CONNECTIVITY AND MINORS

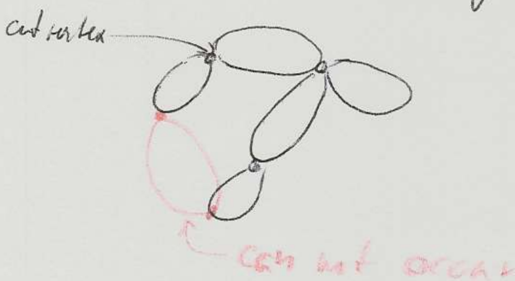
k-connected (vertex)

2-connected graph - properties:

- every two edges/vertices are on same cycle
- have an ear decomposition

- every connected graph have block-cut-tree

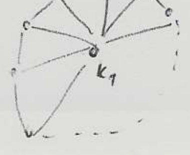
block $\cong$ 2-conn subgraphs



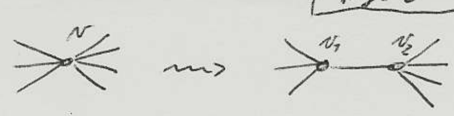
3-connected graphs - structure:

- Tutte's wheel theorem: every 3-connected graph is obtained from K_4 by ~~generalized~~ loose splitting of vertices

Def: <u>wheel</u> $= C_n \oplus K_1 = W_n$



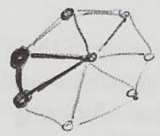
Def: splitting a vertex v



= make new v_1, v_2 and edge edge$\{v_1, v_2\}$ each neighbour of v adj to precisly one of v_1, v_2 and $d(v_1), d(v_2) \geq 3$

loose splitting = each neighbour ~~of~~ of v is adj to at least one of v_1, v_2

(used for wheels



Def: contraction of an edge $e = uv$

= replace u, v by new vertex v_e, remove e, and make all other edges of u, v incident to v_e. (creates multigraph)

loose contraction of $e = uv$

= make ~~x~~ v_e adj to each neighbour of u or v.

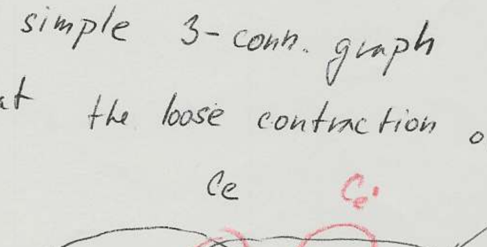
Fact: loose contractions makes simple graph an is "inverse" ~~x~~ of loose split.

notation G/e (loose?) or $G/\!/e$

– THM: In a simple 3-conn. graph there exists edge e such that the loose contraction of e (G/e) is 3-connected graph

Ce C_e'

as small as possible



- ALT. Tutte – every 3-connected graph is obtained from a wheel W_k by (strict) splitting.

- <u>Tutt connectivity</u> (Remark): instead of cut study separations: partition of edges $E(G) = F_1 \dot\cup F_2$

$$|F_1|, |F_2| \geq k$$

F_1

F_2

↑
size $k=3$

~~Ab~~ ~~14~~ is preserved undr duality.