(A) **Teorie**: a) NE b) NE c) NE d) NE e) NE f) NE

(1) $$0{,}95 = P(X_n \ge 50) = P\Bigg(\underbrace{\frac{X_n - n\cdot 0{,}8}{\sqrt{n\cdot 0{,}2\cdot 0{,}8}}}_{N(0,1)} \ge \frac{50-n\cdot 0{,}8}{\sqrt{n\cdot 0{,}2\cdot 0{,}8}}\Bigg) \Leftrightarrow$$

$$0{,}05 = \Phi\left(\frac{50-n\cdot 0{,}8}{0{,}4\sqrt{n}}\right) \Leftrightarrow 0{,}95 = \Phi\left(\frac{n\cdot 0{,}8-50}{0{,}4\sqrt{n}}\right) \Leftrightarrow$$

$$\Leftrightarrow 1{,}65 = \frac{n\cdot 0{,}8-50}{0{,}4\sqrt{n}} \Leftrightarrow 0{,}8n - 0{,}66\sqrt{n} - 50 = 0$$

$\Leftrightarrow$ kvadr. rce v $\sqrt{n} = t$, $t \doteq 8{,}32$ $\Rightarrow n \doteq 69{,}4 \Rightarrow n \ge 70$.

Musí vzít aspoň 70 konzerv.

(2) $x^2+ax+b=0$, $|a|\le 1$, $|b|\le 2$

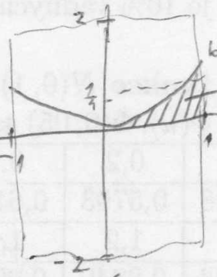
reálné kořeny $\Leftrightarrow D = a^2-4b \ge 0 \Leftrightarrow a^2 \ge 4b$

záporné kořeny: $\left.\begin{array}{l}-a+\sqrt{D}<0\\ -a-\sqrt{D}<0\end{array}\right\} \Leftrightarrow -a+\sqrt{D}<0 \Leftrightarrow \sqrt{D}<a \Rightarrow a>0$

$\sqrt{a^2-4b}<a \quad |^2$

$a^2-4b<a^2 \Leftrightarrow b>0$

Polynom má oba kořeny záporné $\Leftrightarrow a>0, b>0, a^2>4b$



$$\mu(A) = \int_0^1 \frac{a^2}{4}\,da = \left[\frac{a^3}{12}\right]_0^1 = \frac{1}{12}$$

$$P(A) = \frac{\mu(A)}{\mu(\Omega)} = \frac{1/12}{8} \approx 0{,}0104$$

(3) a) $r=(1,2,3)(4,5,6,7)$, $r^2=(1,3,2)(4,6)(5,7)$, $r^3=(4,7,6,5)$, $r^4=(1,2,3)$

$r^5=(1,3,2)(4,5,6,7)$, ..., $r^{11}=(1,3,2)(4,7,6,5)$, $r^{12}=id$

b) $\frac{7!}{12}$

c) není normální, např. $(1,4)\circ(1,2,3)(4,5,6,7)\circ(1,4)^{-1} = (1,5,6,7)(4,2,3) \notin \langle r\rangle$

(4) $h(x) = x^2+x-2 = (x+2)(x-1)$

$f(x) = (x+2)^3(x-1)$

$g(x) = (x-1)(x+2)^3$

$$h(x) = \left(-\frac{2}{27}x - \frac{5}{27}\right)f(x) + \left(\frac{2}{27}x + \frac{7}{27}\right)g(x)$$

(B) Teorie: a) NE b) NE c) NE d) ANO e) NE f) ANO

(1) $0{,}9 = P(X_n \ge 10) \iff 0{,}1 = P(X_n < 10) = P\left(\frac{X_n - n\cdot\frac{1}{26}}{\sqrt{n\cdot\frac{1}{26}\cdot\frac{25}{26}}} < \frac{10 - \frac{n}{26}}{\sqrt{n\cdot\frac{1}{26}\cdot\frac{25}{26}}}\right) =$

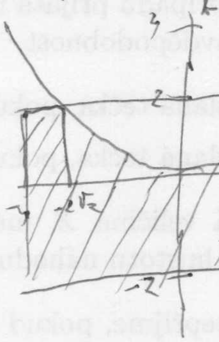
$\doteq \Phi\left(\frac{260-n}{5\sqrt{n}}\right) \iff 0{,}9 = \Phi\left(\frac{n-260}{5\sqrt{n}}\right) \iff 1{,}3 = \frac{n-260}{5\sqrt{n}}$

$\iff n - 260 - 6{,}5\sqrt{n} = 0$ ... kvadr. rovnice pro $k = \sqrt{n}$: $k^2 - 6{,}5k - 260 = 0$

+ kladné řešení $k \doteq 19{,}7$, odkud $n \doteq 388{,}04$.

Věrka si musí koupit aspoň 389 jogurtů.

(2) $x^2 + ax + b = 0$, $|a| \le 4$, $|b| \le 2$.



reálné kořeny $\iff D = a^2 - 4b \ge 0 \iff a^2 \ge 4b$

$\mu(A) = \mu(\Omega) - \int_{-2\sqrt{2}}^{2\sqrt{2}} \left(2 - \frac{a^2}{4}\right) da = 32 - 2\cdot\frac{8}{3}\sqrt{2}$

$P(A) = \frac{\mu(A)}{\mu(\Omega)} = 1 - \frac{\frac{16}{3}\sqrt{2}}{32} = 1 - \frac{\sqrt{2}}{6} \approx 0{,}764$

$\int_0^{2\sqrt{2}} \left(2 - \frac{a^2}{4}\right) da = \left[2a - \frac{a^3}{12}\right]_0^{2\sqrt{2}} = 4\sqrt{2} - \frac{4}{3}\sqrt{2} = \frac{8}{3}\sqrt{2}$

(3) a) $|\mathbb{Z}_{81}^{*}| = \varphi(81) = \varphi(3^4) = 2\cdot 3^3 = 54$

b) $10 \nmid 54 \Rightarrow$ nemá prvek řádu 10, prvek řádu 9 je např. $[2^6]$, protože $[2]$ je generátor (řádu 54).

c) $|H| = 9$, $|G/H| = \frac{54}{9} = 6$

d) G cyklická $\Rightarrow$ G/H cyklická $\Rightarrow$ $G/H \cong (\mathbb{Z}_6, +)$

(4) $x^2 + 2x - 3 = (x+3)(x-1)$

$f(x) = (x+3)^3(x-1)$

$g(x) = (x-1)^3(x+3)$

$h(x) = \left(-\frac{1}{32}x + \frac{3}{32}\right) f(x) + \left(\frac{1}{32}x + \frac{5}{32}\right) g(x)$

(C) Teorie: a) ANO b) NE c) NE d) ANO e) ANO f) ANO

(1) Levostranným intervalem spolehlivosti pro $\mu_1 - \mu_2$ je

$$\left(M_1 - M_2 - \sqrt{\frac{\sigma^2}{m} + \frac{\sigma^2}{n}} \cdot u_{1-\alpha} ; \infty\right)$$

kde $M_1 - M_2 = 0{,}2$, $\sigma^2 = 0{,}25$, $m = 40$, $n = 50$, $u_{0{,}95} = 1{,}65$.

Vyjde $0 \notin (0{,}2 - 0{,}175 ; \infty)$, proto na hladině 0,05 hypotézu $H_0 : \mu_1 = \mu_2$ zamítáme oproti alternativě $\mu_1 > \mu_2$.

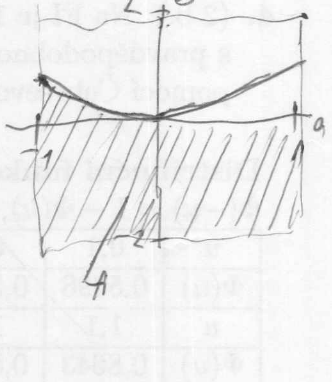
(tedy víme, že přípravek snižuje tvrdost vody)

(2) $x^2 + ax + b = 0$ $|a| < 1$, $|b| < 2$

reálné kořeny $\Leftrightarrow D = a^2 - 4b \geq 0 \Rightarrow a^2 \geq 4b$

kladné $\Leftrightarrow -a + \sqrt{a^2 - 4b} > 0$

$\sqrt{a^2 - 4b} > a \Leftrightarrow a < 0 \vee b < 0$



$\mu(A) = \int_{-1}^{0} \frac{a^2}{4} da + 4 = \left[\frac{a^3}{12}\right]_{-1}^{0} + 4 = 4\frac{1}{12}$

$P(A) = \frac{\mu(A)}{\mu(\Omega)} = \frac{49/12}{8} \doteq 0{,}510$

(3) a) $|\mathbb{Z}_{121}^{\times}| = \varphi(121) = \varphi(11^2) = 110$

b) $9 \nmid 110 \Rightarrow a$ neex. ; např. $b = [-3]$ (řádu 5, -1 řádu 2)

nebo $[2]^{11} = [-9]$ neboť $\langle [2] \rangle = 6$

c) $|H| = |\langle b \rangle| = 10$

$|G/H| = 110/10 = 11$

d) $G/H \cong \mathbb{Z}_{11}$ (jediná grupa řádu 11)

(4) $x^2 - x - 2 = (x-2)(x+1)$

$f(x) = (x-2)^3 (x+1)$

$g(x) = (x-2)(x+1)^3$

$h(x) = \left(\frac{2}{27} x + \frac{5}{27}\right) f(x) + \left(-\frac{2}{27} x + \frac{7}{27}\right) g(x)$

(D) Teorie: a) ANO, b) ANO c) NE d) ANO ~~e) NE~~ f) NE

(1) $H_0: \mu_1 = \mu_2$ oproti $H_1: \mu_1 \neq \mu_2$ při $\alpha = 0{,}1$.

Dvouvýběrový t-test: $m = 16, n = 25$, $S_*^2 = \left((m-1)S_1^2 + (n-1)S_2^2\right)/(m+n-2) =$

$= 1{,}3515 \Rightarrow S_* = 1{,}1626.$

Hodnota statistiky $\dfrac{M_1 - M_2}{S_* \sqrt{\frac{1}{m} + \frac{1}{n}}} \doteq 1{,}8807 > t_{0{,}95}(39) = 1{,}6839$ (aprox.)

Proto H_0 zamítáme – Součástky nepocházejí se stejnou střední hodnotou.

(2) $x^2 + ax + b = 0$ $\quad |a| \leq 4, \ |b| \leq 2$

reálné kořeny $\Leftrightarrow D = a^2 - 4b \geq 0 \Leftrightarrow a^2 \geq 4b$

$\mu(\Omega) = 32$

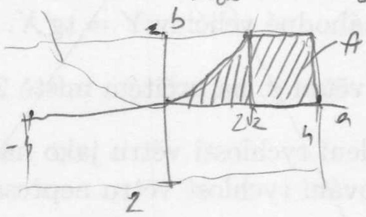
záporné $\Leftrightarrow \begin{cases} -a + \sqrt{D} < 0 \\ -a - \sqrt{D} < 0 \end{cases} \Leftrightarrow -a + \sqrt{D} < 0 \Leftrightarrow \sqrt{D} < a$

$\sqrt{a^2 - 4b} < a$

$a > 0 \wedge a^2 - 4b < a^2$

$b > 0$

$\mu(A) = \int_0^{2\sqrt{2}} \frac{a^2}{4}\, da + (4 - 2\sqrt{2}) \cdot 2 =$



$= \left[\frac{a^3}{12}\right]_0^{2\sqrt{2}} + (4 - 2\sqrt{2}) \cdot 2 =$

$= \frac{16}{12} \cdot \sqrt{2} + 8 - 4\sqrt{2} = 8 - \frac{8}{3}\sqrt{2}$

$P(A) = \frac{\mu(A)}{\mu(\Omega)} = \frac{8 - \frac{8}{3}\sqrt{2}}{32} \doteq 0{,}132$

(3) a) $s = (1,2)(3,4,5)(7,8)$, $s^2 = (3,5,4)$, $s^6 = id$

b) $8!/6$

c) $\langle s \rangle \not\triangleleft S_8$ neboť např. $(1,3) \circ s \circ (1,3)^{-1} = (1,4,5)(2,3)(7,8) \notin \langle s \rangle$

(4) $h(x) = x^2 - 2x - 3 = (x-3)(x+1)$

$f(x) = (x-3)^3 (x+1)$

$g(x) = (x-3)(x+1)^3$

$h(x) = \left(\frac{1}{32} x + \frac{3}{32}\right) \cdot f(x) + \left(-\frac{1}{32} + \frac{5}{32}\right) g(x)$