$(a+0)\cdot v = a\cdot v + 0\cdot v = a\cdot v$

$=a$ $\Rightarrow$ $0\cdot v = 0$ ✓

$u + (-1)\cdot u = \underbrace{(1+(-1))}_{=0}\cdot u = 0\cdot u = 0$

✓ (2)

$a\cdot(u+(-1)\cdot v) = a\cdot u + (-a)\cdot v = a\cdot u + (-1)\cdot(a\cdot v)$

$= a\cdot(u-v)$ ✓ (3)

$(a-b)\cdot v = a\cdot v + (-b)\cdot v = a\cdot v - b\cdot v$ ✓ (4)

$a\cdot 0 = a\cdot(v-v) = a\cdot v - a\cdot v = 0$ ✓

$a\cdot u = 0 \wedge a \neq 0 \Rightarrow u = 1\cdot u = (a^{-1}\cdot a)\cdot u$

$= a^{-1}\cdot\underbrace{(a\cdot u)}_{=0} = \underline{0}$ ✓ (1) (2)

$v_1, \dots, v_k$ lin. nezávislé

$$a_1 v_1 + \dots + a_k v_k = b_1 v_1 + \dots + b_k v_k$$

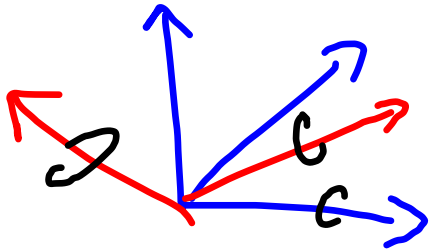
$$\Updownarrow$$

$$a_1 = b_1, \dots, a_k = b_k$$

Lemma

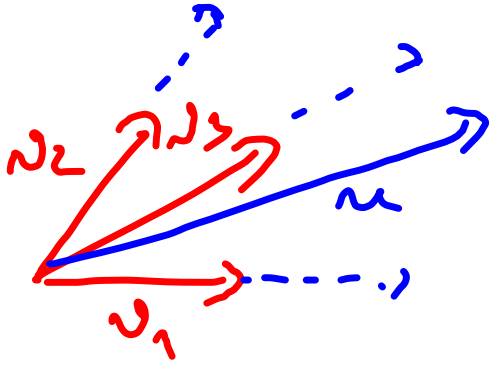
Důk. $\Rightarrow (a_1 - b_1) \cdot v_1 + (a_2 - b_2) \cdot v_2 + \dots + (a_k - b_k) v_k = 0$

$\Rightarrow$ $a_i = b_i$ ✓



Souřadnice vektoru

báze



$$u = x_1 v_1 + x_2 v_2 + x_3 v_3$$

$$u \xmapsto{\varphi} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$$

$$\varphi : V \longrightarrow \mathbb{K}^n \qquad \mathcal{A} = (v_1, \dots, v_n)$$

$$\varphi(u+u') = \overbrace{x_1 v_1 + x_2 v_2 + x_3 v_3}^{\varphi(u)} + \underbrace{x_1' v_1 + x_2' v_2 + x_3' v_3}_{\varphi(u')}$$

$$= (x_1 + x_1') v_1 + (x_2 + x_2') v_2 + (x_3 + x_3') v_3 = \varphi(u) + \varphi(u')$$

$$\varphi(u+u') = \begin{pmatrix} x_1 + x_1' \\ x_2 + x_2' \\ x_3 + x_3' \end{pmatrix}$$

$$f(0) = f(x - x) = f((1-1)\cdot x) = 0 \cdot f(x) = 0$$ ✓

$$f(-x) = f((-1)\cdot x) = (-1)\cdot f(x) = -f(x)$$ ✓

$$f((x+x')+x'') = f(x+x') + f(x'') = f(x) + f(x') + f(x'')$$