Timetabling at Purdue University

March 31, 2010

Part II: Iterative Forward Search

Iterative Forward Search

$\mathrm{IFS}(P, F_{\mathrm{wcsp}}, <_{\mathrm{wcsp}}, \emptyset) \qquad P = (V, \mathcal{D}, C, w_c, w_\theta)$

function $\mathrm{IFS}(P, F, <, \delta)$
    $i = 0$
    $\omega = \emptyset$
    $\sigma = \emptyset$
    while $\mathrm{canContinue}(\omega,\ i)$ **do**
        $i = i + 1$
        $v = \mathrm{selectVariable}(P, \omega)$
        $d = \mathrm{selectValue}(P, \omega, \delta, F, <, v)$
        $\gamma = \mathrm{hardConflicts}(P, \omega, v/d)$
        $\omega = \omega \backslash \gamma \cup \{v/d\}$
        if $F(\omega, \delta) < F(\sigma, \delta)$ **then** $\sigma = \omega$
    end while
    return σ
end function

Function for computing conflicting variables

function hardConflicts($P, \omega, v/d$)
    if $\exists d_v : v/d_v \in \omega$ **then** $\gamma = \{v/d_v\}$
    else $\gamma = \emptyset$
    for $c \in C_h \wedge v \in scope(c)$ **do**
        $\beta = \omega \backslash \gamma \cup \{v/d\}$
        if $\beta \models \neg c$ **then**
            find $\alpha \subseteq \omega \backslash \gamma$ **such that** $\beta \backslash \alpha \models c$
            $\gamma = \gamma \cup \alpha$
        end if
    end for
    return γ
end function

Conflict-based statistics for class CS 101 Lab 2

Current Assignment of C S 101 Lab 2

Not assigned.

Room Locations: 1 (EDUC 108)

Time Locations: 3 (M 9:30a, M 11:30a, M 1:30p)

Conflict-based Statistics

- 2123× Room EDUC 108
  - 718× M 11:30a - 1:20p Full Term EDUC 108
    - 260× C S 101 Lab 3 ← M 11:30a - 1:20p Full Term EDUC 108
    - 235× C S 101 Lab 1 ← M 11:30a - 1:20p Full Term EDUC 108
    - 222× C S 101 Lab 4 ← M 11:30a - 1:20p Full Term EDUC 108
    - 1× ENGR 101 Lab 2 ← M 11:30a - 1:20p Full Term EDUC 108
  - 718× M 1:30p - 3:20p Full Term EDUC 108
    - 256× C S 101 Lab 1 ← M 1:30p - 3:20p Full Term EDUC 108
    - 235× C S 101 Lab 4 ← M 1:30p - 3:20p Full Term EDUC 108
    - 226× C S 101 Lab 3 ← M 1:30p - 3:20p Full Term EDUC 108
    - 1× ENGR 101 Lab 2 ← M 1:30p - 3:20p Full Term EDUC 108
  - 687× M 9:30a - 11:20a Full Term EDUC 108
    - 252× C S 101 Lab 1 ← M 9:30a - 11:20a Full Term EDUC 108
    - 240× C S 101 Lab 4 ← M 9:30a - 11:20a Full Term EDUC 108
    - 192× C S 101 Lab 3 ← M 9:30a - 11:20a Full Term EDUC 108

Conflict-based statistics

- Example

$$x = 1 \;\Rightarrow\; 3 \times \neg\, y = 2, 4 \times \neg\, y = 3, 2 \times \neg\, z = 1, 120 \times \neg\, v = 1$$

- $CBS[x = d_x \;\rightarrow\; \neg\, y = d_y] = c_{xy}$: the assignment $x = d_x$ caused a hard conflict with the assignment $y = d_y$ c_{xy} times in the past.

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- If a value d is selected for a variable v within the IFS, then hardConflicts$(P, \omega, v/d)$ chooses previous assignments $\gamma = \{v_1/d_1, v_2/d_2, \ldots v_n/d_n\}$ to be unassigned in order to enforce consistency of the new partial assignment.
  As a consequence, the counters are incremented

$$CBS[v = d \;\rightarrow\; \neg\, v_1 = d_1], CBS[v = d \;\rightarrow\; \neg\, v_2 = d_2], ...,$$
$$CBS[v = d \;\rightarrow\; \neg\, v_n = d_n]\;.$$

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$$CBS[v = d \;\rightarrow\; \neg\, v_1 = d_1], CBS[v = d \;\rightarrow\; \neg\, v_2 = d_2], \ldots, \\ CBS[v = d \;\rightarrow\; \neg\, v_n = d_n]\ .$$

- Conflict-based statistics are used as part of the value selection criterion. Evaluation for value d of v

$$\sum_{v_i/d_i \in \omega \,\wedge\, v_i/d_i \in \mathrm{hardConflicts}(P,\omega,v/d)} CBS[v = d \rightarrow \neg v_i = d_i]$$

Particular runs with results for student enrollments (S), time preferences (T), room preferences (R), distribution preferences (D)

	Final				Run separately				Run combined				Bound		
Problem	S	T	R	D	S	T	R	D	S	T	R	D	S	T	R
pu-spr07-llr	98.63	89.71	92.86	66.67	98.86	93.20	86.90	72.22	97.67	90.11	84.86	63.47	99.42	98.82	99.4
pu-fal07-llr	98.76	81.77	91.69	96.55	99.00	89.49	78.05	93.56	98.25	88.18	74.98	63.69	99.54	96.29	98.9
pu-spr07-ms	99.37	68.34	76.22	57.14	99.62	75.45	75.96	48.39	98.28	77.42	73.98	55.98	99.87	86.11	94.5
pu-fal07-ms	97.80	71.48	80.00	72.73	99.62	71.88	86.33	63.68	98.99	70.60	85.42	52.86	99.71	84.39	97.5
pu-spr07-cs	94.85	83.86	100.00	33.33	97.15	73.91	100.00	33.33	93.56	70.92	100.00	44.17	98.55	86.27	100.0
pu-fal07-cs	94.14	83.20	90.48	100.00	98.53	76.40	83.49	89.67	97.15	76.09	83.17	70.19	99.28	96.91	100.0
pu-spr07-cfs	93.72	87.32	94.44	100.00	97.66	89.74	82.22	82.50	93.70	93.32	79.63	64.17	98.42	99.12	100.0
pu-fal07-cfs	94.09	96.30	50.00	86.67	98.12	97.14	85.00	92.67	94.70	94.97	66.67	84.30	98.52	100.00	100.0
pu-spr07-vpa	92.70	2.44	40.00	100.00	97.08	88.62	76.67	100.00	95.41	81.30	76.00	100.00	97.69	90.24	100.0
pu-fal07-vpa	93.19	0.00	100.00	100.00	96.79	0.00	100.00	100.00	95.06	0.00	100.00	100.00	96.79	0.00	100.0
pu-spr07-lab	97.45	87.60	75.76	68.02	99.39	94.08	69.19	50.30	97.71	94.58	68.15	57.00	99.82	97.67	83.3
pu-fal07-lab	85.42	89.74	71.46	77.03	97.71	84.73	44.25	38.15	97.29	85.95	39.32	22.00	98.12	93.69	87.6
pu-spr07-c8	97.99	84.87	82.81	61.39	98.69	90.16	77.37	50.70	98.16	89.91	75.79	56.58	98.95	97.55	91.9
pu-fal07-c8	98.35	83.01	87.55	78.00	98.63	86.70	73.49	61.04	98.55	86.62	70.43	54.18	99.35	95.76	96.3

IFS as a Generic Framework

- IFS for classical initial problem:

$$\mathsf{IFS}(P, F_{\mathrm{wcsp}}, <_{\mathrm{wcsp}}, \emptyset)$$

- IFS for feasibility problem:

$$\mathsf{IFS}(P, F_{\mathrm{csp}}, <_{\mathrm{csp}}, \emptyset)$$

- IFS for minimal perturbation problem:

$$\mathsf{IFS}(P, F_{\mathrm{mpp}}, <_{\mathrm{mpp}}, \delta)$$

Feasibility Problem

- Feasibility problem allows the detection of possible inconsistencies in hard constraints
- Any inconsistencies must be removed from the problem by the human schedule manager

- Cost of feasibility problem

$$F_{\mathrm{csp}}\omega = \|\omega\| \ .$$

- The ordering $\leq_{\mathrm{csp}}$ between costs of feasibility problem of two consistent assignments:

$$F_{\mathrm{csp}}\omega \leq_{\mathrm{csp}} F_{\mathrm{csp}}\eta \equiv (\|\omega\| \geq \|\eta\|) \ .$$

- Any solution σ has the best possible value $F_{\mathrm{csp}}\sigma$ corresponding to the number of the variables in the problem.

Minimal Perturbation Problem (MPP)

Existing timetable + requests for changes

MPP: minimization of changes to the original solution (perturbations)

Definition of MPP:

- constraint satisfaction problem $(V, \mathcal{D}, C)$ to be solved
- initial assignment δ: consistent assignment of the original problem
- distance function Φ: evaluates the number of changes between two assignments

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Distance functions:

- $\Phi_{\mathrm{var}}(\omega, \delta) = \|\{x/d_1 | x/d_1 \in \omega \wedge x/d_2 \in \delta \wedge d_1 \neq d_2\}\|$
- $\Phi_{\mathrm{time}}(\omega, \delta) = \|\{x/d_1 | x/d_1 \in \omega \wedge x/d_2 \in \delta \wedge \mathrm{time}(d_1) \neq \mathrm{time}(d_2)\}\|$

Optimal solution of MPP:

- solution σ of $(V, \mathcal{D}, C)$ having minimal distance $\Phi(\sigma, \delta)$

MPP in IFS

Weighted constraint satisfaction problem $P = (V, \mathcal{D}, C, w_c, w_\theta)$

Cost of MPP

$$F_{\mathrm{mpp}}(\omega, \delta) = (\|\omega\|, F_s\omega + w_{\mathrm{mpp}}\Phi(\omega, \delta)) \ .$$

The ordering between costs of MPP for two consistent assignments ω and η wrt. the same initial assignment δ:

$$F_{\mathrm{mpp}}(\omega, \delta) \leq_{\mathrm{mpp}} F_{\mathrm{mpp}}(\eta, \delta) \equiv$$
$$((\|\omega\| > \|\eta\|) \vee ((\|\omega\| = \|\eta\|) \wedge (F_s\omega + w_{\mathrm{mpp}}\Phi(\omega, \delta) \leq F_s\eta + w_{\mathrm{mpp}}\Phi(\eta, \delta))$$

MPP and Value Ordering

Consistent assignment ω and an initial assignment δ with
$F_{\mathrm{mpp}}\omega = (\|\omega\|, F_s\omega + w_{\mathrm{mpp}}\Phi(\omega, \delta))$

New assignment v/d, v not in ω

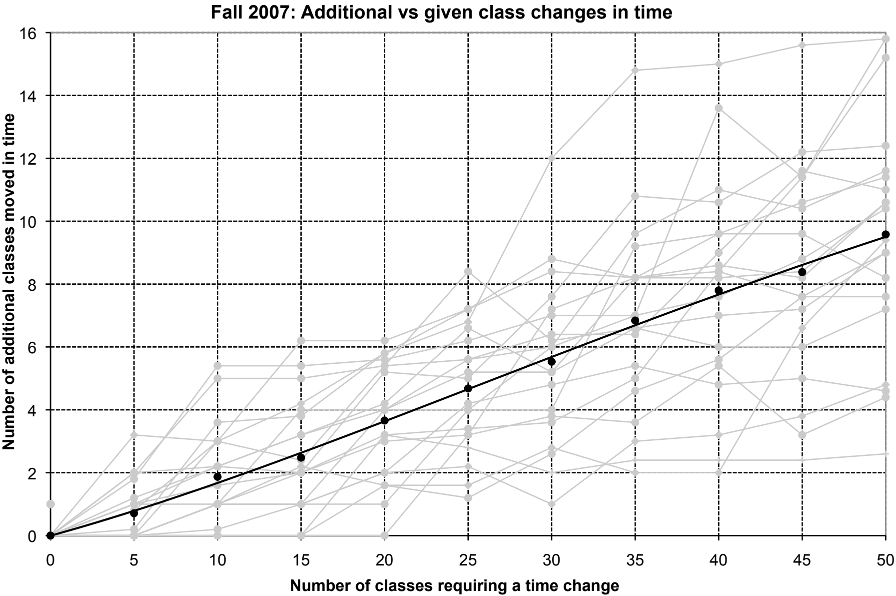
The possible contribution to $F_s(\omega)$ is $\Delta F_s(\omega, v/d)$:
same as for the initial problem

The contribution to the distance function Φ_{var}:

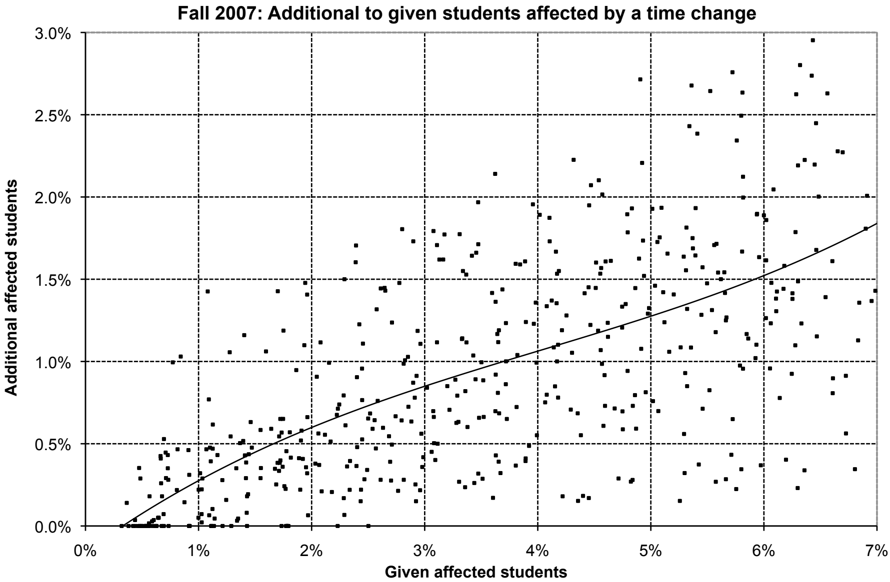
$$\Delta\Phi_{\mathrm{var}}(\delta, v/d) = \begin{cases} w_{mpp} & \exists v/d_i \in \delta \wedge d_i \neq d \\ 0 & \text{otherwise} \end{cases} .$$

The total contribution: $\Delta F_s(\omega, v/d) + \Delta\Phi(\delta, v/d)$
used as a value ordering heuristic

Evaluation of MPP: Additional Classes Moved in Time



Evaluation of MPP: Additional Affected Students



Evaluation of MPP: Changes in Criteria