Cauchyova nerovnost

$$|(x,y)|^2 \leq \|x\|^2 \cdot \|y\|^2$$

odchylka $\cos\sphericalangle(x,y) = \dfrac{(x,y)}{\|x\| \cdot \|y\|}$

$$(x_1y_1 + x_2y_2 + \dots + x_ny_n)^2 \leq (x_1^2 + x_2^2 + \dots + x_n^2)(y_1^2 + \dots + y_n^2)$$

$Y = a + bX$

$$M_Y(t) = Ee^{t\cdot Y} = E\cdot e^{t(a+b\cdot X)} =$$

$$= Ee^{t\cdot a}\cdot e^{t\cdot b\cdot X} = e^{t\cdot a}\cdot \underline{Ee^{t\cdot b\cdot X}} =$$

$$= e^{t\cdot a} M_X(t\cdot b)$$

$\Rightarrow Z \sim N(0,1)$

$Y \sim \mu + \sigma Z$

$$M_Z(t) = e^{\frac{t^2}{2}}$$

$$M_Y(t) = e^{\mu t}\cdot e^{\frac{\sigma^2 t^2}{2}}$$

$$Z \sim N(0,1)$$

$$f_Z(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{t^2}{2}} \qquad -\frac{(t-\mu)^2}{2\sigma^2}$$

$$M_Z(t) = E\, e^{t \cdot Z} = \int_{-\infty}^{\infty} e^{t \cdot z} \cdot \varphi_Z(t)\, dz$$

$$= \int_{-\infty}^{\infty} e^{t \cdot z} \cdot \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}\, dz$$

$$\Longrightarrow M_Z(t) = e^{\frac{t^2}{2}}$$

$EZ = \mu'_1$ … koef. u $\frac{t^1}{1!}$ v rozvoji $e^{\frac{t^2}{2}}$ do exp. řady

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\Rightarrow e^{\frac{t^2}{2}} = 1 + \frac{t^2}{2\cdot 1!} + \frac{t^4}{4\cdot 2!} + \dots + \frac{t^{2n}}{2^n \cdot n!}$$

$$\Rightarrow \mu'_1 = 0 \quad \left(\text{koef. u } \frac{t^1}{1!}\right) \Rightarrow EZ = 0$$

$$\mu'_2 = 1 \quad \left(\text{koef. u } \frac{t^2}{2!}\right) \Rightarrow DZ = E(Z^2) - E(Z)^2 = \mu'_2 - (\mu'_1)^2 = \underline{\underline{1}}$$

$$\Gamma\left(\frac{1}{2},\frac{1}{2}\right) = \sqrt{\frac{1}{2}} \cdot \frac{1}{\sqrt{\pi}} \cdot x^{-\frac{1}{2}} e^{-\frac{1}{2}x} =$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{x}} e^{-\frac{x}{2}}$$

$\varepsilon := k \cdot \sqrt{DX}$ $\qquad EX =: \mu$

σ ... sm. odchylka

$$P(|X-\mu| \geq k \cdot \sigma) \leq \frac{\sigma^2}{k^2 \cdot \sigma^2} = \frac{1}{k^2}$$

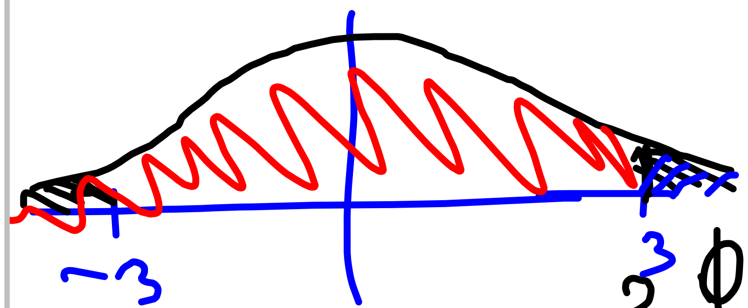
$\underline{k=3}$: $P(|X-\mu| \geq 3\sigma) \leq \frac{1}{9}$

$X \sim N(0,1)$: $P(|X-\mu| \geq 3\sigma) =$

$P(|X| \geq 3) = 2\Phi(-3) = 2(1-\Phi(3)) =$

$= 2(0{,}00135) =$

$= \underline{0{,}0027}$



$2\phi(-3) = 2(1-\Phi(3))$

Věta (Bernoulliova)

Pro náhodnou veličinu s binomickým rozdělením $Y_n \sim \mathrm{Bi}(n,p)$ *a pro libovolné* $\epsilon > 0$ *platí*

$$P\left(\left|\frac{Y_n}{n} - p\right| > \epsilon\right) \leq \frac{p(1-p)}{n\epsilon^2}.$$

plyne z Čeb. nerovnosti

$$\lim_{n\to\infty} P\left(\left|\frac{Y_n}{n} - p\right| > \varepsilon\right) \leq \lim_{n\to\infty} \frac{p(1-p)}{n\varepsilon^2} = 0$$

$$\Downarrow$$

$$\lim_{n\to\infty} P\left(\left|\frac{Y_n}{n} - p\right| \leq \varepsilon\right) = 1$$

$$Y_n = \sum_{k=1}^{n} A \quad , \text{kde } A \sim Bi(1, p) = A(p)$$

$$EA = p$$
$$DA = p(1-p)$$

$$\lim_{n\to\infty} P\left(\left|\frac{1}{n}\cdot\sum_{k=1}^{n} A - p\right| < \varepsilon\right) = 1$$

$$\parallel$$

$$\lim_{n\to\infty} P\left(\left|\frac{\overset{Y_n}{\sum_{k=1}^{n} A} - np}{n}\right| < \varepsilon\right) = 1$$

$$\parallel$$

$$P\left(\left|\frac{Y_n}{n} - p\right| < \varepsilon\right) \to 1$$

Bernoulliova

Př.: X_n ... # studentů, neznajících malou násobilku

$X_n \sim Bi(n, p)$ $\qquad$ p ... neznámé $\qquad n = 600$

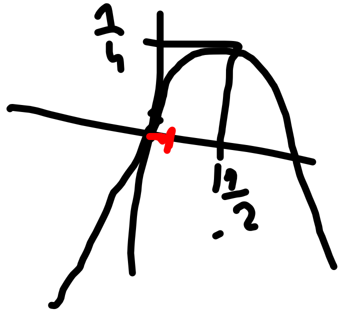
$p \leq 0{,}02 =: p_0$ $\qquad \varepsilon = 0{,}01 = \frac{1}{100}$

$\Rightarrow p(1-p) \leq p_0(1-p_0) = 0{,}02 \cdot 0{,}98$

Bernoulli

$$\boxed{P\left(\left|\frac{X_n}{n} - p\right| > \varepsilon\right) \leq \frac{p(1-p)}{n \cdot \varepsilon^2}}$$

$$P\left(\left|\frac{X_n}{n} - p\right| > \frac{1}{100}\right) \leq \frac{p(1-p)}{n \cdot 10^{-4}} \leq$$



$f(x) = x(1-x)$

roste pro $x \in (0, \frac{1}{2})$

$$\leq \frac{2 \cdot 10^{-2} \cdot 98 \cdot 10^{-2}}{600 \cdot 10^{-4}} =$$

$$= \frac{2 \cdot 10^{-2}}{6 \cdot 10^{-2}} = \frac{1}{3}$$

$$P\left(\left|p - \frac{5}{600}\right| > \frac{1}{100}\right) \leq \frac{1}{3}$$

$Y_1, Y_2, \dots, Y_n, \dots$

$X_1 = Y_1,\ X_2 = Y_1 + Y_2,\ \dots,\ X_n = \sum_{i=1}^{n} Y_i$

$EX_n = n \cdot \mu$

$DX_n = n \cdot \sigma^2$

$D(X+Y) = DX + DY$

normování:

$$\frac{X_n - n\mu}{\sqrt{n \cdot \sigma^2}} = \frac{1}{\sqrt{n}} \cdot \frac{X_n - n\mu}{\sigma} =$$

$$= \frac{1}{\sqrt{n}} \cdot \frac{\sum_{i=1}^{n} Y_i - n\mu}{\sigma} = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{Y_i - \mu}{\sigma}$$

Centrální limitní $\overset{?}{\sim}$ $N(0,1)$