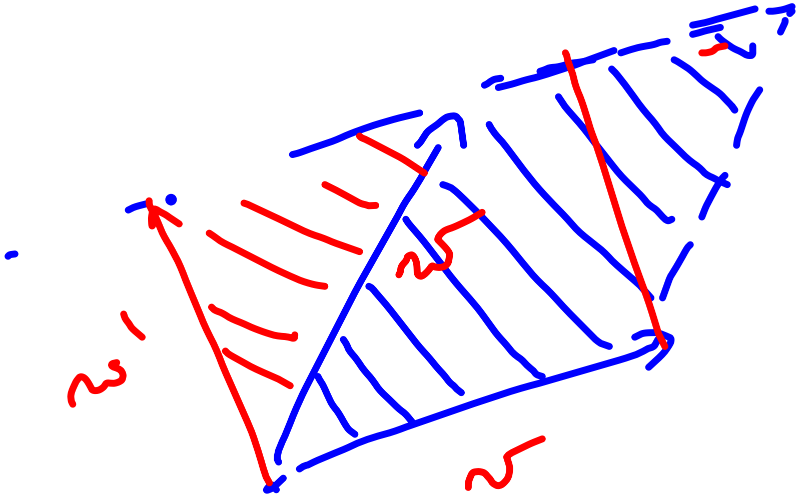
$$1+4 = (x-1)^2+(y-1)^2 = (x-2)^2+(y-3)^2$$



$R_{\pi/3}$ je dána maticí

$$\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

výsledek: $R_{\pi/3} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$ pomocí



obsah $\Delta(v, w) =$

$\|v\| \cdot \|\text{výška}\|$

$\|$

w'

$$\det \begin{pmatrix} x(v) & x(w) \\ y(v) & y(w) \end{pmatrix}$$

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

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$$v = \begin{pmatrix} 8-2 \\ 8-2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$w = \begin{pmatrix} 3-2 \\ 5-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Δ má obsah $\pm \left| \begin{pmatrix} 6 & 1 \\ 6 & 3 \end{pmatrix} \right| =$

$$= 6 \cdot 3 - 6 \cdot 1 = 12$$

[illegible]