Gaussian quantum marginal problem

Jan Vlach

June 7, 2011

Quantum marginal problem

- quantum ellectromagnetic oscillator with N modes
- ith mode is characterised by $\hat{x}_i$ (position) and $\hat{p}_i$ (momentum)
- reduction
  - we measure only subset of all modes
  - 2 reduced systems S_1 and S_2
- **solution:** Characterize possible states of reduced systems if global state is given.

Quantum marginal problem

- **Simplifications**
  - Gaussian states
  - characterise possible states of only one of the reduced subsystems

Mathematical formulation

- vector space V of dimension $2n$
- $V = span(\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2, \ldots, \hat{x}_n, \hat{p}_n)$
- symplectic form $\omega \ldots$ matrix Ω
- symmetric positive definite form $\gamma \ldots$ covariance matrix K
- $F = K^{-1}\Omega \ldots$ operator on V
- eigenvalues of $\mathrm{i}F \ldots$ symplectic eigenvalues
- symplectic eigenvalues determine state of the system

State of the art

- Eisert, J., Tyc, T., Rudolph, T., Sanders, B.C.: Gaussian Quantum Marginal Problem. Commun. Math. Phys. 280, 263-280 (2008)
- global system reduced to N 1-mode systems
- $c_j \ldots$ symplectic eigenvalues of the ith mode
- $d_k \ldots$ kth global symplectic eigenvalue

State of the art

- necessary and sufficient condition $\ldots n+1$ inequalities

$$\sum_{j=1}^{k} c_j \geq \sum_{j=1}^{k} d_j, k = 1, \ldots, N$$

and

$$c_n - \sum_{j=1}^{N-1} c_j \leq d_n - \sum_{j=1}^{N-1} d_j$$

Williamson theorem

- there exists a basis such that

$$K = diag(\frac{1}{\mu_1}, \frac{1}{\mu_2}, \ldots, \frac{1}{\mu_n}, \frac{1}{\mu_1}, \frac{1}{\mu_2}, \ldots \frac{1}{\mu_n})$$

and

$$\Omega = \bigoplus_{j=1}^{N} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

where $\mu_1, \mu_2, \ldots, \mu_n$ are symplectic eigenvalues

Restrictions on subsystems of $M = N - 1$ modes

- $M = N - 1$:
  - $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_N \ldots$ global symplectic eigenvalues
  - $\nu_1 \geq \nu_2 \geq \cdots \geq \nu_{N-1} \ldots$ local symplectic eigenvalues
  - solution:

$$\mu_1 \geq \nu_1 \geq \mu_2 \geq \cdots \geq \nu_{N-1} \geq \mu_N$$

Restrictions on subsystems of $M < N$ modes

- $M < N$:
  - $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_N \ldots$ global symplectic eigenvalues
  - $\nu_1 \geq \nu_2 \geq \cdots \geq \nu_M \ldots$ local symplectic eigenvalues
  - solution:

$$\mu_j \geq \nu_j \geq \mu_{N-M+j}, j < M + 1$$

Future work

- find restrictions for reduced and complementary subsystem together
- consider arbitrary number of subsystems of arbitrary size
- consider non-Gaussian states