Baf! Opět experimentálně!
nevěřím!...

a) $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0$

$a_{21}x_1 + a_{21}x_2 + a_{23}x_3 = 0$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A \cdot x = 0$$

$$M = \{ x \in \mathbb{R}^3 ; \ A \cdot x = 0 \}$$

$$x, y \in M, \quad c, b \in \mathbb{R} \ \overset{?}{\underset{?}{\Longrightarrow}}$$

$$c \cdot x + b \cdot y \in M \qquad \text{ANO !}$$

$$A \cdot (c \cdot x + b \cdot y) \overset{!}{=} A \cdot (c \cdot \cancel{x})^{0} + A(b \cdot y) = c \cdot (A \cdot x) + b \cdot (A \cdot y)^{0} = 0$$

a) $A \cdot x = b \qquad b \neq 0$

$A \cdot y = b$

$A \cdot (x+y) = b + b = 2 \cdot b \neq 0$

(jsme v $\mathbb{R}^n$)

b) $x_{n+k} = a_0 x_n + a_1 x_{n+1} + \dots a_{k-1} x_{n+k-1}$

$x_i'$, $x_i''$ řešení

$\alpha x'_{n+k} = \alpha(\cdot/\cdot) = a_0(\alpha x_n) + \dots$

$(x'_{n+k} + x''_{n+k}) = (\cdot/\cdot) + (\cdot/\cdot)$

$= a_0(x_n' + x_n'') + \dots$

opět M je uzavřené na součet a násobení skalárem $\Rightarrow$ je vekt. prostor

ANO

c) NE

d) NE pokud $c \neq 0$.

a) $\mathbb{K}^n$, $\mathbb{K} = \mathbb{R}, \mathbb{Q}, \mathbb{Z}_p, \dots$

báze: **standardní**: $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $\dots$ $e_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

$$x = a_1 e_1 + \dots + a_n e_n = \begin{pmatrix} a_1 \cdot 1 + a_2 \cdot 0 + \dots + a_n \cdot 0 \\ a_2 \cdot 1 \\ \vdots \\ a_n \cdot 1 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$x = 0 \iff a_1 = 0 \wedge a_2 = 0 \wedge \dots \wedge a_n = 0$

i lineárně nezávislé, "generují vše" $\Rightarrow$ báze

jiné volby:

$f_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $\dots$ $f_n = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$

$f_i = e_1 + e_2 + \dots + e_i$; $e_1 = f_1$, $e_2 = f_2 - f_1$, $\dots$

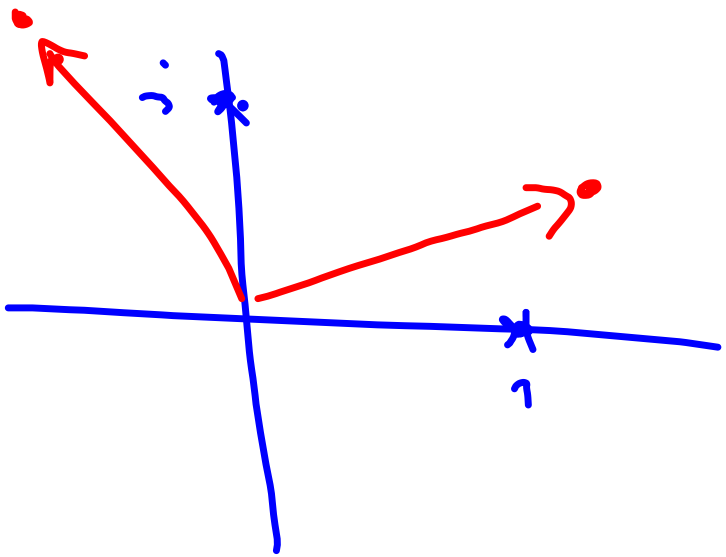
b) $V = \mathbb{C} = \{ x + iy \; ; \; x, y \in \mathbb{R} \}$

$a(x+iy) = ax + i(ay)$, sčítání po složkách

$\Rightarrow$ vekt. prostor nad $\mathbb{R}$.

$e_1 = 1$, $e_2 = i$

$x + iy = x \cdot e_1 + y \cdot e_2$



c) $\mathbb{K}_n[x] = \{a_0 + a_1 x + \dots + a_n x^n ;\ a_i \in \mathbb{K}\}$

– všech polynomů stupně nejvýše $n$.

$\mathbb{K}_n[x] \cong \mathbb{K}^{n+1}$ – izomorfní vekt. prostory

(pozor např. $\mathbb{Z}_2$ : $x + x^2$)

báze: $\{1, x, x^2, \dots, x^n\}$

$\mathbb{K}[x]$ např. báze:

$\{1, x, x^2, \dots, x^k, \dots\}$

d) $\mathbb{R}$ nad $\mathbb{Q}$ nemá spočetnou bázi.

$1, \sqrt{2}$ jsou lineárně nezávislé, ale ...

e) podobně ...

Příklad 3: v $\mathbb{R}_3[x]$:

$$\left.\begin{aligned} v_1 &= ax^2+x+2 \\ v_2 &= -2x^2+ax+3 \\ v_3 &= x^2+2x+a \end{aligned}\right\}$$ pro jaké $a$ jsou závislé

$x_1 \cdot v_1 + x_2 v_2 + x_3 v_3 = 0$ pro netriviální $x_i$.

srovnání u mocnin:

$x^2$: $a x_1 - 2x_2 + x_3 = 0$

$x^1$: $x_1 + a x_2 + 2x_3 = 0$

$x^0$: $2x_1 + 3x_2 + a x_3 = 0$

$$\begin{vmatrix} a & -2 & 1 \\ 1 & a & 2 \\ 2 & 3 & a \end{vmatrix} = 0$$

$= a^3 + 3 - 8 - 2a + 2a - 6a =$

$= a^3 - 6a - 5$

$(a^3 - 6a - 5) : (a+1) = a^2 - a - 1$

$-a^3 - a^2$

$-a^2 - 6a - 5$

$a^2 + a$

$-5a - 5$

$\boxed{a_1 = -1}$

$$\boxed{a_{2,3} = \frac{1 \pm \sqrt{5}}{2}}$$

$V_1, V_2 \subset \mathbb{R}_3$ $\quad$ mezi

$$V_1 = \left\langle \overset{=v_1}{\begin{pmatrix}1\\1\\0\end{pmatrix}}, \overset{=v_2}{\begin{pmatrix}1\\0\\1\end{pmatrix}} \right\rangle$$

$$V_2 = \left\{ x = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix},\ x_1 + x_2 + x_3 = 0 \right\}$$

$\begin{pmatrix}1\\0\\-1\end{pmatrix} \in V_2$

$\notin V_1$

$V_1 \cap V_2$ ?

$$v = a \cdot v_1 + b \cdot v_2 \overset{?}{\underset{!}{\in}} V_2$$

$$= \begin{pmatrix}a+b\\a\\b\end{pmatrix}$$

$$\Rightarrow \quad a + b + a + b = 0 \quad \Rightarrow \quad a = -b \quad \checkmark$$

$$2a + 2b = 0$$

$$V_2 \cap V_1 = \left\langle \begin{pmatrix}0\\1\\-1\end{pmatrix} \right\rangle$$

$$e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$f_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad f_2 = \begin{pmatrix}1\\1\\0\end{pmatrix} \quad f_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

identické zobrazení

$$\begin{array}{ccc} v \in \mathbb{R}^3 & = & \mathbb{R}^3 \\ \underline{f} \downarrow & & \downarrow \underline{e} \\ \mathbb{R}^3 & & \mathbb{R}^3 \end{array}$$

$$\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \overset{A\cdot y}{\longmapsto} \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$v = y_1 f_1 + y_2 f_2 + y_3 f_3$$

$$= y_1(e_1) + y_2(e_1+e_2) + y_3(e_1+e_2+e_3)$$

$$= x_1 e_1 + x_2 e_2 + x_3 e_3$$

$$(y_1+y_2+y_3)\, e_1 + (y_2+y_3)\, e_2 + y_3 e_3$$

$$x = \begin{pmatrix} \boxed{\begin{matrix}1\\0\\0\end{matrix}} & \boxed{\begin{matrix}1\\1\\0\end{matrix}} & \boxed{\begin{matrix}1\\1\\1\end{matrix}} \end{pmatrix} \cdot y$$

$$= A \cdot y$$

$$y = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot x$$

$$= A^{-1} \cdot x$$

# Lineární zobrazení v souřadnicích:

$$\begin{tikzcd} v \in V \arrow[r, "L"] \arrow[d, "\underline{e}"'] & W \arrow[d, "\underline{f}"] \\ \mathbb{R}^m \arrow[r, dashed] & \mathbb{R}^n \end{tikzcd} \Rightarrow L(v)$$

$$\underline{e}(v) = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \xmapsto{A \cdot x} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$L(e_1) = a_{11} f_1 + a_{21} f_2 + \dots + a_{n1} f_n$$

$$\vdots$$

$$L(e_m) = a_{1m} f_1 + \dots + a_{nm} f_n$$

$$L(v) = x_1 L(e_1) + \dots + x_m L(e_m)$$

$$A = \begin{pmatrix} \boxed{\begin{matrix} a_{11} \\ \vdots \\ a_{n1} \end{matrix}} & \begin{matrix} \dots \\ \\ \dots \end{matrix} & \begin{matrix} a_{1m} \\ \\ a_{nm} \end{matrix} \end{pmatrix}$$

$\sim L(e_1)$

$$\mathbb{R}^3 = \left\langle e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rangle$$

$$x = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \longmapsto A\cdot x = \begin{pmatrix}1&-1&0\\0&1&1\\2&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1-x_2\\x_2+x_3\\2x_1\end{pmatrix}$$

$$f_1 = \begin{pmatrix}1\\1\\0\end{pmatrix} = e_1 + e_2$$

$$f_2 = \begin{pmatrix}-1\\1\\1\end{pmatrix} = -e_1 + e_2 + e_3$$

$$f_3 = \begin{pmatrix}2\\0\\1\end{pmatrix} = 2e_1 + e_3$$

$$y = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \overset{T}{\longmapsto} \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \overset{A}{\longmapsto} \begin{pmatrix}x_1'\\x_2'\\x_3'\end{pmatrix} \overset{T^{-1}}{\longmapsto} \begin{pmatrix}y_1'\\y_2'\\y_3'\end{pmatrix}$$

$$\tilde{A} = T^{-1}\cdot A\cdot T$$

$$y_1(e_1+e_2) + y_2(-e_1+e_2+e_3) + y_3(2e_1+e_3)$$

$$\Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}y_1-y_2+2y_3\\y_1+y_2\\y_2+y_3\end{pmatrix} \longmapsto \begin{pmatrix}1&-1&0\\0&1&1\\2&0&0\end{pmatrix}\begin{pmatrix}\cdot&&\\&\ddots&\\&&\cdot\end{pmatrix}$$

$$x = T\cdot y \qquad T = \begin{pmatrix}1&-1&2\\1&1&0\\0&1&1\end{pmatrix} \qquad T^{-1} = \begin{pmatrix}1&3&-2\\-1&1&2\\1&-1&2\end{pmatrix}\cdot\frac{1}{4}$$