

Cvičení 3 – opakování

1) Najděte body nespojitosti a určete jejich druh:

a) $f(x) = \frac{x^3 - x^2}{x - 1}$

b) $f(x) = \frac{\ln(x+1)}{|x|}$

c) $f(x) = e^{\frac{1}{x-3}}$

Řešení:

a) v bodě $x = 1$ je odstranitelná nespojitost

b) v bodě $x = 0$ je nespojitost 1. druhu

c) v bodě $x = 3$ je nespojitost 2. druhu

2) Zderivujte funkce:

(a) $f(x) = x^5 - 4x^3 + 2x - 3;$

(b) $f(x) = \frac{\pi}{x} + \ln 2;$

(c) $f(x) = 3x^{\frac{2}{3}} - 2x^{\frac{5}{2}} + x^{-2};$

(d) $f(x) = x^2 \sqrt[3]{x^2};$

(e) $f(x) = \frac{x+1}{x-1};$

(f) $f(x) = \frac{2x+3}{x^2-5x+5};$

(g) $f(z) = \frac{1+\sqrt{z}}{1-\sqrt{z}};$

(h) $f(x) = \frac{a}{\sqrt[3]{x^2}} - \frac{b}{x\sqrt[3]{x}}, \quad a, b \text{ konst.};$

(i) $f(x) = 5 \sin x + 3 \cos x;$

(j) $f(x) = \operatorname{tg} x - \operatorname{cotg} x;$

(k) $f(t) = 2t \sin t - (t^2 - 2) \cos t;$

(l) $f(x) = x \arcsin x;$

(m) $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x};$

(n) $f(x) = \frac{\arccos x}{\arcsin x};$

(o) $f(x) = x^7 \cdot e^x;$

(p) $f(x) = e^{-x} \operatorname{arctg} x;$

(q) $f(x) = x^3 \ln x - \frac{x^3}{3};$

(r) $f(x) = \frac{1}{x} + 2 \ln x - \frac{\ln x}{x};$

(s) $f(x) = \ln x \log x - \ln a \log_a x;$

(t) $f(t) = 5^t \operatorname{tg} t;$

Výsledek:

(a) $5x^4 - 12x^2 + 2;$

(b) $-\frac{\pi}{x^2}, \quad x \neq 0;$

(c) $2x^{-\frac{1}{3}} - 5x^{\frac{3}{2}} - 2x^{-3}, \quad x \neq 0;$

(d) $\frac{8}{3}x^{\frac{5}{3}};$

(e) $-\frac{2}{(x-1)^2}, \quad x \neq 1;$

(f) $\frac{-2x^2-6x+25}{(x^2-5x+5)^2}, \quad x \neq \frac{5 \pm \sqrt{5}}{2};$

(g) $\frac{1}{\sqrt{z}(1-\sqrt{z})^2}, \quad z > 0, z \neq 1;$

(h) $\frac{4b}{3x^2\sqrt[3]{x}} - \frac{2a}{3x\sqrt[3]{x^2}}, \quad x \neq 0;$

(i) $5 \cos x - 3 \sin x;$

(j) $\frac{4}{\sin^2 2x}, \quad x \neq k\frac{\pi}{2};$

(k) $t^2 \sin t;$

(l) $\arcsin x + \frac{x}{\sqrt{1-x^2}},$
 $x \in (-1, 1);$

(m) $\frac{-2}{(\sin x - \cos x)^2}, \quad x \neq \frac{\pi}{4} + k\pi;$

(n) $-\frac{\pi}{2\sqrt{1-x^2}\arcsin^2 x},$
 $x \in (-1, 0) \cup (0, 1);$

(o) $x^6 e^x (x + 7);$

(p) $e^{-x} \left(\frac{1}{1+x^2} - \operatorname{arctg} x \right);$

(q) $3x^2 \ln x, \quad x > 0;$

(r) $\frac{2}{x} + \frac{\ln x}{x^2} - \frac{2}{x^2}, \quad x > 0;$

(s) $\frac{2 \ln x}{x \ln 10} - \frac{1}{x}, \quad x > 0;$

(t) $\frac{5^t (\ln 5 \sin 2t + 2)}{2 \cos^2 t}, \quad t \neq (2k+1)\frac{\pi}{2},$
 $k \in \mathbb{Z};$