$[0,1)$ je nespočetná

Sporem: $a: \mathbb{N} \to [0,1)$ bijekce

$a_0, a_1, a_2, \ldots$

$0{,}\underline{1}230\,1;\ 0{,}3\underline{7}5\ldots$

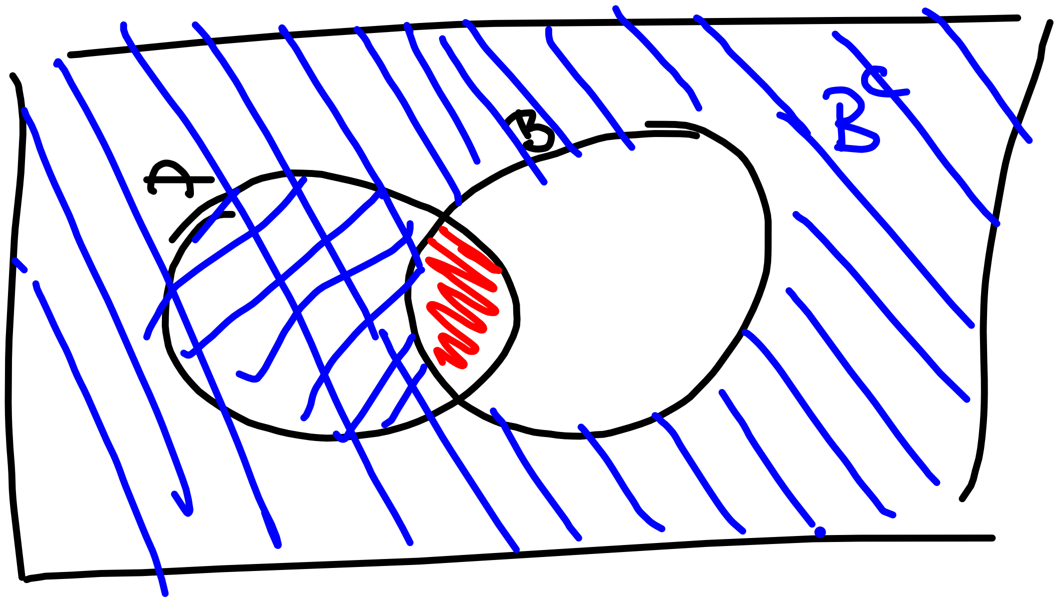
$\phantom{0{,}}1.\qquad\qquad\qquad 2.$

$$b_x := \begin{cases} 1 & \text{pokud } a_{xx} \neq 1 \\ 2 & \text{pokud } a_{xx} = 1 \end{cases}$$

Pak $b = 0{,}b_1 b_2 b_3 \ldots$

se liší od $\forall a_z$: $b_z \neq a_{zz}$

$A \cap B = A \setminus (\Omega \setminus B)$



$P(\emptyset)=0$ | $0 \leq P(A) \leq 1$

$\Omega, \emptyset$ neslučitelné $\Rightarrow P(\Omega \cup \emptyset) = P(\Omega) + P(\emptyset)$

$\underset{1 = P(\Omega)!}{\shortparallel}$ $\qquad \underset{1}{\shortparallel}$

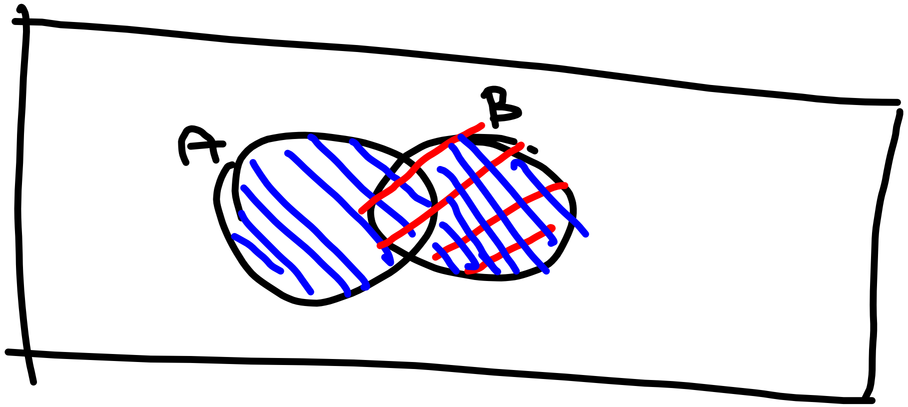
$\Rightarrow P(\emptyset) = 0$

$\Omega = A \cup (\Omega \setminus A)$, kde $A, \Omega \setminus A$ jsou neslučitelné!

$1 = P(\Omega) = P(A) + \underbrace{P(\Omega \setminus A)}_{\geq 0} \Rightarrow P(A) \leq 1$

$\Rightarrow P(A^c) = 1 - P(A)$

$A \subseteq B \Rightarrow B = A \underset{\text{nesluč.}}{\cup} (B \setminus A) \Rightarrow P(B) = P(A) + P(B \setminus A)$



$A \cup B = A \cup (B \setminus A)$

nesluč.

$B \setminus A = B \setminus \underbrace{(A \cap B)}_{\subseteq B}$

$P(A \cup B) = P(A) + \underline{P(B \setminus A)} =$

$= \underline{P(A) + P(B) - P(A \cap B)}$

• $A_1 \subseteq A_2 \subseteq A_3 \subseteq \dots$ $\quad P\left(\bigcup_{i=1}^{\infty} A_i\right) = ?$

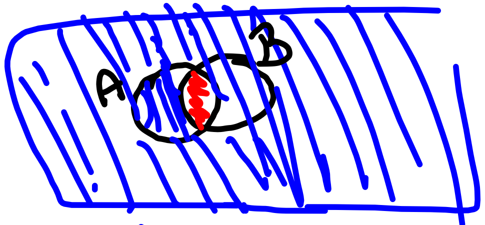
$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup (A_2 \setminus A_1) \cup (A_3 \setminus A_2) \cup \dots$$

po 2 nesl.

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A_1) + \underline{P(A_2 \setminus A_1)} + P(A_3 \setminus A_2) + \dots$$

$$= P(A_1) + P(A_2) - P(A_1) + P(A_3) - P(A_2) + \dots$$

$$= \lim_{n\to\infty} P(A_n)$$



• $\bigcap_{i=1}^{\infty} A_i = \left(\bigcup_{i=1}^{\infty} A_i^c\right)^c$

de Morganova pravidla

$P(A_2 \setminus A_1) \le P(A_2)$

• $P\left(\bigcup_{i=1}^{\infty} A_i\right) \le P(A_1) + P(A_2 \setminus A_1) + P(A_3 \setminus (A_1 \cup A_2)) + \dots$

$\le P(A_1) + P(A_2) + P(A_3) + \dots$

• $P\left(\bigcap_{i=1}^{\infty} A_i\right) = 1 - \underline{P\left(\bigcup_{i=1}^{\infty} A_i^c\right)} \ge 1 - \sum_{i=1}^{\infty} P(A_i^c) = 1 - \sum_{i=1}^{\infty} (1 - P(A_i))$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A\cap B) = P(B)\cdot \underbrace{P(A|B)}_{A,B \text{ nez.}} = P(B)\cdot P(A)$$

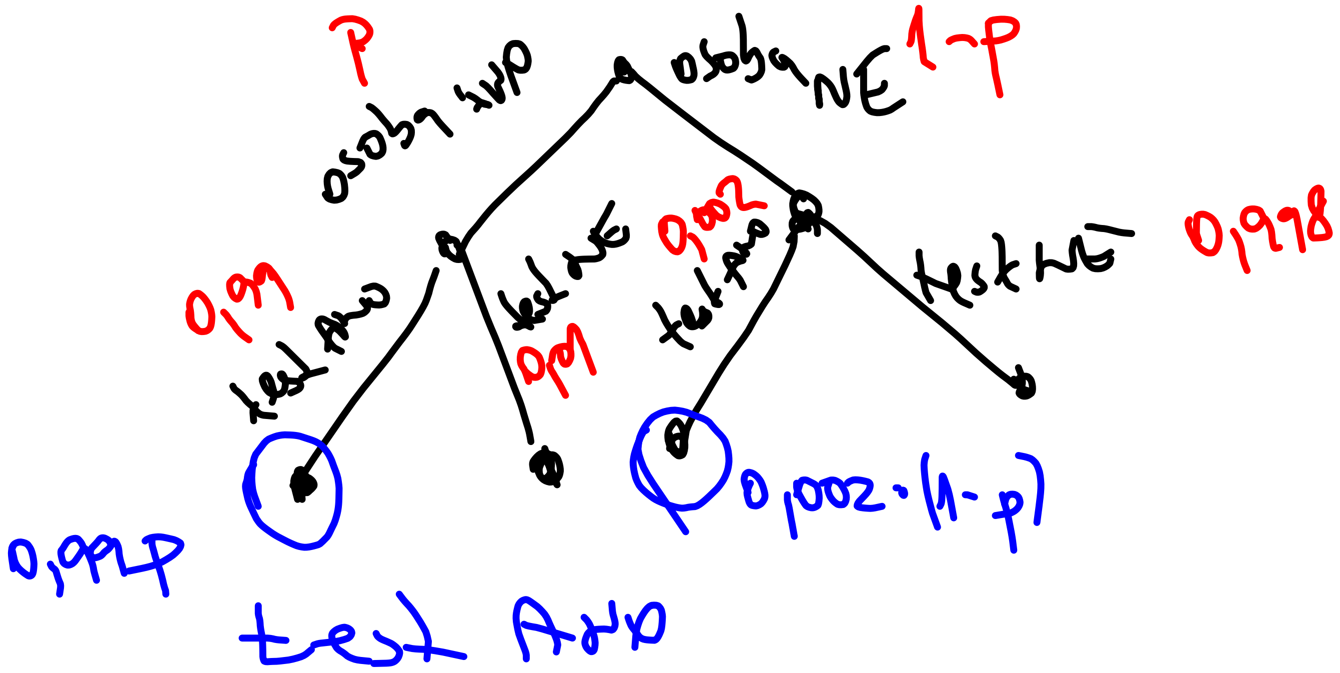
$$\parallel$$

$$P(B\cap A) = P(A)\cdot \underbrace{P(B|A)}_{B \text{ nez. na } A} = P(A)\cdot P(B)$$

$$\underbrace{P(A)\cdot P(B|A)}_{P(A\cap B)} + \underbrace{P(A^c)\cdot P(B|A^c)}_{P(A^c\cap B)}$$

nesluč.

$$= P\big(\underbrace{(A\cap B) \cup (A^c\cap B)}_{B}\big) = P(B)$$


p
osoba ano
osoba NE
1-p
0,99
test ANO
test NE
0,01
0,002
test ANO
test NE
0,998
0,99p
0,002·(1-p)
test ANO